GRADE 12 APPLIED

UNIT C

FUNCTIONS

WORKBOOK

A clearly communicated answer

— is easily identified in the response space
Box it, label it, state in plain words even; include units!

— includes the parameters in the equation, and "y =", "sin", "ln", or "x", as applicable
ie: show proper formula, not just chicken scratch

— includes the units of measure, where applicable

— includes labels, units, and scales for the axes on graphs

— is expressed as an exact value or is appropriately rounded

Directing Words
Some questions may include directing words such as explain, state, and calculate. These words are explained below.

The word	The question is asking for…
identify/choose	the appropriate answer(s) from a given list of choices
state	a word, sentence, or number, without an explanation
describe/explain	words or symbols, diagrams, charts or graphs, or other methods that clearly show what you are thinking
justify/support	an explanation, information, or evidence that shows why your method, idea, or answer is correct
sketch/illustrate	a reasonably neat picture or diagram (not necessarily to scale) that shows or explains an idea, concept, or method

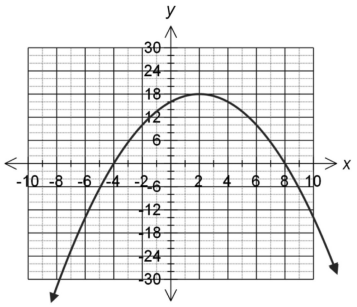
GRADE 12 APPLIED
UNIT C
WORKBOOK

Try to solve questions manually where possible *and* with technology, especially the TI-83 Graphing Calculator

Round all decimal answers to the nearest 0.01 (hundredth) unless otherwise indicated or appropriate.

QUADRATIC FUNCTIONS

1-1. Given the graph:



a) State the vertex.

b) State the axis of symmetry.

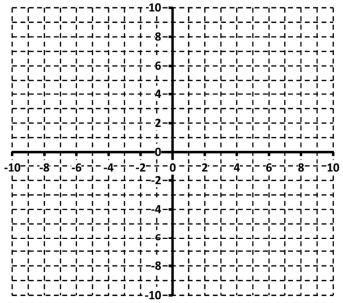
c) State the values of the intercepts.

d) What is the domain and range of the function?

1-2. Graph $y = -x^2 - 5x - 8$.

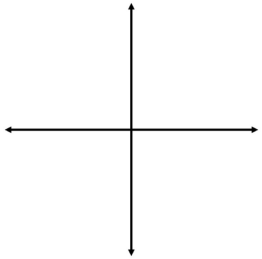
a) Label the vertex, the zeros and the axis of symmetry.

b) What is the domain and range?



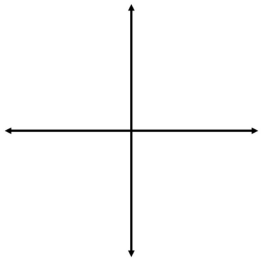
Sketch

1-3. *Sketch* a graph of a parabola that points up and has one zero.

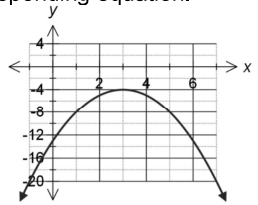


1-4. *Sketch* a graph of a parabola with an axis of symmetry of x = 3 and has two zeros, one of which is 0.

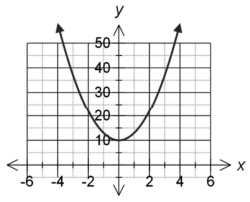
Sketch



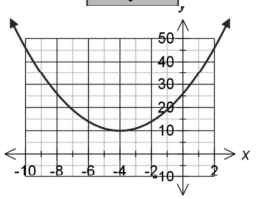
1-5. There are four parabolas graphed below. Match each of them to their corresponding equation.



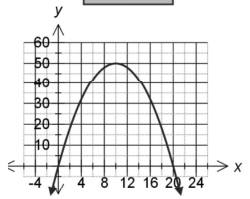
Graph 1



Graph 2



Graph 3



Graph 4

a) $y = -0.5x^2 + 10x$ b) $y = 3x^2 + 10$

c) $y = x^2 + 8x + 26$ d) $y = -x^2 + 6x - 13$

1-6. A toy rocket's height after being launched on is given by the following formula:

$$h(t) = -4.9t^2 + 24t ;$$

where h(t) represents height in metres and t represents the number of seconds.

a) When does the rocket reach a maximum height? What is the maximum height?
b) After how many seconds does the rocket reach a height of 18 m?
c) How long is the rocket in the air?

