M8LE4 Module Review Sample Solutions

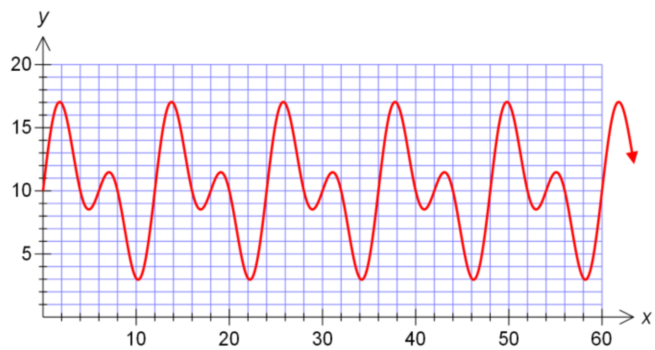
Compare the results you recorded with the possible solutions provided in this document. Take note of where there are differences.

You may want to make adjustments to your work based on what you see here, and save your revisions, so that you can refer to it later.

Remember that you were provided the choice on which method to use to solve the problem so your work may look different.

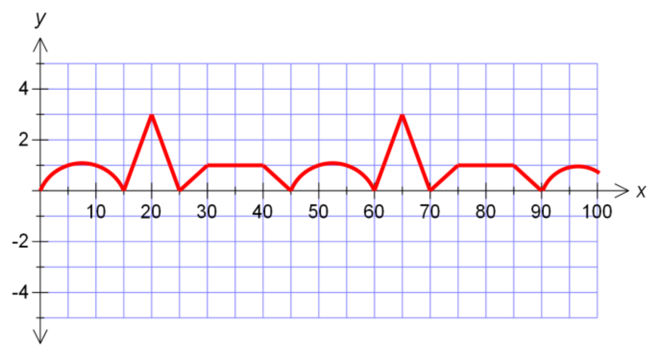
1. State whether each of the following graphs is periodic or not. If they are, state their period.

a)



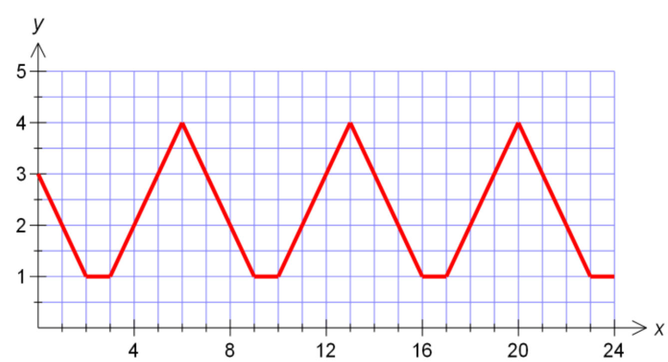
Yes, it's periodic and the period is 12

b)



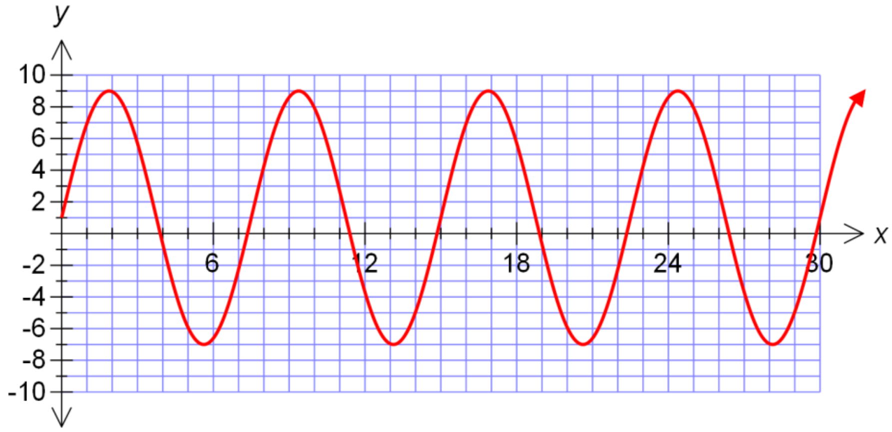
Yes, it's periodic and the period is 45.

c)



Yes, it's periodic and its period is 7.

2. Sketch any periodic graph that has an x-scale that goes from 0 to 30 and has 4 complete periods.



3. Convert the following into degrees to the nearest degree.

a) $\frac{5\pi}{6}$ a) $\frac{5\pi}{6} \times \frac{180}{\pi} = 150°$

b) 10.5 radians b) $10.5 \times \frac{180}{\pi} = 602°$

4. Convert the following into radians in terms of π.

a) 270° a) $270° \times \frac{\pi}{180} = \frac{3\pi}{2}$

b) 105° b) $105° \times \frac{\pi}{180} = \frac{7\pi}{12}$

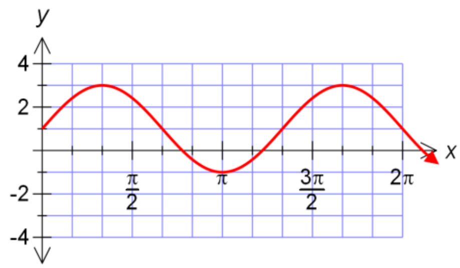
5. Convert the following into radians to 3 decimal places.

a) 50° a) $50° \times \frac{\pi}{180} = 0.873$

b) 175° b) $175° \times \frac{\pi}{180} = 3.054$

6. Find the equation of the following graphs

a)



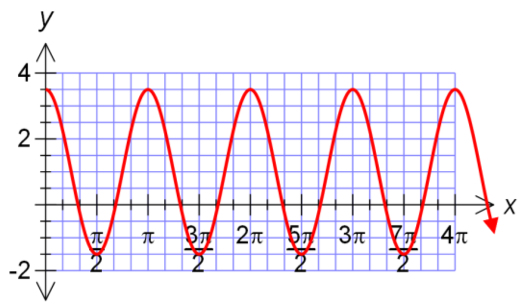
a) d is 1.
a is 2.

