GRADE 11 PRE-CALCULUS COURSE REFERENCE NOTES

Gr11Precalc_Ref_Notes.doc

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C-QUADRATIC FUNCTIONS

Vertex: Point where curve reaches max or min. **Y-intercept**: where $x = 0$, just evaluate for $x = 0$. **x-intercept (aka: zeros or roots)**: where $y = 0$. **Axis of Symmetry**: x={the x part of vertex). **Min or Max**: y = {the y part of vertex}. **Domain**: $-\infty < x < \infty$ for all quadratics. **Range**: what f(x) or y can be; from vertex and range. EG: Range: $-\infty < y \le 4$.

Quadratic function forms

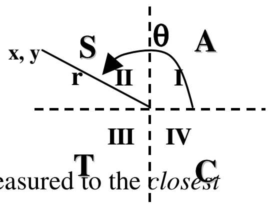
Vertex form: $y=f(x)= a(x-h)^2 + k$. (*aka Standard Form*)
(h, k) is the vertex.
'**x = h**' is the axis of symmetry. **k** is the max or min. **Convert** standard form to general form by multiplying
(eg: $3(x + 2)^2 + 1 = 3x^2 + 12x + 13$)
Factored form: $a(x - r_1)(x - r_2)$.
Eg: $2x^2 - 2x - 12 = 2(x + 2)(x - 3)$

Example complete the square with a ≠ 1:
$y = 2x^2 - 6x - 4$ **Divide by 'a';**
$y/2 = x^2 - 3x - 2$ ***magic nbr* is** $(-3/2)^2$ **or 9/4**
$y/2 = (x^2 - 3x + 9/4) - 9/4 - 2$ **Add& subtract magic #**
$y/2 = (x - 3/2)^2 - 9/4 - 8/4$ **Factor square trinomial**
$y/2 = (x - 3/2)^2 - 17/4$
$y = 2[(x - 3/2)^2 - 17/4] = 2(x - 3/2)^2 - 17/2$
or for earthlings: $y = 2(x - 1.5)^2 - 8.5$

General form. $y=f(x)=ax^2+bx+c$. **Convert** General to Vertex Form by completing square. To '**complete the square**' for **a=1** in $y = ax^2+bx+c$; add and subtract $(b/2)^2$. But need to have an 'a' of 1 for this. To find vertex directly from general form without having to complete the square: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$. Just calculate **[–b/(2a)]** to find the x of the vertex and plug that into the function **f(x)** to find the vertex y.

Example find Vertex from General form
$f(x) = 2x^2 - 6x - 4$ **put in the form** $y = ax^2 + bx + c$
$a = 2, b = -6 \therefore$ **x coordinate of vertex** $= (-b/2a) = (6/4) = (3/2) = 1.5$
Plug the 3/2 or 1.5 back into the function
$y = 2(3/2)^2 - 6(3/2) - 4 = 2(9/4) - 18/2 - 4 = -17/2$
or in decimal $2(1.5)^2 - 6(1.5) - 4 = -8.5$

F-TRIGONOMETRY



y**=Asin(B(**x **– C)) + D**. **A**=Amplitude. **B**: Number of cycles in 360°.
C = Phase shift. **D** = Vertical Shift of Median.

$\sin(\theta) = \frac{y-coordinate}{r} = \frac{y}{r}$; $\cos(\theta) = \frac{x-coordinate}{r} = \frac{x}{r}$, $\tan(\theta) = \frac{y-coordinate}{x-coordinate} = \frac{y}{x}$

where $r = \sqrt{x^2 + y^2}$ is the length of the terminal arm. Reference Angle, θ_{ref}, positive and acute angle measured to the *closest* x-axis. **Sine Law**: $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$. Watch for the ambiguous case. **Cosine Law**: $a^2 = b^2 + c^2 - 2bc * \cos(A)$. In all of these **A, B, C** is the corner opposite sides **a, b, c**. **C.A.S.T**: **C**osine only **+**, **A**ll **+**, **S**ine only **+**. **T**angent only **+**.