QUADRATIC REGRESSION

2-1. A hockey team keeps records that indicate the revenue they earn and the ticket price they charge for the last several home games. The results are in the table:

Ticket Price ($)	Revenue ($)
15	11 200
18	17 000
25	26 000
30	30 000
33	31 100
38	30 800
45	26 000

a) Find the quadratic regression equation modeling the revenue the team earns at different ticket prices.
b) How well does the data model a quadratic fit?
c) At what price is revenue maximized? What is the maximum revenue?
d) The team wants to set a price of $50 for a certain game. What revenue can they expect?
e) Explain limitations with the model.

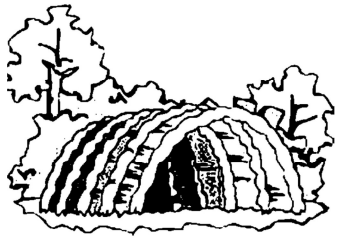
2-2. A farmer has a rectangular garden that he wishes to enclose with 200 m of fencing.

a) Use the table below to fill in 5 possible pairs of lengths and widths and their corresponding areas.

Length (m)					
Width (m)					
Area (sq m)					

b) Find the quadratic regression equation that models the length of the garden and the area enclosed.
c) What dimensions maximize the area?

2-3. The entrance way to a tunnel is shown. The far left of the tunnel opening (on the ground) is said to be the origin
(*ie*: the point (0, 0)).



Here is more information:

- The height of the tunnel 3 feet to the right of the origin is 4 ft.
- The height of the tunnel 4.5 feet to the right of the origin is 5.50 ft.
- The height of the tunnel 7 feet to the right of the origin is 7.50 ft.

You need at least 3 points to get a quadratic regression.

a) Find the quadratic regression equation that models the height, h, of the tunnel (h), x feet to the right of the origin. *ie*: h= f(x)
b) How wide is the tunnel?
c) What is the maximum height of the tunnel?
d) An object that is 8 feet wide and 8 feet tall is to be pulled through the tunnel. It cannot be tipped. Will it fit? Justify your answer.

2-4. A tour boat experiments with their pricing during their last 6 day cruises. The results are in the table below:

Price ($)	10	15	18	25	30	40
# Tickets Sold	50	40	35	25	20	8
Revenue						

a) Fill in the revenue portion of the table.
b) Using the Price and the Revenue, find the quadratic regression equation.
c) What price maximizes the revenue? What is the maximum revenue?
d) What factors, other than price, may affect ticket sales?

2-5. A canon is used to launch a stuntman at a fair.
The following heights of the stuntman are observed over prescribed times.

Time (s)	0.1	0.4	1.1	2.7	4.2
Height (ft)	19.8	40.0	68.3	88.1	30.2

a) Find the quadratic regression equation that relates the height of the stuntman versus time.
b) What is the height of the stuntman at 1.5 s?
c) At what time is the stuntman at a height of 50 feet?
d) What is the maximum height of the stuntman? After how many seconds does he reach that height?
e) After how much time does the stuntman return to the ground?

CUBIC FUNCTIONS

Third degree polynomial functions

3-1. Consider the cubic function **$y = 5x^3 - 3x^2 - 4x - 7$**.

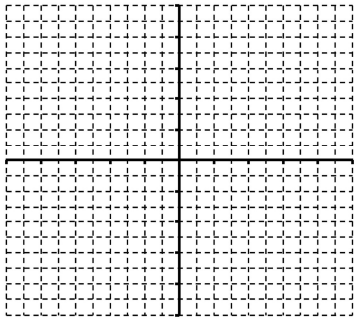
a) What, if any, are the coordinates of the relative maximum and minimum points?

b) What are the coordinates of the x-intercept(s)?

c) What are the coordinates of the y-intercept?

d) Discuss the end behavior.

e) Graph the function. Label all key significant points found in a) – c).