The period is $\frac{4\pi}{3}$ and so the b value is 1.5

The function starts at the beginning, and so the c value is 0.

$y = 2\sin(1.5x) + 1$

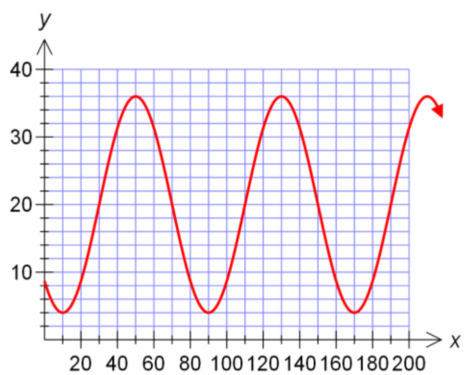
b)



b) D is 1.
A is 2.5.
The period is π and so the B value is 2
The function starts at the max, and so the C value is $-\frac{\pi}{4}$.

$y = 2.5\sin 2\left(x + \frac{\pi}{4}\right) + 1$

c)



c) d is 20.
a is 16.
The period is 80 and so the b value is $\frac{2\pi}{80}$
The beginning of the function is at x = 30, and so the c value is 30.

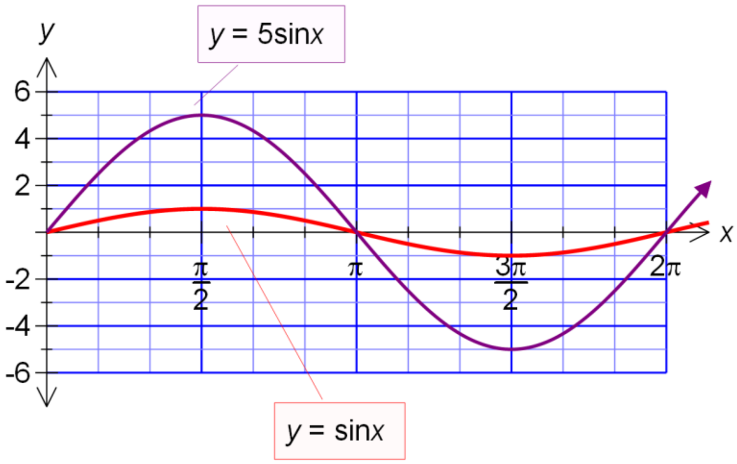
$y = 16\sin\left(\frac{2\pi}{80}(x - 30)\right) + 20$

7. Graph at least one period of each of the following sinusoidal graphs.

a) $y = 5\sin\left(x + \frac{\pi}{3}\right) + 3$

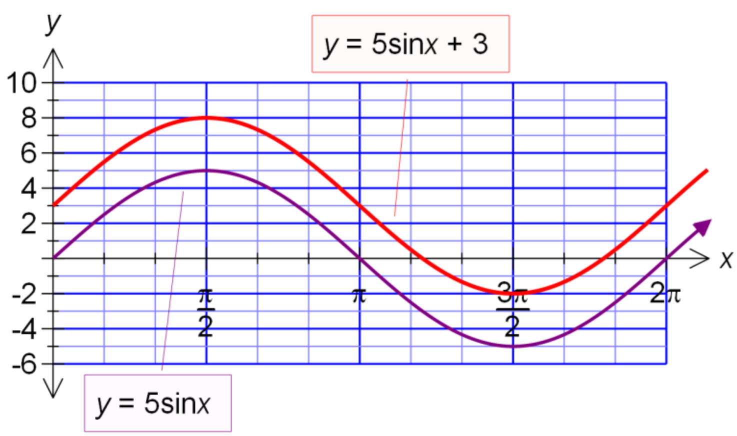
First, you need to apply a value of 5 which will multiply the y-values by a factor of 5:

Point	**y = sinx**	**y = 5sinx**
1	(0, 0)	(0, 0)
2	$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{\pi}{2}, 5\right)$
3	$(\pi, 0)$	$(\pi, 0)$
4	$\left(\frac{3\pi}{2}, -1\right)$	$\left(\frac{3\pi}{2}, -5\right)$
5	$(2\pi, 0)$	$(2\pi, 0)$



