GRADE 12 APPLIED
SINUSOIDAL FUNCTIONS
CLASS NOTES

Introduction

1. To date we have studied several 'functions':

Function	General Equation	Graph; Diagram	Usage; Occurence
linear	$y = mx + b$		
quadratic	$y = ax^2 + bx + c$		
cubic (different unit)	$y = ax^3 + bx^2 + cx + d$		
exponential (different unit)	$y = ab^x$		

Examples of Periodic Functions

2. Periodic functions have a repeating pattern called a cycle. Some examples from real-life that have repeating patterns might include:

Angles Greater Than 90°

3. To date you have only studied angles that go up to 90°, maybe sometimes all the way up to 180° in some 'obtuse' triangles. It is possible to have angles that go past 90° or all the way to infinity even. In this unit we measure angles differently than from vectors (if you have done that unit). Proper mathematicians measure their angles using the right-handed rule and measure ***counter-clockwise from the positive x-axis***!

Measuring Angles using the 'right handed' rule.

4. You **mark** and **label** the following angles of the rotating arm:

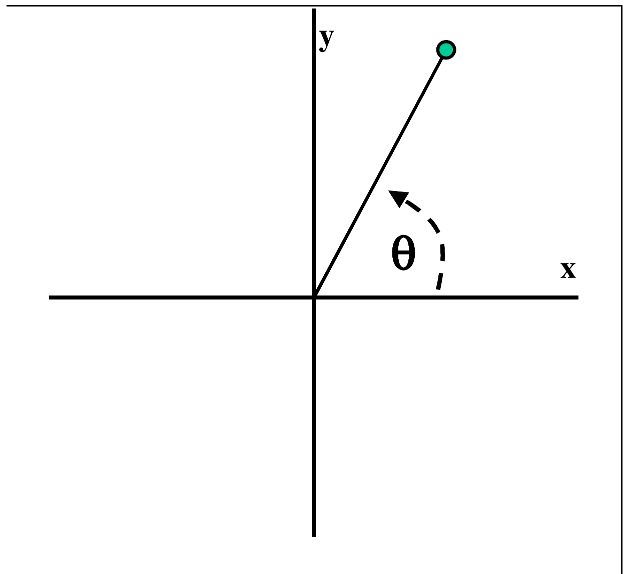
a. 90° b. 120°

c. 180° d. 240°

e. 330° f. 400°

g. 540° h. –60°

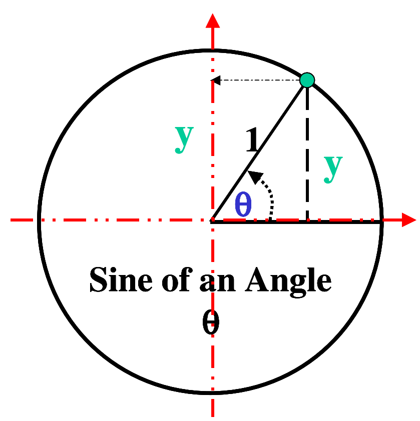
i. –240°



The Unit Circle

5. Have you ever thought of a circle as just a lot of right-angle triangles with the same length hypotenuse?

6. We define the sine of an angle to be its y-value (or component) on a *unit* circle.
A *unit* circle has a radius of 1.
The *y* value, or sine of the angle, is just how high up (or how far below) the point on the end of the angle arm gets as it rotates around all angles.

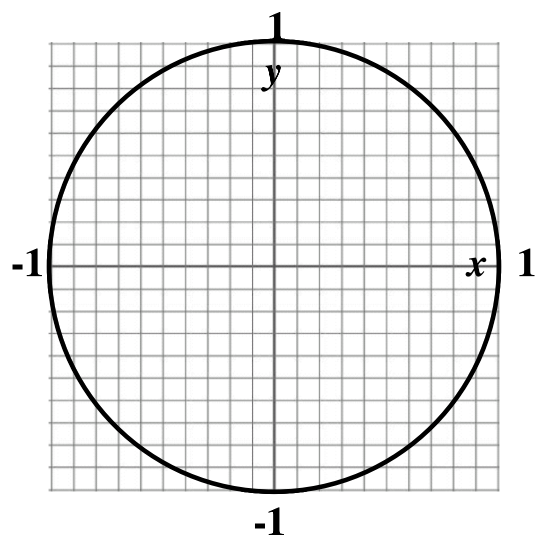


Estimating the Value of the Sine of an Angle on A Unit Circle.

7. When you type in: '**sin(45)'** on your calculator and it tells you: "**0.7071**"; what does that mean? One possible interpretation, besides that Opposite / Hypotenuse stuff, is how high you are on a ferris wheel! 0.7071 would mean you are 70.71% of the way to the top!

8. Being able to estimate a sine of an angle is very important (you have probably done it many times in your life without realizing it!)

9. Estimate the value of the sine of the following angles (use a protractor for the angle if you want to be a bit more accurate) by finding the height of a point (the y-coordinate) on the circle at the desired angle:



a. sin (30°) =

b. sin (90°) =

c. sin (120°) =

d. sin (345°) =

e. sin(–150°) =

f. sin(180°) =

g. you try a few!

10. If you want to use this above method for finding sines (and cosines) of angles a blank template is attached at the end of these class notes.