3-2. Consider the cubic function **$y = {}^{-}3x^3 + 4x^2 + 6x - 5$**

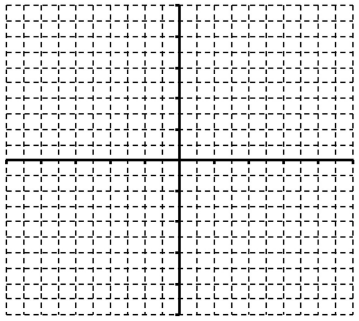
a) What, if any, are the coordinates of the relative maximum and minimum points?

b) What are the coordinates of the x-intercept(s)?

c) What are the coordinates of the y-intercept?

d) Discuss the end behavior.

e) Graph the function. Label all key points found in a) – c).



3-3. What are the zeros of the function **$y = x^3 + x^2 - 5x - 10$**? Round all answers to two decimal places.

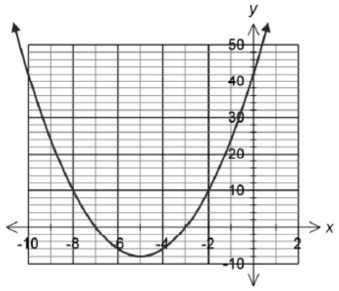
3-4. You have the following data:

k	-3	0	3	5	8
P	-6.00	1.75	-0.90	3.10	34.90

a) Find the cubic function of best fit for the data.
b) How well does a cubic function model the data?
c) Find the coordinates of the relative minimum.

REVIEW OF POLYNOMIAL FUNCTIONS

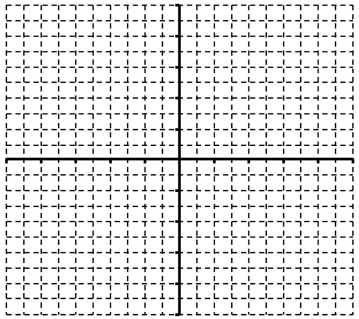
4-1. Given the graph



a) State the vertex.
b) State the axis of symmetry.
c) State the values of the intercepts.
d) State the domain and range of the function.

work

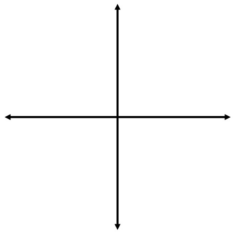
4-2. Represent a function:



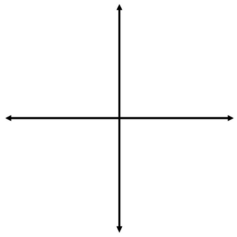
a) Graph $y = -3x^2 - 12x - 6$. Label the vertex, the zeros and the axis of symmetry.

b) State the domain and range.

4-3. Sketch a graph of a parabola that has no zeros. Give its equation.

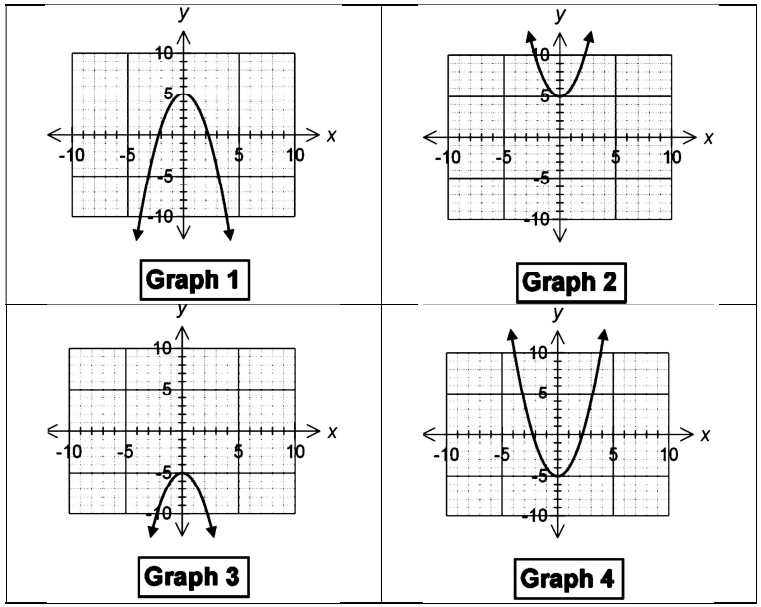


4-4. Sketch a graph of a parabola zeros 4 and -3 with a minimum value of -12.