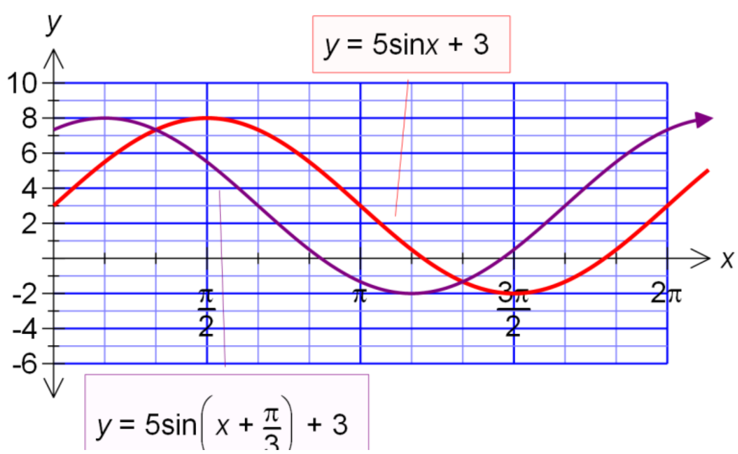
Next, you need to apply the d value of 3 which will add 3 to the y-values

Point	y = 5sinx	y = 5sinx + 3
1	(0, 0)	(0, 3)
2	$\left(\frac{\pi}{2}, 5\right)$	$\left(\frac{\pi}{2}, 8\right)$
3	$(\pi, 0)$	$(\pi, 3)$
4	$\left(\frac{3\pi}{2}, -5\right)$	$\left(\frac{3\pi}{2}, -2\right)$
5	$(2\pi, 0)$	$(2\pi, 3)$



Finally, you need to apply the c value of $-\frac{\pi}{3}$ which will subtract $\frac{\pi}{3}$ from the x-values.

Point	y = 5sinx + 3	$y = 5\sin\left(x + \frac{\pi}{3}\right) + 3$
1	(0, 3)	$(-\frac{\pi}{3}, 3)$
2	$\left(\frac{\pi}{2}, 8\right)$	$\left(\frac{\pi}{6}, 8\right)$
3	$(\pi, 3)$	$\left(\frac{2\pi}{3}, 3\right)$
4	$\left(\frac{3\pi}{2}, -2\right)$	$\left(\frac{7\pi}{6}, -2\right)$
5	$(2\pi, 3)$	$\left(\frac{5\pi}{3}, 3\right)$



b)

$$y = 2\sin\left(\frac{1}{2}(x-\pi)\right) - 3$$

b) Using the b value of ½, the period is found to be 4π. You can start by considering the graph of the function y = sin½x.

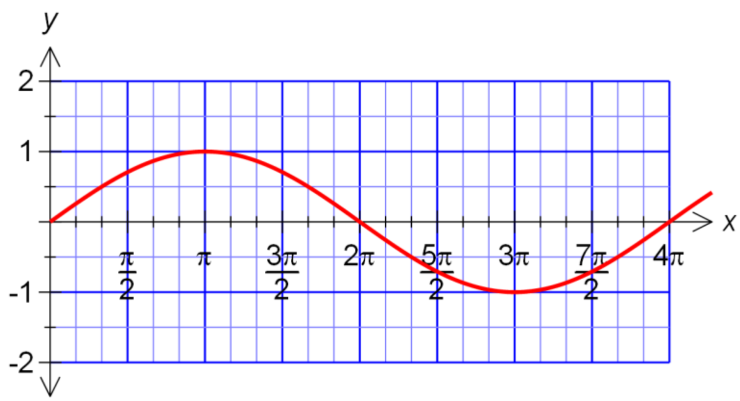
1 is at the beginning: (0, 0)

2 is at one quarter of the period: $\frac{1}{4}(4\pi) = \pi$ $(\pi, 1)$

3 is at the half way point: $\frac{1}{2}(4\pi) = 2\pi$ $(2\pi, 0)$

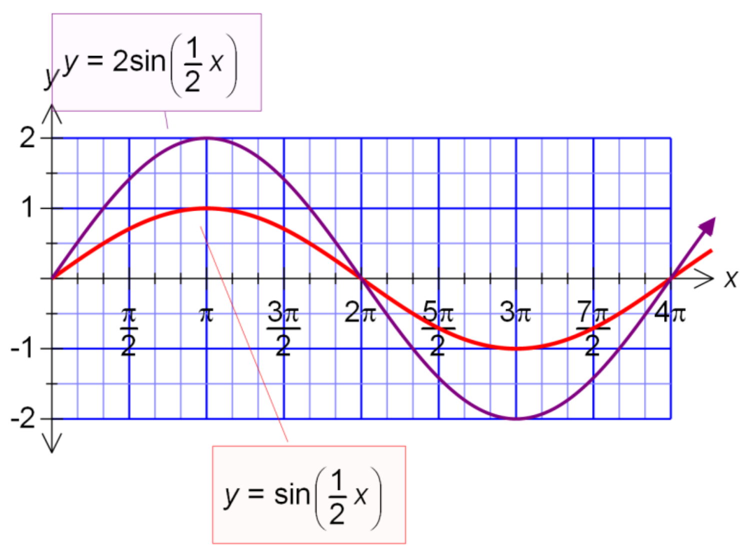
4 is at the three quarter point: $\frac{3}{4}(4\pi) = 3\pi$ $(3\pi, -1)$

