Rate of Change and Slope (for Holt Algebra 1, Lesson 5-3)

- A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

The table shows the cost of mailing a 1-ounce letter in different years.

Year	1985	1988	1990	1991
Cost (cents)	22	25	25	29

The independent variable is the year.
The dependent variable is the cost.

The rates of change for each interval are found as follows.

1985 to 1988: $\frac{\text{change in cost}}{\text{change in years}} = \frac{25-22}{1988-1985} = \frac{3}{3} = 1 \Rightarrow \frac{1\text{ cent}}{\text{year}}$

1988 to 1990: $\frac{\text{change in cost}}{\text{change in years}} = \frac{25-25}{1990-1988} = \frac{0}{2} = 0 \Rightarrow \frac{0\text{ cents}}{\text{year}}$

1990 to 1991: $\frac{\text{change in cost}}{\text{change in years}} = \frac{29-25}{1991-1990} = \frac{4}{1} = 4 \Rightarrow \frac{4\text{ cents}}{\text{year}}$

When graphed, each line segment would have a different steepness, corresponding to the rate of change for that interval.

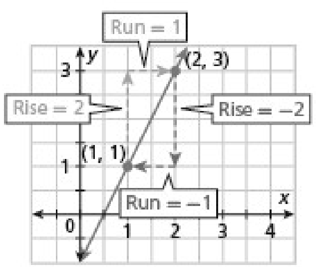
- If all the segments of a graph have the same steepness, they form a straight line. This constant rate of change is called the **slope** of a line.
- The **rise** is the difference in the *y*-values of two points on a line. The **run** is the difference in the *x*-values of two points on a line.
- The slope of a line is the ratio of rise to run for any two points on the line. $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

Examples

FINDING SLOPE

Find the slope of each line.

1.

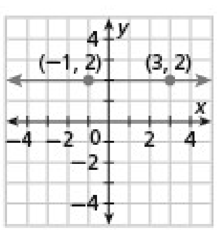


Begin at one point and count vertically to find the rise.
Then count horizontally to the second point to find the run.
It does not matter which point you start with. The slope is the same.

$\text{slope} = \frac{2}{1} = 2$

$\text{slope} = \frac{-2}{-1} = 2$

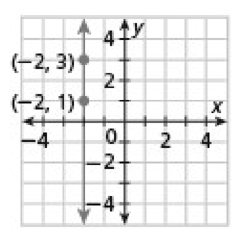
2.



$\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0$

The slope of all horizontal lines is 0.

3.



$\frac{\text{rise}}{\text{run}} = \frac{2}{0}$ *You cannot divide by 0.*

The slope of all vertical lines is undefined.

The Slope Formula (for Holt Algebra 1, Lesson 5-4)

- The slope m of a line can be found using a formula. The formula can be used whether you are given a graph, a table, or an equation. All you need is the coordinates of two different points on the line.

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in y-values to the difference in x-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If (6, 2) and (8, 4) are two points on a line, the slope of the line is $m = \frac{4-2}{8-6} = \frac{2}{2} = 1$.

Example

FINDING SLOPE BY USING THE SLOPE FORMULA

1. Find the slope of the line that contains (4, –2) and (–1, 2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$m = \frac{2-(-2)}{-1-4}$ *Substitute (4, –2) for (x_1, y_1) and (–1, 2) for (x_2, y_2).*

$m = \frac{4}{-5}$ *Simplify.*

The slope of the line is $-\frac{4}{5}$.

- If you are given a table or graph that describes a line, you can find the slope by substituting any two different points into the slope formula.
- If you are given an equation that describes a line, you can find the slope from any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

Example

FINDING SLOPE FROM AN EQUATION

2. Find the slope of the line described by $6x - 5y = 30$.

Step 1: Find the x-intercept.

$6x - 5y = 30$

$6x - 5(0) = 30$ *Let $y = 0$.*

$6x = 30$ *Simplify.*

$x = 5$

Step 2: Find the y-intercept.

$6x - 5y = 30$

$6(0) - 5y = 30$ *Let $x = 0$.*

$-5y = 30$ *Simplify.*

$y = -6$

The x-intercept is 5. The y-intercept is –6.