4-5. There are four parabolas graphed below. Match each of them to their corresponding equation.

A. $y = x^2 - 5$ B. $y = -x^2 - 5$ C. $y = -x^2 + 5$ D. $y = x^2 + 5$



4-6. A cannon shoots a cannonball! The cannonball's height after being shot is given by the following formula:

$$h = -4.9t^2 + 100t + 50$$

where **h** represents height in metres and **t** represents the number of seconds.

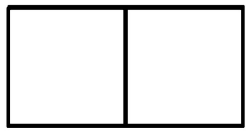
a) From what initial height is the cannonball fired?
b) **When** does the cannonball reach a maximum height? **What** is the maximum height?
c) How many seconds is the cannonball above a height of 300 m?
d) How long is the cannonball in the air?

4-7. A movie theatre wants to determine the price they should charge for a movie ticket to maximize the revenue. They experiment with different prices and the revenue earned. The results are in the table.

Ticket Price ($)	Revenue ($)
5.00	660.00
5.50	693.00
6.00	720.00
6.50	741.00
7.25	761.25
8.50	765.00
9.00	756.00

a) Find the quadratic regression equation modeling the revenue the theatre earns at different ticket prices.
b) How well does the data model a quadratic fit?
c) At what price is revenue maximized? What is the maximum revenue?
d) If the theatre charges $15 for a particular movie, what revenue should they expect? How many people purchased tickets at that price?

4-8. A man has 200 m of fencing he can use to make the following rectangular shape to enclose animals.

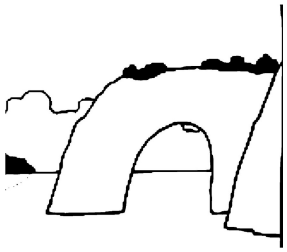


a) Use the table below to fill in 5 possible pairs of lengths and widths and their areas.

Length (m)					
Width (m)					
Area (sq m)					

b) Find the quadratic regression equation that models the length of the garden and the area enclosed.
c) What dimensions maximize the area?

4-9. The inner part of a rock formation in a body of water forms a parabolic shaped arch, as shown below. Here is information about the arch:

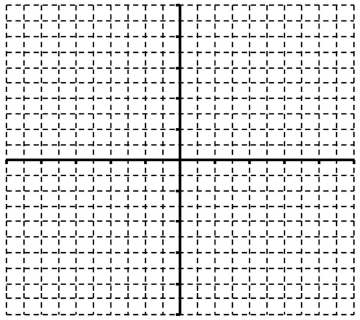


- The height of the arch above the water 4 feet from the leftmost part of the arch is 20.8 feet
- The height of the arch above the water 7 feet from the leftmost part of the arch is 32.2 feet
- The height of the arch above the water 25 feet from the leftmost part of the arch is 25 feet

a) Find the quadratic regression equation that models the height of the arch x feet from the leftmost part of the arch.
b) How wide is the arch?
c) A boat is travelling down the middle of the arch. The top of the boat is 10 feet off the surface of the water. What is the maximum height of the flag that the boat can have to fit under the arch?

4-10. Consider the cubic function $y = 2x^3 + 8x^2 - 22x - 60$.

a) What are the coordinates of the relative maximum and minimum points?
b) What are the coordinates of the x-intercepts?
c) What are the coordinates of the y-intercept?
d) Discuss the end behavior.
e) Graph the function. Label all key points found in a) – c).