(A cosine by the way is just the x-coordinate on the circle as you move around an angle of the circle)

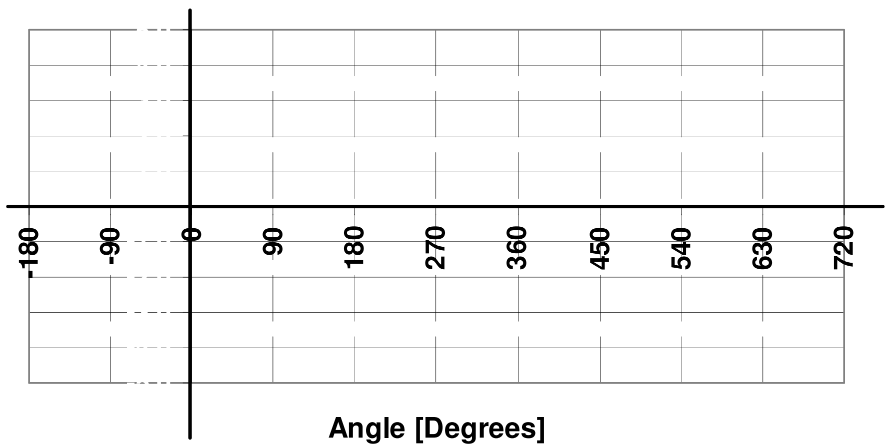
The Sinusoidal Curve from the Sine Function

5. Instead of measuring the angle and sine in a circle, let us plot the angle out along a line and see what the value of the sine function (the y-value of the unit circle) does.

Calculate the following values of **sin(x)** on a calculator

Plot the values of sin(x) (scale the vertical gridlines by 0.2 intervals)

x [°]	Sin(x)
–180	
–90	
0	
30	
45	
60	
75	
90	
120	
150	
180	
210	
240	
270	
300	
330	
360	
450	
540	
630	
720	



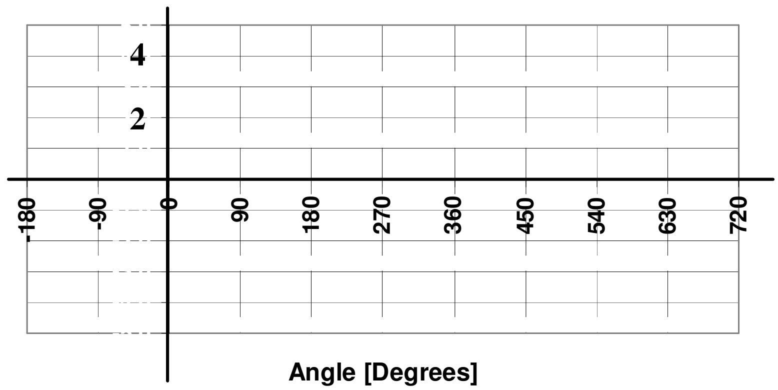
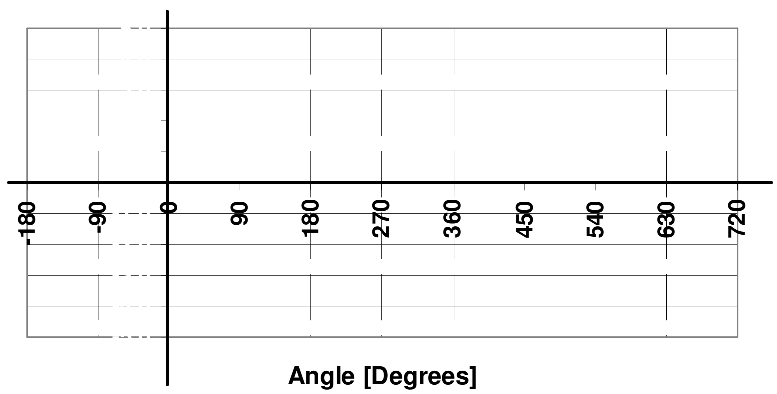
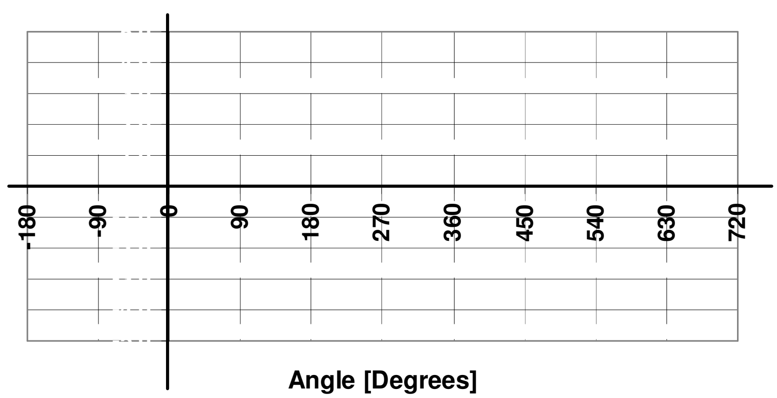
The Periodic Function Equation