5 is at the end of one period: $(4\pi, 0)$



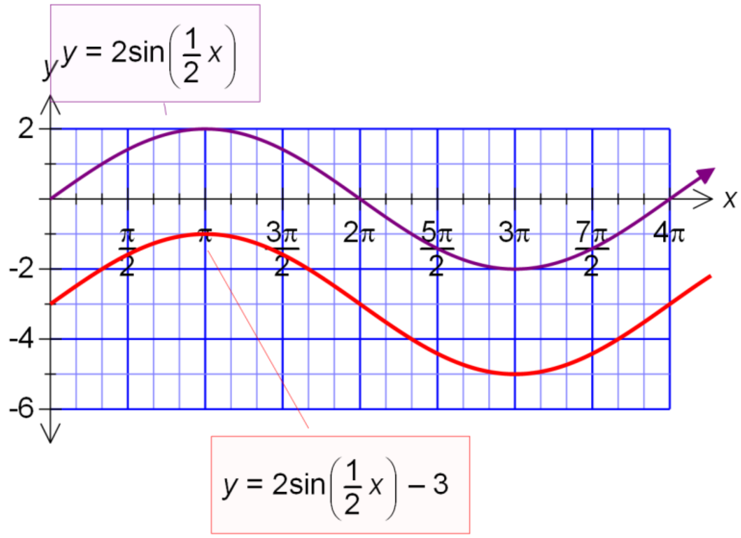
Next, you need to apply the a value of 2 which will multiply the y-values by 2.

Point	**y = sin½x**	**y = 2sin½x**
1	(0, 0)	(0, 0)
2	(π, 1)	(π, 2)
3	(2π, 0)	(2π, 0)
4	(3π, -1)	(3π, -2)
5	(4π, 0)	(4π, 0)



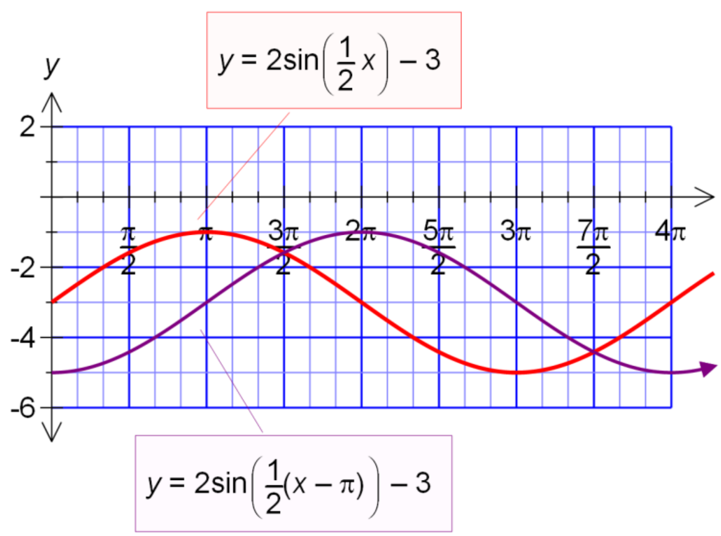
Next, you need to apply the d value of -3 which will subtract 3 from the y-values.

Point	**y = 2sin½x**	**y = 2sin½x - 3**
1	(0, 0)	(0, -3)
2	(π, 2)	(π, -1)
3	(2π, 0)	(2π, -3)
4	(3π, -2)	(3π, -5)
5	(4π, 0)	(4π, -3)



Finally, you need to apply the c value of π which will add π to each of the x-values:

Point	y = 2sin½x - 3	y = 2sin½(x-π) - 3
		(0, -5)
1	(0, -3)	(π, -3)
2	(π, -1)	(2π, -1)
3	(2π, -3)	(3π, -3)
4	(3π, -5)	(4π, -5)
5	(4π, -3)	(5π, -3)



c) $y = 40\sin\left(\frac{2\pi}{160}(x-20)\right)+25$

c) Using the b value of $\frac{2\pi}{160}$, the period is found to be 160. You can start by considering the graph of the function y = sin $\frac{2\pi}{160}$x.

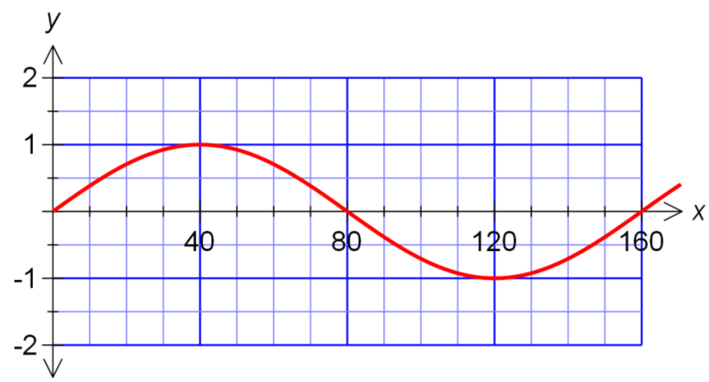
1 is at the beginning: (0, 0)

2 is at one quarter of the period: $\frac{1}{4}(160)=40$ (40, 1)

3 is at the half way point: $\frac{1}{2}(160)=80$ (80, 0)