A- QUADRATIC EQUATIONS

Solving Quadratic Equations. Quadratic Equation: $ax^2 + bx + c = 0$. Interested in when a quadratic function or machine or model has an output equal zero. *Usually* 0 to two answers!! Example: Solve $x^2 + 2x - 15 = 0$ *(ie: find x that makes it true)*

Factoring
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
true only when $x = -5$ **or** $x = +3$

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Make sure equation is in the form $ax^2 + bx + c = 0$
Plug in: $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)} = \frac{2 \pm \sqrt{64}}{2} = 1 \pm 4 = 5\, or -3$

Completing the square for a =1
$x^2 + 2x - 15 = 0$ magic # is $(b/2)^2 = (2/2)^2 = 1$
$(x^2 + 2x + 1) - 1 - 15 = 0$ add & subtract magic #
$(x + 1)^2 - 16 = 0$ Factor the square trinomial
$(x+1)^2 = 16$
$x+1 = \pm\sqrt{16} = \pm 4$
$x = -1 \pm 4$ **so** $x = -5$ **and** $x = +3$

Discriminant in Quadratic Formula: $b^2 - 4ac$

If $b^2 - 4ac$ is:	**Then the solution has:**
> 0 and a *non*-perfect square	**2 real irrational roots (√)**
> 0 and perfect square	**2 real rational roots**
= 0	**1 real rational root**
< 0	**0 real roots, roots are 'imaginary' [$i^2 = -1$]**

Solve Absolute Value Equations
Definition: $|a| = a \;\; if \;\; a \ge 0; \;\; |a| = -a \;\; if \;\; a < 0$
Always check solutions, some may be extraneous
Isolate the **| stuff |** , make two equations
1) stuff = + blah and 2) **stuff = – blah**.

Example
|x + 2| – 4 = 2 Isolate the | | part → |x+2| = 6 Make two equations and solve:

Eqn$_1$: x + 2 = +6 **x = 4 Check the answer in original equation. It is good**	**Eqn$_2$: x+2 = –6** **x = -8. Check in original** **|-8+2| - 4 = |-6| - 4 = 6-4 =2 It is good**

UNIT D-SEQUENCES. Arithmetic. Successive terms change by common difference, d (step size). **Geometric** successive terms change by common ration, **r**. **Arithmetic**. **Recursion**: $t_{(n+1)} = t_n + d$. **Explicit**: $t_n=a+(n-1)*d$. **Sum**:

$S_n = \frac{n}{2}(2a + (n-1)d)\cdot$ $S_n = \frac{n}{2}(a + l)$ **Geometric**: **Recursion**: $t_{(n+1)} = t_n*r$. **Explicit**: $t_n=t_1r^{(n-1)}$. Sum: $S_n = \frac{a(1-r^n)}{1-r}$ or

$S_n = \frac{a - rl}{1 - r}$ and $S_\infty = \frac{a}{1-r}$ for infinite convergent. '**n**' is the term number, '**d**' the common difference of successive terms, **r** the common ratio of successive terms (**r ≠ 1**), **a** is the first term. '**a**' **is first term** (sometimes called t_1). '**l**' is last term.

UNIT G - SYSTEMS

SOLVING SYSTEMS OF TWO EQUATIONS BY SUBSTITUTION (Grade 10 & 11 for Quadratic)

Label the equations. Express one variable in terms of the other ('isolating' a variable). ***Substitute*** that expression into the ***other*** equation. Solve for the single variable. Substitute the found variable ***back into either equation*** to find the remaining variable. ***Check*** the answer in both original equations

SOLVING SYSTEMS OF TWO LINEAR EQUATIONS BY ELIMINATION (Grade 10)

Put equations into a '*standard*' order:(**ax + by = c**). Label the equations. Multiply either, or both, equations by some number so that one variable can be eliminated when the equations are added or subtracted. Add or subtract the equations to eliminate one variable and find the other. Find the eliminated variable now by substituting the solved variable back into either equation of the system. Check your answer (***x, y***) in both original equations.

UNIT H INEQUALITIES

Example inequalities: **4>2** (4 is **greater than** 2, or 2 is **less than** 4); **7 ≤ 7** (seven is **less than *or* equal to** seven), etc.

Inequality Algebra. ***Don't forget*** to ***change direction*** of inequality when **multiply** or **divide** by a negative number. Eg: **–2x > 6→ x < –3.**

Graphing Inequalities: ***Solve*** for ***y***; ***plot*** the line on graph; use broken line if no equality involved; ***shade*** the half of the universe that makes the inequality true (use ***the test point*** method *if necessary*).

UNIT B - RADICALS

Squares and Cubes: x*x = x^2. x*x*x = x^3. $\sqrt{x^2}=x$. $\sqrt[3]{x^3}=x$. '*Roots*' undo '*exponents*'.