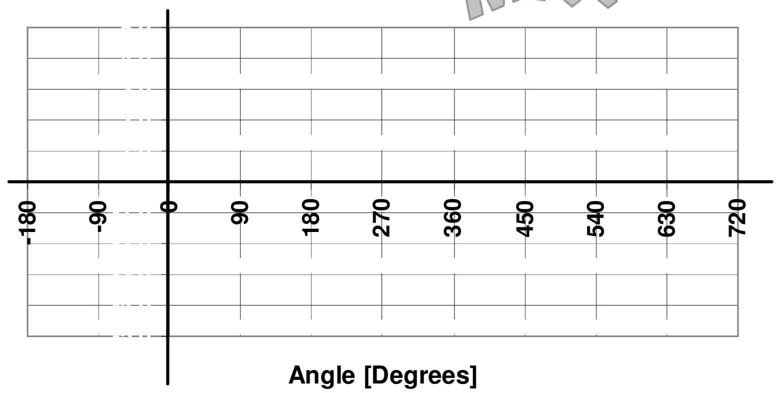
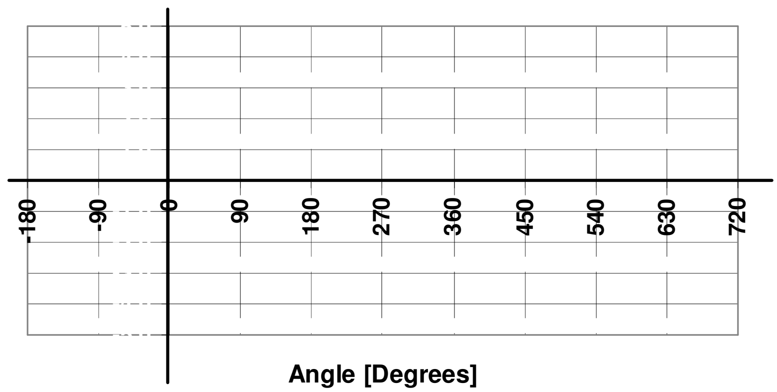
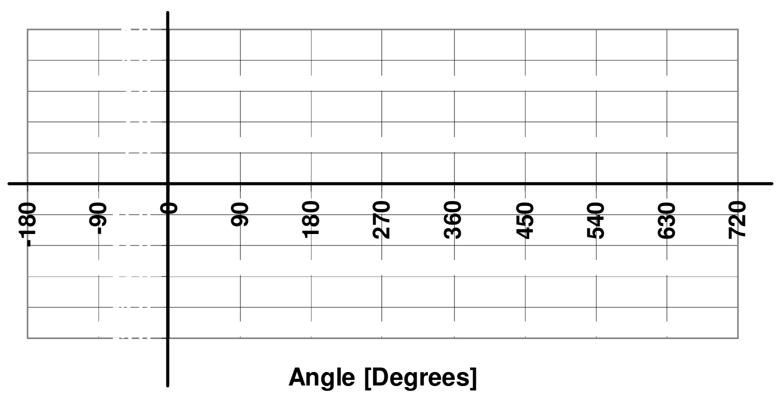
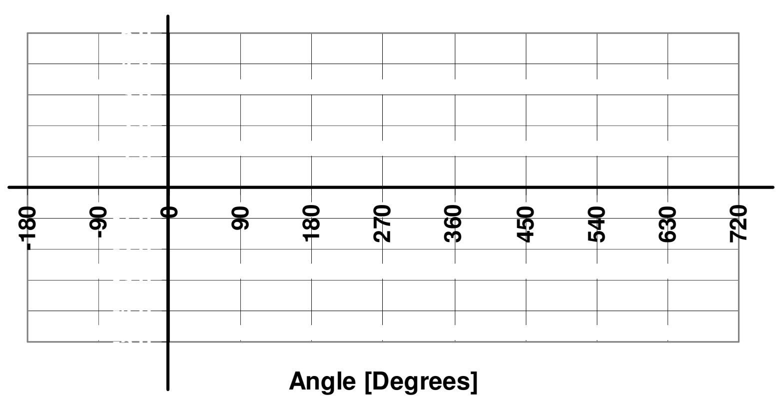
6. A special and fundamental case of periodic functions is the Sine function. It can be shown (in university) that every repeating function can be made from the sine function. So we will study the 'sinusoidal function'.

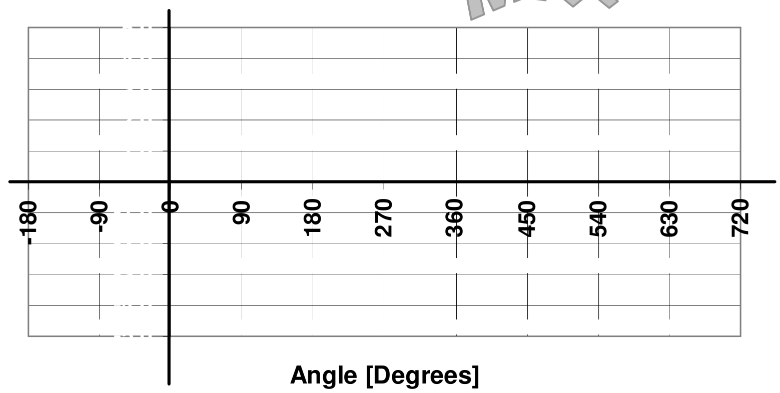
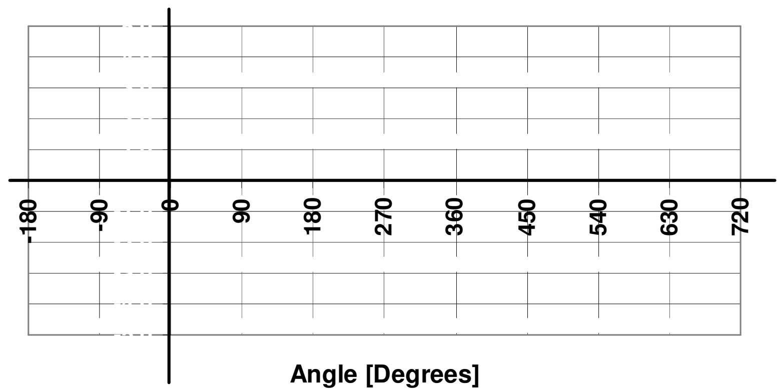
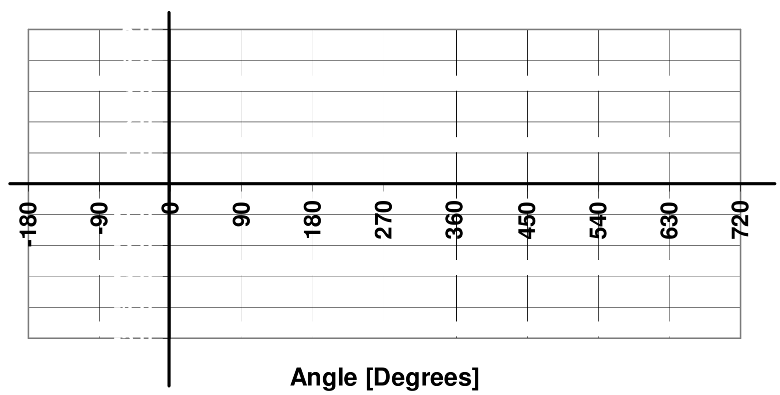
$$y = A\sin(Bx + C) + D$$

Let's sketch a few of these before we study the equation in detail. Use the TI-83 of course to make the sketches really easy.