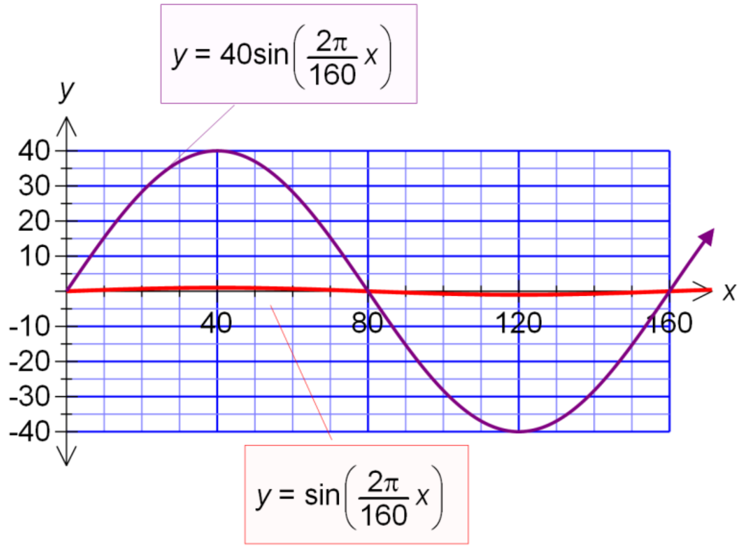
4 is at the three quarter point: $\frac{3}{4}(160)=120$ (120, -1)

5 is at the end of one period: (160, 0)



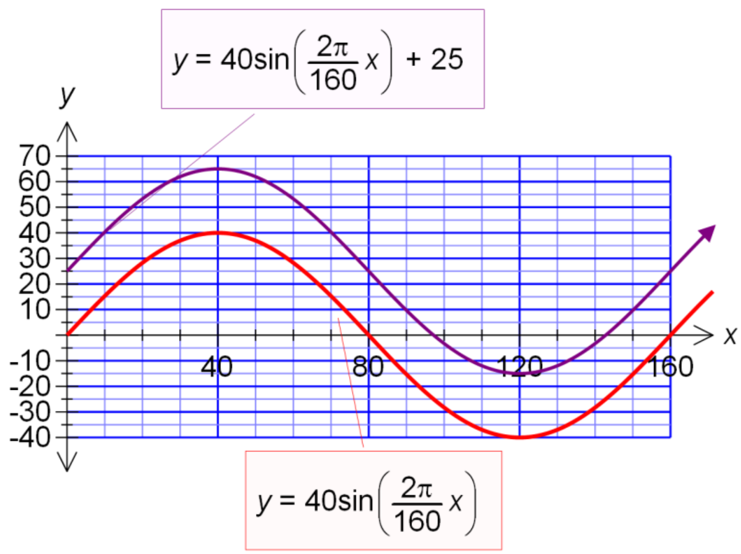
Next, you need to apply the a value of 40 which will multiply the y-values by 40.

Point	**y = sin $\frac{2\pi}{160}$x**	**y = 40sin $\frac{2\pi}{160}$x**
1	(0, 0)	(0, 0)
2	(40, 1)	(40, 40)
3	(80, 0)	(80, 0)
4	(120, -1)	(120, -40)
5	(160, 0)	(160, 0)



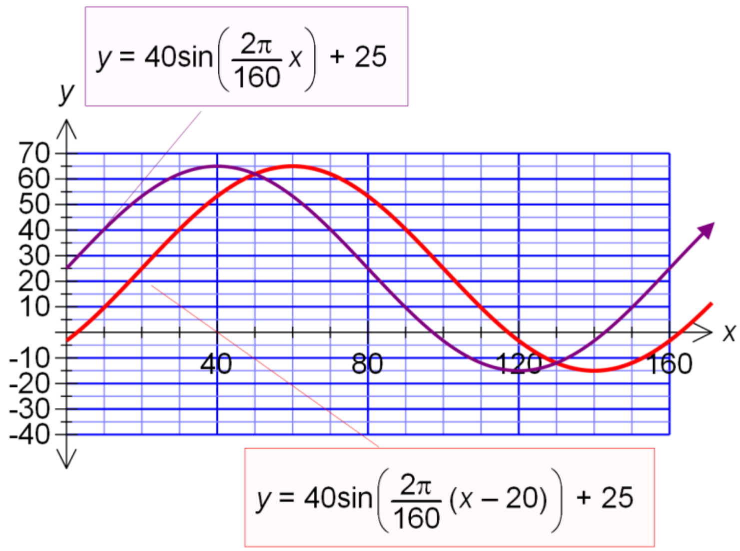
Next, you need to apply the d value of 25 which will add 25 to the y-values

Point	**$y = 40\sin\frac{2\pi}{160}x$**	**$y = 40\sin\frac{2\pi}{160}x + 25$**
1	(0, 0)	(0, 25)
2	(40, 40)	(40, 65)
3	(80, 0)	(80, 25)
4	(120, -40)	(120, -15)
5	(160, 0)	(160, 25)



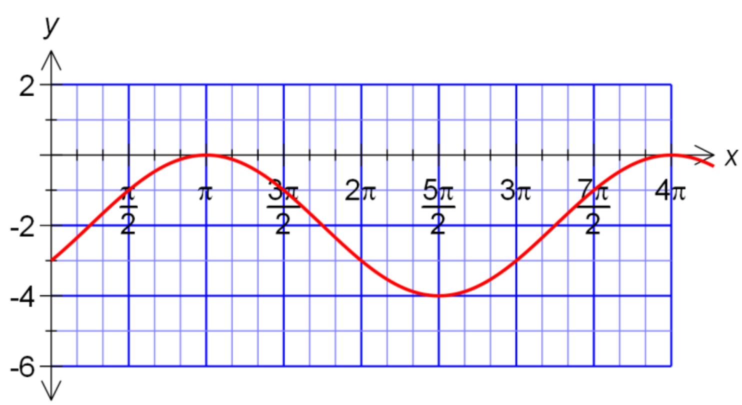
Finally, you need to apply the c value of 20 which will add 20 to the x-values