Rational numbers. Can be made into fractions or terminate in repeat decimal pattern:

$3/8=0.375000\overline{0},\ 4\,{}^{5}\!/_{9}=4.5555\overline{5}$. **Irrational Numbers.** Not fractions. No computer can show them. $\sqrt{5},\ \pi, etc.$

No repeat pattern in decimal; not a fraction**. Which is bigger?** $\sqrt{14}\ or\ 4$? Square both! Is 14 **>** 16?

Rationalize Denominator: $\dfrac{a}{\sqrt{b}}\equiv\dfrac{a\sqrt{b}}{b}$; Complex (mixed) denominator: $\dfrac{a}{\sqrt{b}+c}\equiv\dfrac{a(\sqrt{b}-c)}{(\sqrt{b}+c)(\sqrt{b}-c)}=\dfrac{a(\sqrt{b}-c)}{b-c^2}$

Solving Radical Equations: Isolate the radical part (radicand). Will always get a plus and minus version when you take a **square** root or any **even** root.

UNIT E – RATIONALS

Solve Rational Equations

Note first what solutions are forbidden! Forbidden solutions are those where there is a division by zero. Multiply both sides by a common denominator (preferably lowest common denominator) to get rid of the ugly denominators. Solve as usual

Always check answers for ***extraneous*** solutions

Example: x cannot be 1 and x cannot be –2 (x ≠ 1 or –2)

$\dfrac{2}{2x-2}=\dfrac{6}{x+2}$ Multiply by Common Multiple of denominators.

In this case the only common thing the denominators go into is:

(2x-2)*(x+2). So multiply both sides by that

$(2x-2)(x+2)\dfrac{2}{2x-2}=\dfrac{6}{x+2}(2x-2)(x+2)$

$(x+2)*2=6*(2x-2)$ solve from there

UNIT B – RADICALS

Exponent Rules:

Product: $a^m * a^n \equiv a^{(m+n)}$

Quotient: $\dfrac{a^m}{a^n}\equiv a^{(m-n)}$

Power: $(a^m)^n \equiv a^{(m*n)}$

More Rules:

$(xz)^n \equiv x^n z^n$

$\left(\dfrac{x}{z}\right)^n \equiv \dfrac{x^n}{z^n}$

Negative Exponent: $a^{-n}\equiv\dfrac{1}{a^n}$

Zero Exponent:

$x^0 \equiv 1$ (***anything*** to the zero exponent is **1**)

Fractional (Rational) Exponents:

$x^{1/2}\equiv\sqrt{x}$, $x^{1/3}\equiv\sqrt[3]{x}$, $x^{1/n}\equiv\sqrt[n]{x}$

$x^{m/n}=\sqrt[n]{x^m}\equiv\sqrt[n]{x}^{\,m}$. Eg: $4^{3/2}=\sqrt[2]{4}^{\,3}=8$

$\sqrt{ab}=\sqrt{a}\sqrt{b}$ $\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{a^2}=a$ $\sqrt{a}^{2}=a$	**conjugate:** Multiply t&b by $a-\sqrt{b}$ $\dfrac{c}{a+\sqrt{b}}=\dfrac{c(a-\sqrt{b})}{(a+\sqrt{b})(a-\sqrt{b})}=\dfrac{ac-c\sqrt{b}}{a^2-b}$	$\sqrt[n]{b}=b^{1/n}$ $\sqrt[n]{b}^{\,m}=b^{m/n}$	$\dfrac{c}{\sqrt{b}}=\dfrac{c\sqrt{b}}{b}$

Reduce Radicals: Factor out perfect numbers. $\sqrt{72}=\sqrt{36}\sqrt{2}=6\sqrt{2}$. Rationalize denominator multiply top and bottom by the radical: $\dfrac{4}{\sqrt{8}}=\dfrac{4}{2\sqrt{2}}*\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{4\sqrt{2}}{4}=\sqrt{2}$

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TI83 Hints

Zoom 6 always when first start to centre. **Zoom 0** fits most Equations. **Zoom 7** for most Trig.

Find A vertex: Use **2nd TRACE 3:minimum** or **4:maximum** . Go a bit left of the vertex **ENTER**, go a bit right of the vertex **ENTER**, then move to approximate guess **ENTER**.

Find a Y-Intercept. Evaluate the function at X=0. **2nd TRACE 1:value** and enter **X = 0**

Find an x-intercept (or Zero). Find the '*zeros*'. **2nd TRACE 2:zero** . A bit left:boom, a bit right:boom, guess: boom

Find the Intersection of two curves (or lines) . **2nd TRACE 5:intersect.**