7. Sketch the following to see what the '**sinusoidal**' function looks like: (make sure the Graphing Calculator is in '**degree**' **mode** just for now). You can use an on-line graphing tool also. Use the '**7:Zoom Trig**' on the TI-83 to make it fit just perfectly in the window.

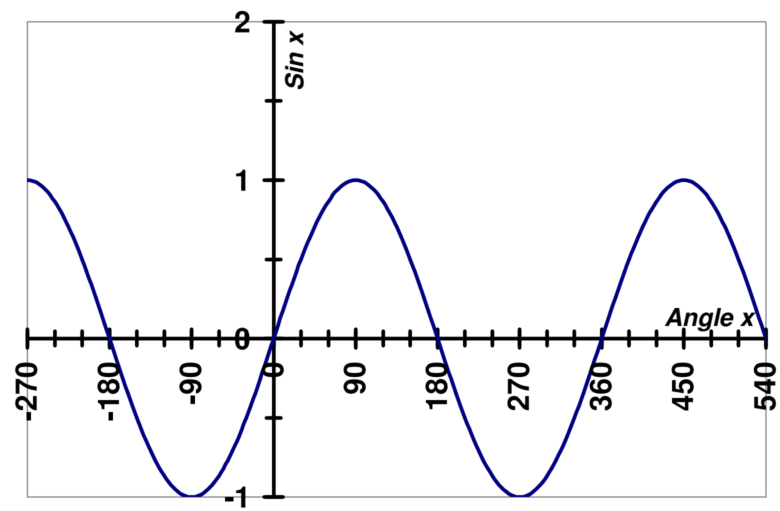
Function	Sketch
a. y = sin(θ) Amplitude: _________ Period: ____________ Median : __________	
b. y = 3 sin(θ) Amplitude: _________ Period: ____________ Median : __________	
c. y = sin(2θ) Amplitude: _________ Period: ____________ Median : __________	

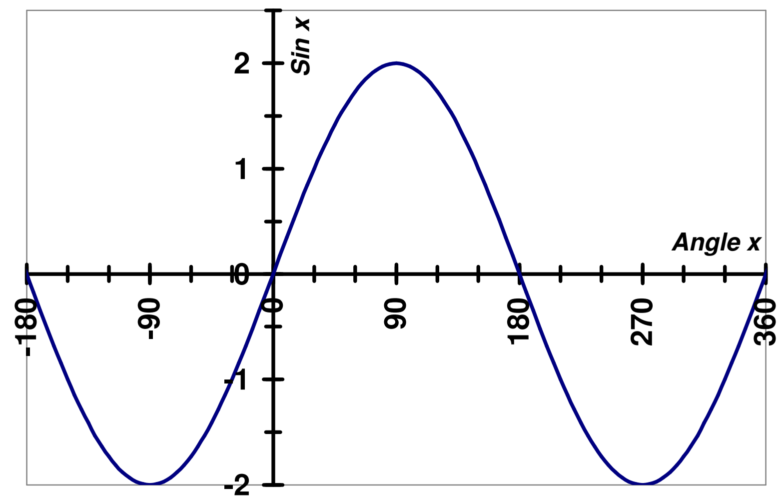
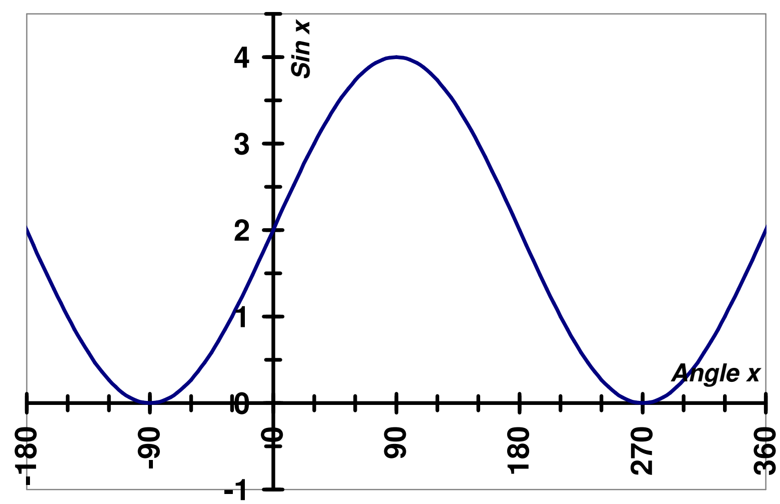
d. $y = \sin(\theta) + 4$ Amplitude: _________ Period: ____________ Median : __________	
e. $y = \sin(\theta + 60)$ Amplitude: _________ Period: ____________ Median : __________	
f. $y = 3\sin(2\theta) + 2$ Amplitude: _________ Period: ____________ Median : __________	
g. $y = 2\sin(4\theta) - 3$ Amplitude: _________ Period: ____________ Median : __________	

Here are some blanks if you want to invent your own!	
	
	

Understanding the ‘Standard’ Equation y = Asin(Bx + C) + D

8. A detailed explanation of some of the characteristics of the sinusoidal equation follows:

Parameter	Diagram examples
9. A cycle. First let’s graph a *cycle* of a sine curve, we graph several cycles in fact**.** The length of a ‘cycle’ along the x-axis is called its ***period***. (or *wavelength* if the x-axis is measuring distance). **Show a ‘cycle’ by highlighting** **Show the period, ‘T’ and it’s value.**	

10. Amplitude: **A** represents **Amplitude**. How much the function swings up and down above and below the **median** (or average). The bigger the **A** the bigger the swings. **A** is always non-negative. **Indicate the Amplitude**	
11. Median **D** represents the **Median**. The curve is half above it and half below it. (Like in statistics). **D** measures how much the average of the curve is displaced vertically. **Mark in the D with a dashed horizontal line.** **Mark in the A.**	