Point	$y = 40\sin\frac{2\pi}{160}x + 25$	$y = 40\sin\left(\frac{2\pi}{160}(x-20)\right)+25$
1	(0, 25)	(20, 25)
2	(40, 65)	(60, 65)
3	(80, 25)	(100, 25)
4	(120, -15)	(140, -15)
5	(160, 25)	(180, 25)

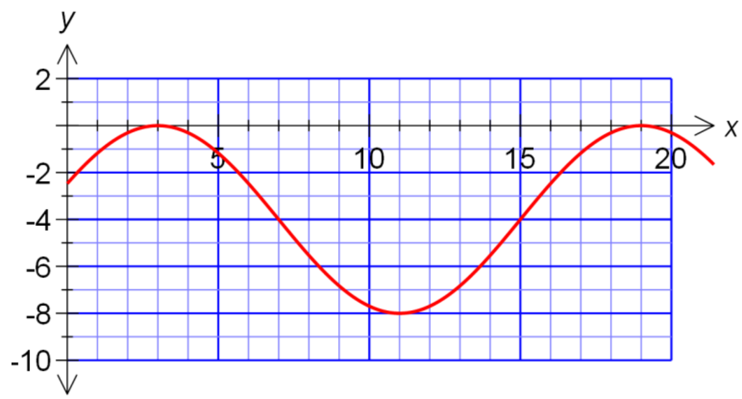


8. Draw at least one period of a sinusoidal graph with these properties:

   a) The period is 3π
      The amplitude is 2
      A maximum value is $(\pi, 0)$



b) A maximum is at (3, 0)
A minimum is at (11, -8)



9. The number of customers at a coffee shop varies according to the equation $N = 25\sin\left(\frac{\pi}{2}(t+1)\right)+40$, where N is the number of customers and t represents the time, measured in hours. Time starts at midnight.

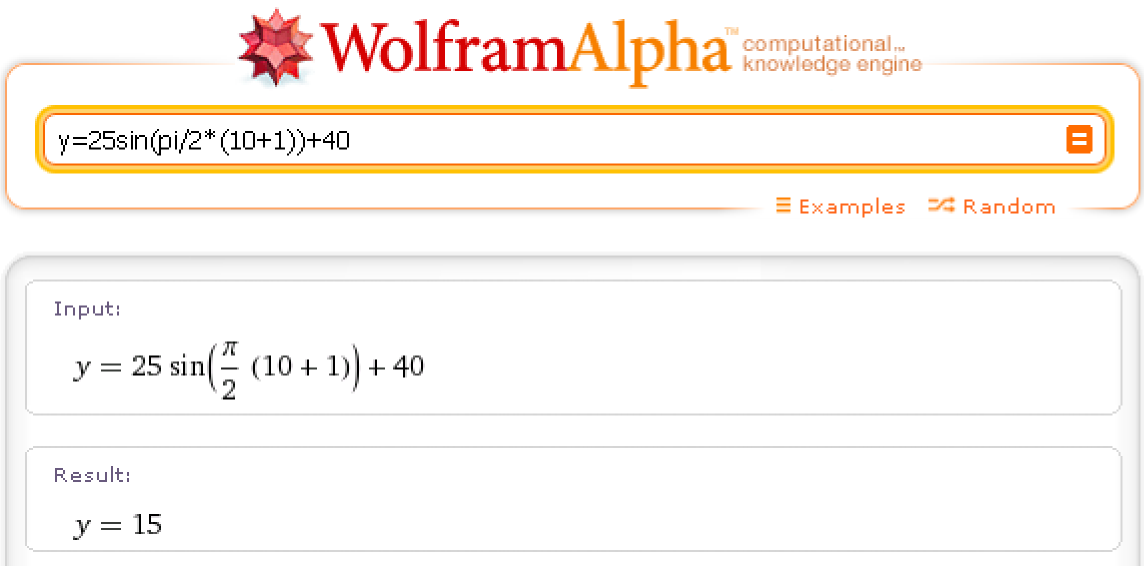
   a) How many customers will there be at 10 am?

   b) State a time there will be 50 customers.

   c) What is the period for this sinusoidal model?

   d) Which time(s) during the day will have the most customers?

a) Substitute 10 in for t.



There are 15 customers at 10 am.

b) Using WinPlot, you can find several times in which there are 50 customers. The x-coordinates of the points of intersection are:

0.74, 3.26, 4.74, 7.26, 8.74, 11.26, 12.74, 15.26, 16.74, 19.26, 20.74, 23.26

Represented as times:

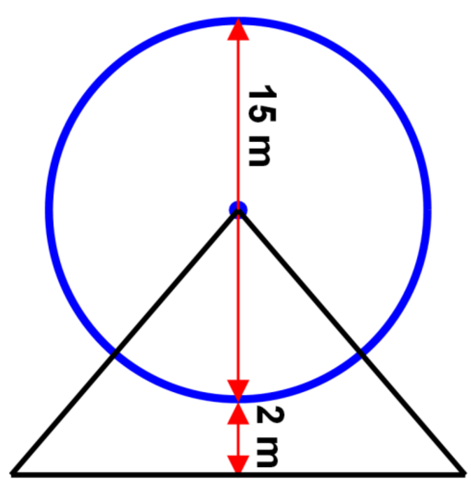
12:44 am, 3:16 am, 4:44 am, 7:16 am, 8:44 am, 11:16 am,

12:44 pm, 3:16 pm, 4:44 pm, 7:16 pm, 8:44 pm, 11:16 pm

c) $2\pi \div \frac{\pi}{2} = 2\pi \times \frac{2}{\pi} = 4$

d) The graph starts at the maximum value and has a period of 4 hours, and so the times at which the maximum is reached: 12 am, 4 am, 8 am, 12 pm, 4 pm, 8 pm.

