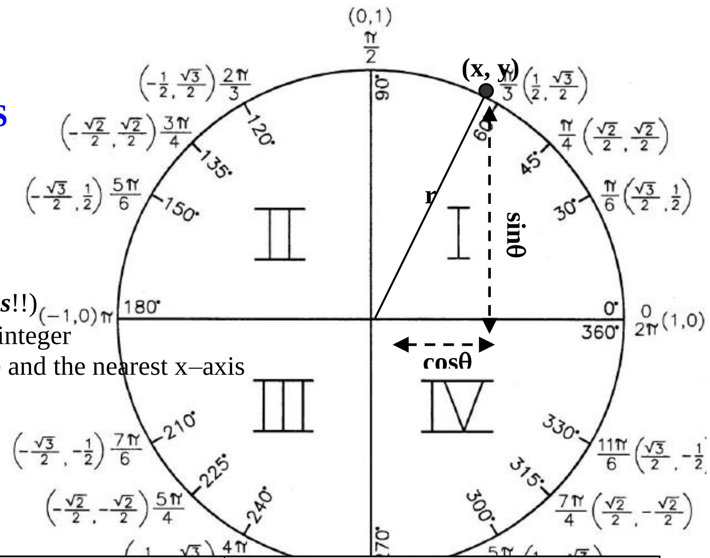

GRADE 12 PRE-CALCULUS REFERENCE NOTES

Unit B: Circular Functions

Radian Conversion: **180° = π^r.** *Eg*: $75^\circ = 75^\circ * \frac{\pi^r}{180^\circ} = \frac{5\pi^r}{12}$

Circumference 'c' = π*d. **Arc Length 'a'** = **r*θ**. (θ ***in radians***!!)
Co-terminal angles of θ have the form **θ+2πn**, where ***n*** is any integer
Reference Angle: the + acute angle formed by the terminal side and the nearest x–axis
$r^2 = x^2 + y^2$. $\sin\theta = \frac{y}{r}$; $\cos\theta = \frac{x}{r}$; $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$; $\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

Unit A: Transformations

New Function	Parent function is:	New Function	Parent function is:
f(x – 2)	Shifted 2 ***right***	**f(x) + 3**	Shifted 3 up
–f(x)	Reflected about x-axis	**f(–x)**	Reflected about y-axis
2*f(x)	Stretched vertically by factor **2**	$\frac{1}{2} * f(x)$	Compressed by a factor of ½ in **y** direction (vertically)
f(2x)	Compressed by a factor of **1/2** horizontally.	$f\left(\frac{x}{2}\right)$	Expanded (stretched) by a factor of **2** horizontally.

$f^{-1}(x)$	**Inverse Function**. Exchange ***x*** for ***y***, then solve for ***y*** again. Reflects about the line **y = x**.
1/f(x)	**Reciprocal Function**. All output **y** values of parent function are divided into one. Big numbers become small fractions, small fractions become big numbers. Zeros become asymptotes.
Even Functions:	Symmetrical in a reflection about the *y* axis**. f(x) = f(–x).** Polynomials with all **even** exponents
Odd Functions:	Symmetrical with a refection about the *x*-axis *and* the *y*-axis. **–f(–x) = f(x)**. Which is really a **180°** rotation around the origin. Polynomials with all **odd** exponents.

Unit C: Trigonometric Identities

Basic Identities	Sum and Difference Identities
Reciprocal Identities $\csc\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta}$ **Quotient Identities** $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$ **Pythagorean Identities** $\sin^2\theta + \cos^2\theta = 1$; $\tan^2\theta + 1 = \sec^2\theta$; $\cot^2\theta + 1 = \csc^2\theta$	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ ***Note $\mp$*** $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ **Note $\mp$
Double Angle Identities $\sin 2\theta = 2\sin\theta\cos\theta$; $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$; $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	

Unit F: Exponents and Logarithms

Laws of Exponents. $a^m a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^m b^m = (ab)^m$; $a^m/b^m = (a/b)^m$; $a^{-m} = 1/a^m$; $a^{1/m} = \sqrt[m]{a}$

Exponential function: **y = b^x**. Asymptote on x-axis. **Solving**: If $x^a = x^b$ then **a = b. Need same base to solve.**

Logarithm. (The inverse function of exponent). ****Given ***x = b^y***, we *define* the variable ***y*** as: $y = \log_b x$ *****

Product Law of Logs: $\log_b(m*n) = \log_b m + \log_b n$ {b>0; b≠1; m and n >0}

Quotient Law of Logs: $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ **{b> 0, b ≠1; m and n >0}**

Power Law for Logs: $\log_b(m)^p = p*\log_b m$ {m, n, b > 0; b≠1; and p ε R} ;

Law of Logs for Roots: $\log_b \sqrt[r]{m} = \frac{1}{r}\log_b m$ **Change of Base Law**: $\log_b a = \frac{\log_c a}{\log_c b}$ Eg: $\log_4 16 = \frac{\log_2 16}{\log_2 4}$

Solving log equations. If **$\log_c a = \log_c b$ then a = b**. Anti-logarithm. $c^{\log_c x} = x$. They '*undo*' each other. **ln(e) = 1**

Unit C: Permutations, Combinations, Binomial Theorem

Fundamental Counting Principle (FCP): If one event can occur in 'a' ways, a second event in 'b' ways, a third event in 'c' ways, and so on, then the number of ways that all events can occur one after the other is the product a*b*c…. ***Eg***: number of license plates we can make 26*26*26*10*10*10. Watch if *repetitions* are allowed or not.

Permutations: ***Order does matter***! $nP_r = \frac{n!}{(n-r)!}$ for how many ***ordered*** arrangements can be made of ***r*** things from ***n*** things. 1st, 2nd, 3rd in a race of 12 runners. 12*11*10 possible choices or $12P_3$. **Counting Non-Distinguishable Objects**: 4 Red balls, 2 Green Balls. Number of *distinguishable* ways to arrange the balls is $\frac{6!}{4!*2!}$. **Combinations**. Arrangements of n objects taken r at a time where ***order*** does **not** ***matter***. $nC_r = \frac{n!}{(n-r)!r!}$. Selecting committees of just members (not special positions), Lotto 6/49, etc. *Eg*: how many ways can a committee of 3 people be formed from 12.

$$12C3 = \frac{12!}{(12-3)!\,3!} = 220$$

Counting Pathways. Use *recursive* Pascal Triangle method. Or just use how many steps as a combination taking a certain number in a certain direction. *Eg*: need to travel 10 blocks, 4 of them must be east and 6 south. Then ${}_{10}C_6$ (or ${}_{10}C_4$) is the answer for # of pathways

Binomial Expansion. $(a + b)^n = {}_nC_0a^n + {}_nC_1a^{(n-1)}b + {}_nC_2a^{(n-2)}b^2 + {}_nC_3a^{(n-3)}b^3 + \ + \ + \ {}_nC_nb^n$

Finding a given Term of an expansion. **The (k + 1)th term of** $(a + b)^n$ **is given by:** $t_{k+1}={}_nC_k * a^{(n-k)}*b^k$

UNIT D: POLYNOMIAL FUNCTIONS

Your own notes here

UNIT G: RADICALS AND RATIONALS

Several elements of this are in Unit A

Your own notes here

Other Hints: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **Quadratic Formula for solution of Quadratic in form:** $ax^2 + bx + c = 0$

Cosine Law: $a^2 = b^2 + c^2 - 2bcCos(A)$. Where side ***a*** is opposite from angle ***A***. Use cosine law when given all three sides ***or*** 2 sides and included angle. **Sine Law**: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ need at least one side with corresponding corner angle across.