12. Maximum and Minimum

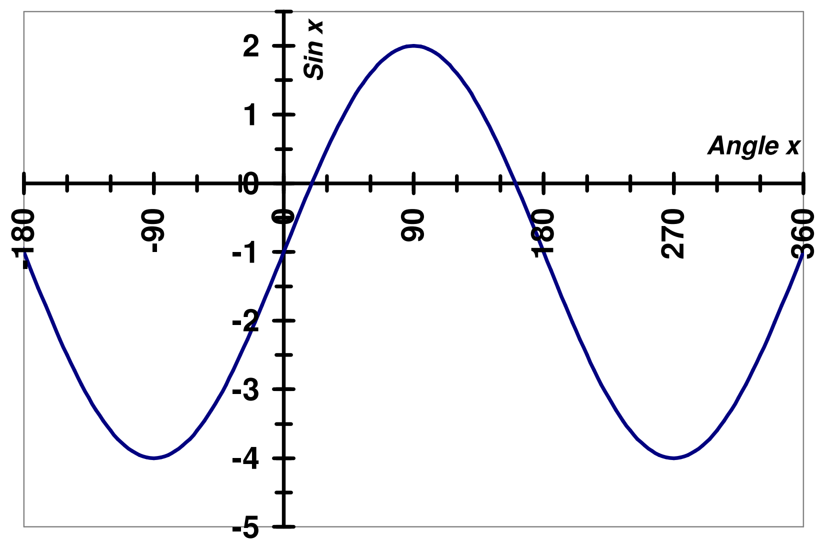
The **Max** and **Min** of the function can be seen from the graph or the function expression.

Mark a Max Value
Mark a Min Value

A formula for Max and Min from the expression:

Max = D +A
Min = D –A

Also: **D = (Max + Min)/2**



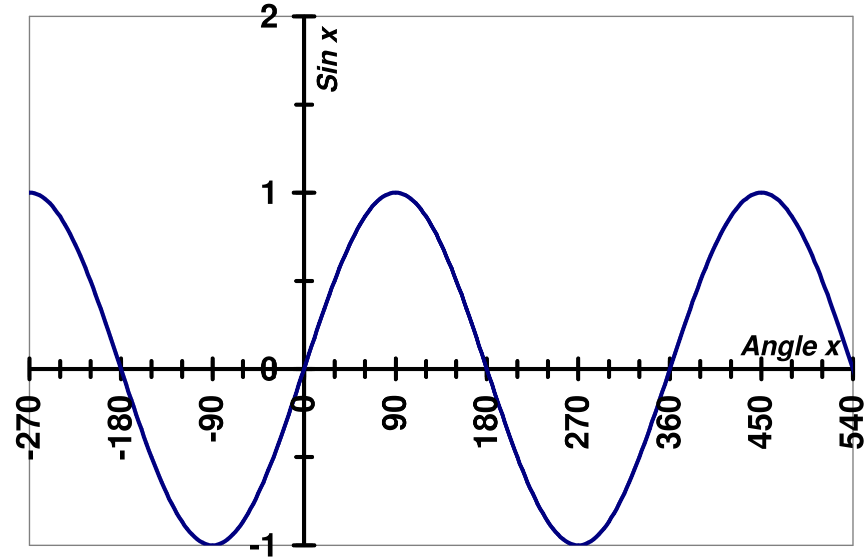
13. **Frequency**

The **B** *relates* to a *Frequency*. A high value of **B** 'squishes' the wave together with cycles of shorter period. A low value of **B** stretches the cycles out. **B** is always non-negative.

Here is a sinusoidal curve or function graph with a familiar and basic **B = 1**. It has a period of 360º.

Sketch in a curve with a B =2. State its period.

Sketch in a Curve with B = 0.5. State its period.



14. **Calculating Period, T**

The **Period, T**, of a sinusoidal function can be calculated by the formula:

$$T = \frac{360}{B}$$

So the **B** really tells you how many cycles squeeze into 360º if you re-write it as: $B = \frac{360}{T}$

Calculate the period of the following sinusoidal functions (check them on a graphing calculator if you want):

a. y = 3 sin (4x):

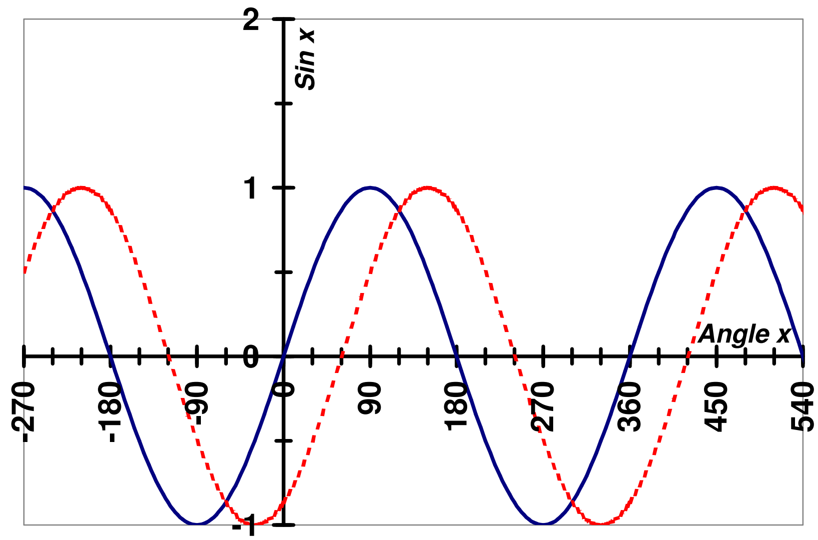
b. y = 2 sin (0.2x) + 2

c. y = 1.5 sin (2x – 60) – 4

15. Phase shift C

C contributes to a Horizontal Shift of the curve. It is called a '*phase shift*' in science and engineering. A **positive value of C** shifts the curve to the **left**, a **negative value of C** shifts it to the **right**. We will not get to in-depth with *phase shift* in Applied Mathematics.

The shift of the curve from a normal C of zero can be calculated from:
Phase Shift = –C/B.