10. A Ferris wheel is pictured below. A complete ride takes 6 minutes.



a) Find an equation that models the height of a passenger on the wheel if they start at the lowest point.
b) What will be the height of a passenger after 2.5 minutes?

a) You need to try to figure out values for a, b, c and d to build the equation $y = a\sin b(x - c) + d$.

The d value is the central axis. This spot is located at the vertical middle. The vertical middle is 9.5 m from the ground.

The a value is the amplitude. The amplitude is the height from the central axis to the minimum or maximum height. The amplitude of the ferris wheel's journey is 7.5 m.

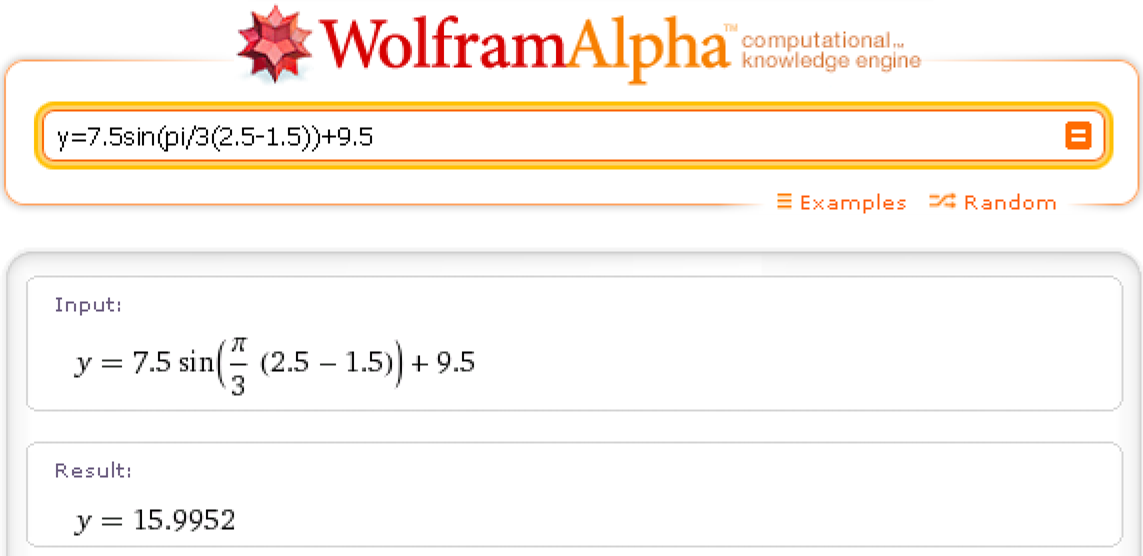
The period is 6 minutes so the b value will be $\frac{2\pi}{6}$ or $\frac{\pi}{3}$.

Since you want to start at the bottom of the sine curve, it can be accomplished with a phase shift of one quarter of the period to the right. The period is 6 minutes, and so the shift needs to be 1.5 minutes.

An equation that models the situation above is

$$y = 7.5\sin\left(\frac{\pi}{3}(x - 1.5)\right) + 9.5.$$

b) To find the height of a passenger after 2.5 minutes:



The height of a passenger after 2.5 minutes is approximately 16 m.

11. The time of sunset in Churchill on the first of each month is given in the table below. (Minutes have been converted to decimals. For example, 5:15 would be written as 5.25 since :15 is 0.25 part of an hour.)

Sunset Time in Churchill			
Month	**Sunset Time (pm)**	**Month**	**Sunset Time (pm)**
Sept '10	8.27	May '11	9.12
Oct '10	6.82	June '11	10.18
Nov '10	5.42	July '11	10.47
Dec '10	3.47	Aug '11	9.65
Jan '11	3.53	Sep '11	8.28
Feb '11	4.60	Oct '11	6.83
Mar '11	5.75	Nov '11	5.42
Apr '11	7.95	Dec '11	3.47