Step 3: The line contains (5, 0) and (0, –6). Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-5} = \frac{-6}{-5} = \frac{6}{5}$$

Slope-Intercept Form (for Holt Algebra 1, Lesson 5-6)

- If you know the slope and *y*-intercept of a line, you can write an equation that describes the line.

Slope-Intercept Form of a Linear Equation

If a line has slope *m* and *y*-intercept *b*, then the line is described by the equation $y = mx + b$.

- Any linear equation can be written in *slope-intercept form* by solving for *y* and simplifying. In this form, you can immediately identify the slope and *y*-intercept and quickly graph the line.

Examples

WRITING LINEAR EQUATIONS IN SLOPE-INTERCEPT FORM

1. Write the equation that describes each line in slope-intercept form.

A slope = $\frac{1}{3}$, *y*-intercept = 6

$y = mx + b$

$y = \frac{1}{3}x + 6$ *Substitute the given values for m and b.*

B slope = 0, *y*-intercept = –5

$y = mx + b$

$y = 0x + (-5)$ *Substitute.*

$y = -5$ *Simplify.*

USING SLOPE-INTERCEPT FORM TO GRAPH

2. Write the equation $3x - 4y = 8$ in slope-intercept form. Then graph the line described by the equation.

Step 1: Solve the equation for *y*.

$3x - 4y = 8$

$-3x \qquad -3x$ *Subtract 3x from both sides.*

$-4y = -3x + 8$

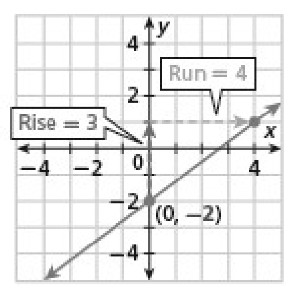
$\frac{-4y}{-4} = \frac{-3x + 8}{-4}$ *Since y is multiplied by –4, divide both sides by –4.*

$y = \frac{3}{4}x - 2$

Step 2: The *y*-intercept is –2, so the line contains (0, –2). Plot the point (0, –2).

Step 3: Slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{3}{4}$. Count 3 units up and 4 units right from (0, –2) and plot another point.

Step 4: Draw the line through the two points.



Point-Slope Form (for Holt Algebra 1, Lesson 5-7)

- You can graph a line if you know its slope and a point on the line.
- If you know the slope and a point on a line, you can write the equation of the line in *point-slope form*.

Point-Slope Form of a Linear Equation

The line with slope m and point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

- An equation written in point-slope form can be written in slope-intercept form and then quickly graphed.

Examples

USING SLOPE AND A POINT TO GRAPH

1. Graph the line that has a slope of $-\frac{1}{2}$ and contains the point (3, –2).

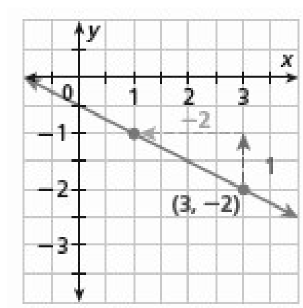
Step 1: Plot (3, –2).

Step 2: Use the slope to move from (3, –2) to another point.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = -\frac{1}{2} = \frac{1}{-2}$$

Move 1 unit up and 2 units left and plot another point.

Step 3: Draw the line connecting the two points.



WRITING LINEAR EQUATIONS IN POINT-SLOPE FORM

2. Write an equation in point-slope form for the line that has a slope of 7 and contains the point (4, 2).

$y - y_1 = m(x - x_1)$ *Write the general point-slope form of an equation.*

$y - 2 = 7(x - 4)$ *Substitute 7 for m, 4 for x_1, and 2 for y_1.*

USING TWO POINTS TO WRITE AN EQUATION

3. Write an equation in slope-intercept form for the line through the points (1, –4) and (3, 2).

Step 1: Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3$$

Step 2: Substitute the slope and one of the points into the point-slope form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$ *Choose (3, 2).*

Step 3: Write the equation in slope-intercept form.

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$ *Distribute.*

$\underline{+2 \quad\quad +2}$ *Add 2.*

$y = 3x - 7$

An equation for the line is $y = 3x - 7$.