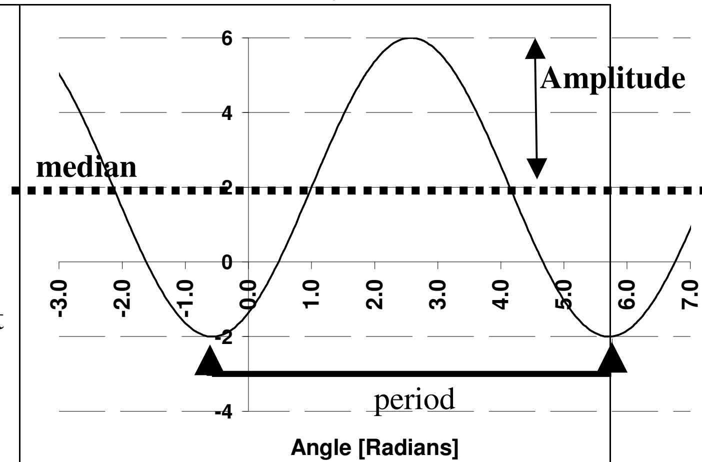
REVIEW

16. The Sine curve has several special characteristics:

A Median. It is the median like in stats, half the curve is above, half is below. When the median is above or below the x-axis, the curve has a 'vertical' displacement

A Period. The distance, or time, from one point on one cycle to the same point on the next. It is given by **Period = $360°/B$ or $2\pi/B$ (*when you learn radians*)**

An Amplitude. The height that the curve goes above, then below the median. It is half of the difference between the **maximum** and **minimum** of the curve.



Phase Shift. (Sometimes called horizontal shift) Sometimes the curve doesn't always start where the x coordinate = 0. In the above figure the **phase shift is +1** to the right. The phase shift can be calculated from the equation to be:
Phase Shift = –C/B

SAMPLE APPLICATION

17. Don is at the Red River Ex. He is on the Ferris wheel. He wants to be able to find his late friends to meet them while he rides. But he can only see out in the crowd towards the gate when he is more than **20** meters above the ground. The expression for the height of Don above the ground is $h = 30\sin(180^\circ * t - 90^\circ) + 32$ where ***'h'*** is height in meters, and ***'t'*** is time in minutes from when Don got on the ferris wheel and it starts to rotate.

a. Sketch a graph of the position of Don as height above the ground, ***h***, for at least two cycles as a function of time (graph paper below)

b. What is the period of the Ferris wheel ride?

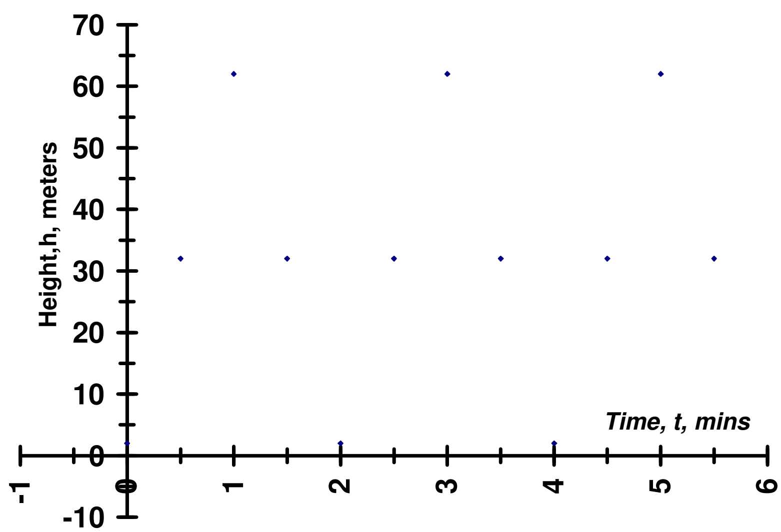
c. What is the length of time on each cycle that Don is below 20 meters?

d. What percentage of the time will Don be unable to watch for his friends?

e. what is the maximum height that Don reaches? The minimum?

f. when **t = 1.2** minutes, how high up is Don?

g. what is the phase shift of the cycle given that Don doesn't start his watch when the ferris wheel goes through the 0° position but at the bottom of the ride.



18. Now check your graph and answers with a graphing tool!

REGRESSION OF DATA – FINDING THE EQUATION FOR A TABLE OF DATA

19. To regress data means to find the equation that best matches the data. Recall that we had done this in Grade 10 and 11 for other functions and statistics. Regressing data to find the equation for periodic and sinusoidal data is exactly the same except for two considerations. The two considerations are mainly related to our funny system of measuring angles.

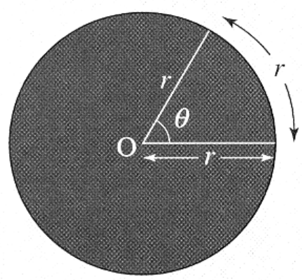
a. The ancients measured angles so that there was 360 degrees of arc in a circle, 60 minutes of arc in a degree, and 60 seconds of arc in a minute. Who says there are 360 degrees in a circle anyway??

b. Further, we measure time like we measure angles (surprise! Since how we used to measure time was based on the angle of the sun in the sky!) But modern mathematics and computers don't handle well AM and PM and 60 minutes in one hour etc.

THE RADIAN-A BETTER WAY TO MEASURE ANGLES

20. One radian is the measure of the central angle of a sector with arc length of that sector equal to the radius. The radian is pure and non-arbitrary measure of angles as it relates solely to the characteristic of a circle. The fact that there are 360 degrees in a circle would be debated by those who use other measures of angles (any army, French, Russians, aliens! Who have chosen other measures of angles; regardless, the radian is universal)

One radian is the measure of the central angle of a sector with arc length of the sector equal to the radius.



$$\theta = 1 \text{ radian}$$

$$2\pi \text{ radians} \doteq 6.28 \text{ radians} = 360^\circ$$

$$1 \text{ radian} \doteq 57.295\ 78^\circ$$

The TI-83 uses a raised r, ^{r}, as the symbol to indicate that an angle measure is expressed in radians. By convention, if no units are shown for an angle, then it is assumed to be in radians.