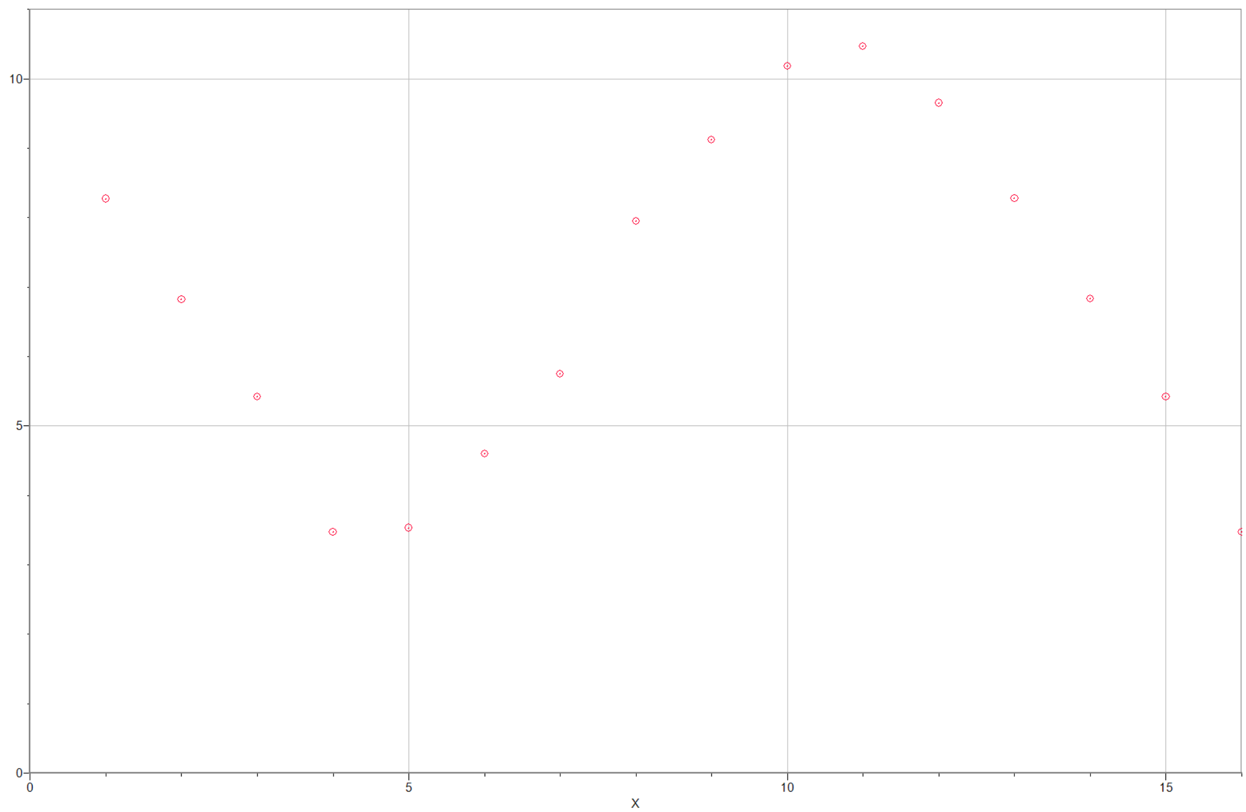
a) Graph the data.

b) Using sinusoidal regression, create an equation that models the data.

c) Using the equation found in b), comment on the trends in the sunset time.

d) Using the equation, estimate the sunset time in June 1, 2012.

a)

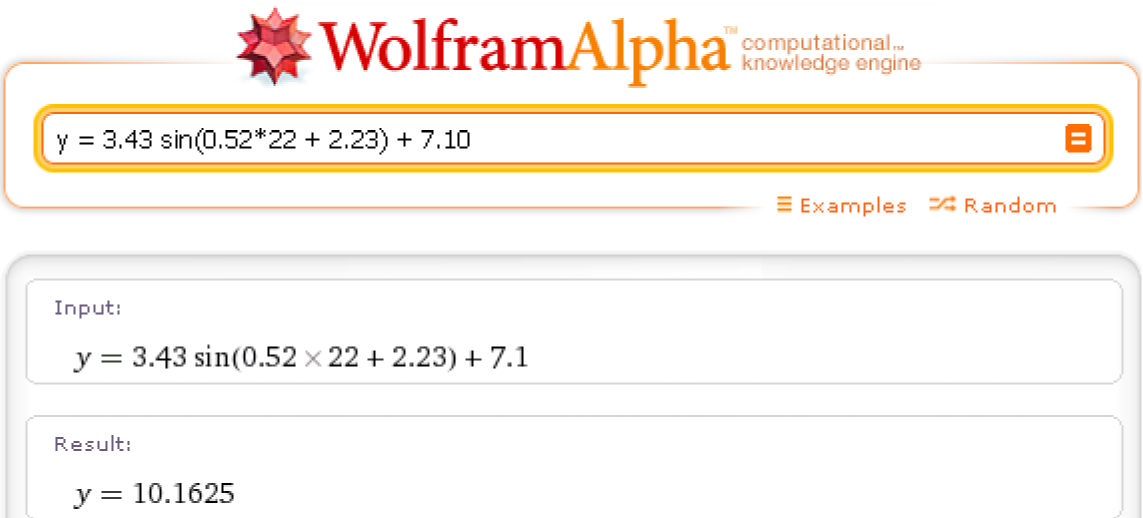


b) $y = 3.43 \sin(0.52x + 2.23) + 7.10$

c) The D value of 7.10 tells you the average time of sunset is 7.10 (or 7:06 pm).The A value of 3.43 tells you that the earliest and latest sunset times vary from the average by 3.43 hours.

The B value of 0.52 tells you that the period is $\frac{2\pi}{0.52} = 12.08$ months.

d) You need to substitute 22 in for x and find y:



The sunset time on June 1, 2012 is estimated to be 10.16 hours which is 10:10 pm.