Don't forget to switch your calculator to radian mode in most applications involving sinusoidal and periodic data when you are using IT such as TI-83 or EXCEL.

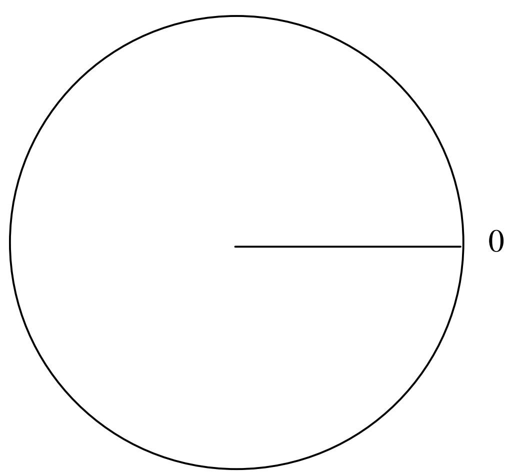
21. Complete the blanks in the following table:

Point	Degrees	π Radians	Radians
A	30		
B	45		
C	60		
D	90		
E	120		
F	180		
G	270		
H	330		
I	360		
J	400		
K			3.141596
L			1
M			2
N			3
O			3.5
P			5^r
Q			6^r
R		π/4	
S		5π/3	

****** Remember the conversion factor: $\frac{180^\circ}{\pi}$. So for example:***

240° = 240°* π/180° = 4.19 radians.   And 2.5 radians = 2.5*180°/π=143.2°

22. Sketch and label the points E to S above on the circle below as an angular measure.



FINDING THE REGRESSION EQUATION

23. Say you have made some observations like this for a vibrating beam:

Time [secs]	Distance [cm]
0.00	3.00
1.00	4.41
2.00	5.00
3.00	4.41
4.00	2.99
5.00	1.58
6.00	1.00
7.00	1.59
8.00	3.00
9.00	4.42
10.00	5.00

24. Look at the data first, get a sense of what it is doing. Hard to do without a graph right? But try anyway. See if you can find the function just from the data table:

Approximate Period: ________

Approximate: "**B**": _________

Approximate **Max**: _________ Approximate **Min**: __________

Approximate Median, **D**: _____________

Approximate Amplitude, **A**: ______________

Approximate Phase Shift; **-C/B**: ___________

So you can pretty well 'eye ball' what the sinusoidal expression looks like!

25. Now try it on the TI-83:

26. **Mode**: Ensure your mode is set to **radians**.

27. **Entering Data**. Enter data points as follows. (You need at least 4 points)

- a. ensure data are in increasing order by independent variable
- b. Press **STAT** Select **1:EDIT**
- c. Clear Lists 1 and 2 (Cursor to the top of each column and press **CLEAR ENTER**)
- d. Entering independent data in $\mathbf{L_1}$. Enter dependent data in $\mathbf{L_2}$. (the data must be in increasing order by dependent variable)

28. **Plotting Data**. Plotting data is similar to graphing data.

e. De-select all **Y=** formulas so they will not graph. The equals sign will be highlighted if they *are* to be graphed.

f. Select **STAT PLOT** by pressing **2nd** **Y=**. Select **Plot 1**. Turn on Plot 1. Put plot into the *Scatter Plot* mode. Select the largest *mark* possible.

29. Press **GRAPH**. You should have a plot of your data. You may need to use **ZOOM 9:ZOOMSTAT** to fit the data.

30. **Sinusoidal Regression Equation**. Now that you have your periodic data entered let the TI83 calculate the Sin Curve and the A,B,C, D of Y=Asin(Bx+C)+D. This is a statistical operation, **fitting the best curve to the data**.

g. Press **STAT** . Select **CALC**. Select **C:SinReg**. Press **ENTER**

h. The operation **SinReg** appears on the home screen.

i. Press **VARS**. Select **Y-VARS**. Select **1:FUNCTION**. Select **1:Y1**.

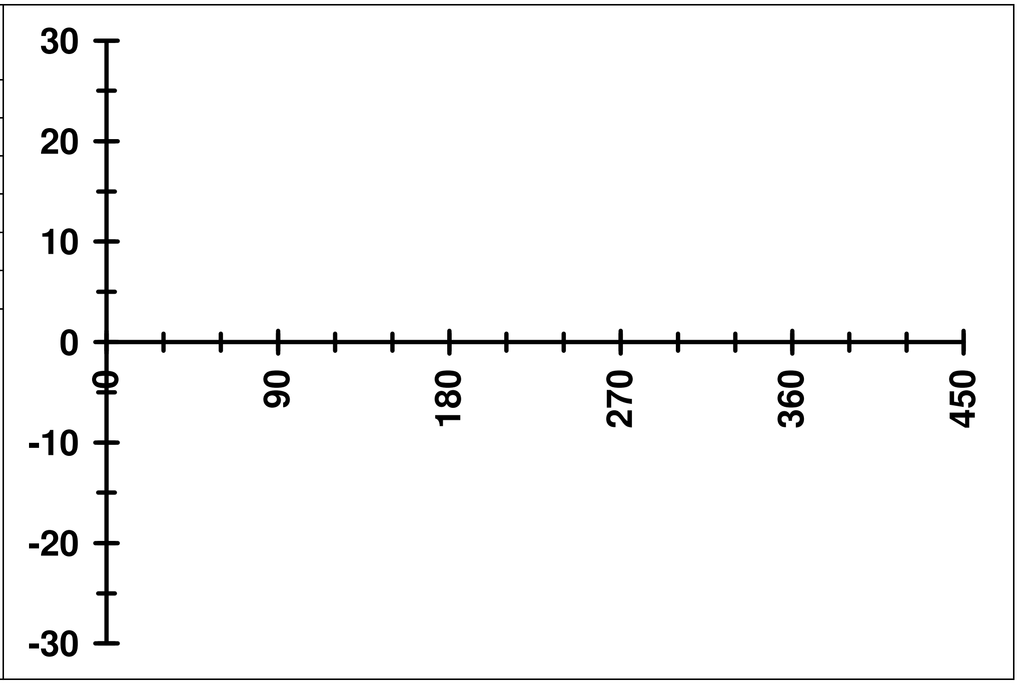
j. Press **ENTER**

k. The screen will show you the A, B, C and D for the equation that best matches the data. Of course you should always check for gross errors, especially for Period (B Factor).

Press **GRAPH**. Both your raw data plot and the best-fit curve graph will appear

31. Try again, plot the given data, find and state the regression sinusoidal equation, graph it on the TI 83 and ***sketch*** it below:

x axis	y axis
0	-17
60	-7
120	13
180	23
240	14
300	-6
360	-17



32. What real life experience do you think this represents?

EXPRESSING TIME

33. Entering time in a computer requires some special adjustments. The way earthmen measure time is based on a 'sexisgesimal' system. We count by 60s. 60 seconds to a minute, 60 minutes to an hour. Plus we throw in this AM and PM stuff too! Entering numbers like that in a calculator is not possible

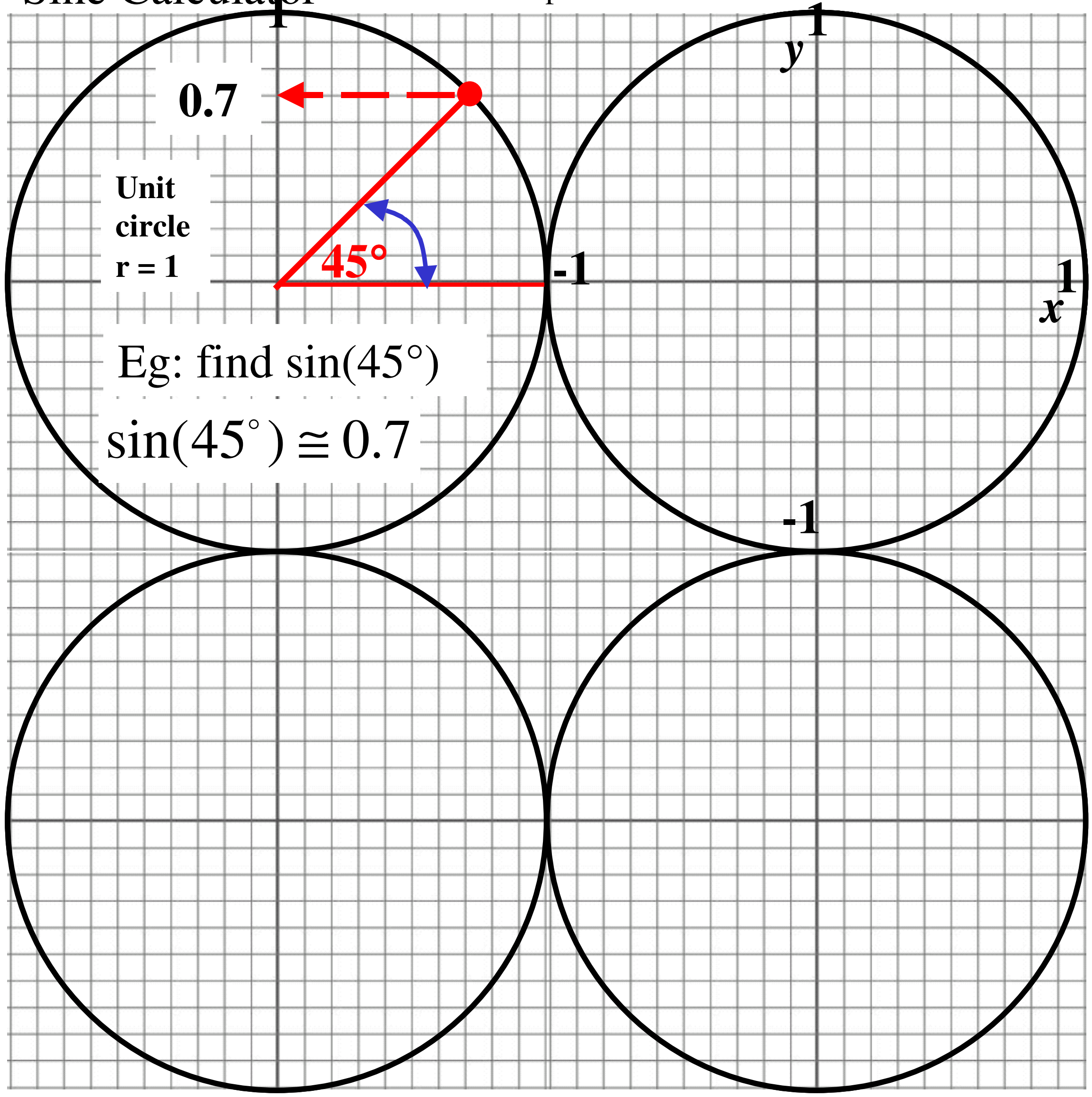
34. **Convert to Decimal Time**. To convert for example 07:30 to decimal time we need to realize we mean 7 hours and 30/60th of an hour. So 7.5 hours if we used a 'decimal time'.

35. **Eliminating AM and PM**. PM means 'post-meridian'. The meridian they mean is the longitude of where you are. So **3PM** means that sun has moved 3 hours past your place on earth, in other words it is three hours past noon. Of course, if we consider midnight to be the start, time zero of a day, that makes **3PM** 15 hours past our start of the day. Most nations on earth write PM times as 12:00 plus the time; ie: 5PM is 17:00. Of course anyone who drives a car must know this!!

CONVERT THESE:		
6PM	07:20 AM	09:24AM
06:36 AM	19:45	7:45PM
09:06PM	09:18AM	10:20 AM

MA40S Periodic Functions
Sine Calculator

Try graphically finding the Sine of different angles and compare it to your calculator values. Don't forget, in circular trig, angles are measured from the positive x-axis



Sin(θ) is the just the y-component or coordinate of the terminal arm angle on the unit circle. Incidentally Cos (θ) is the x-part