WORKSPACE:

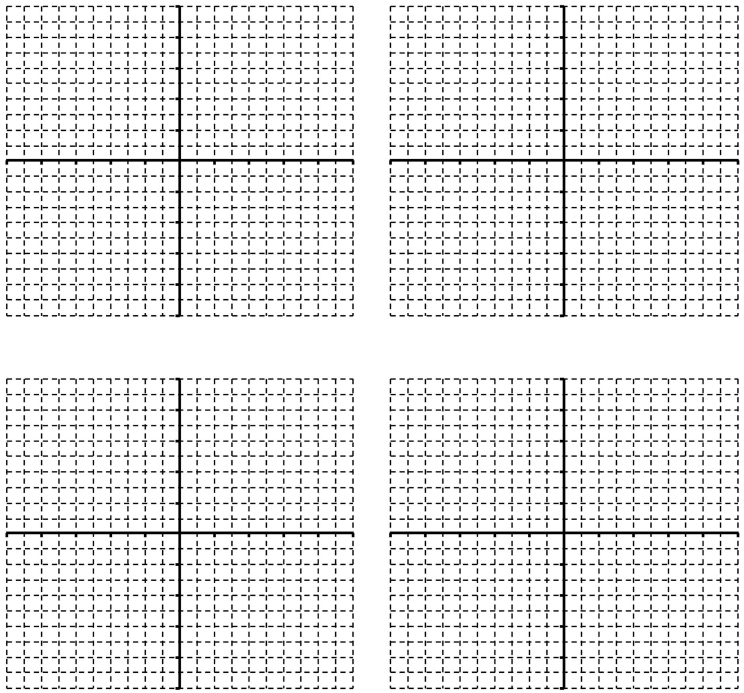
4-11. What are the zeros of the function $y = x^3 - 3x^2 + x - 3$? Round all answers to two decimal places.

4-12. You have the following data. Assume a is the independent variable. [instead of x and y, we are using a and b; big deal]

a	-0.8	3.4	4.4	5	6.8
b	22.5	28.5	-17.0	-36.0	-12.8

a) Find the cubic function of best fit for the data.
b) How well does a cubic function model the data?
c) Find the coordinates of the relative minimum.

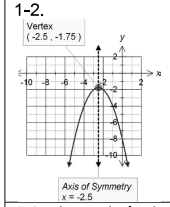
SOME BLANK GRAPH CARTESIAN GRIDS



ANSWER KEY TO UNIT C WORKBOOK

1-1. a) The vertex is (2, 18). b) The axis of symmetry is x = 2.
c) The intercepts are -4 and 8.
d) The domain is at D={x|xεR}; all real numbers.
The range is at R={y|y≤18,yεR}; all real numbers that are less than or equal to 18.

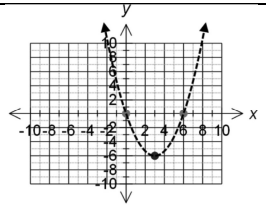
1-2.



b) The domain is at D={x|xεR}.
The range is at R={y|y≤-1.75,yεR}.

1-3. A parabola that points up and has one zero must have its vertex somewhere on the x-axis.

1-4. This question has a few possibilities. The vertex must be somewhere on the given axis of symmetry. You could choose (3, -6) as the vertex. Now, one zero must be at 0. You can notice that the zero is 3 units to the left of the axis of symmetry The other zero will be 3 units to the right of the vertex. This graph is one possibility that has an axis of symmetry at x = 3 and a zero at 0.



One possible graph

1-5 Graph them!
You can see that graph 2 is $y = 3x^2 + 10$ since it is pointing up and has its axis of symmetry on the y-axis, and a vertex of (0,+10)
The only other graph pointing up is $y = x^2 + 8x + 26$ which is graph 3.
The graph pointing down with vertex above the x-axis is $y = -0.5x^2 + 10x$ which is graph 4.
By process of elimination, $y = -x^2 + 6x - 13$ is graph 1.

1-6. a) The rocket reaches its maximum height of 29.39 m after 2.45 s.
b) The rocket reaches a height of 18 m at 0.92 s on its way up and again at 3.97 s on its way down.
c) The rocket is in the air for 4.90 s.

2-1.
a) $y = -30120.4634 + 3513.5578x - 50.3351x^2$
b) The correlation coefficient is 0.9996, and so the model fits very well.
c) A ticket price of $34.90 maximizes the revenue. The maximum revenue is $31 194.05.
d) The revenue expected at a price of $50 is $19,719.61.
e) Other factors than ticket price may affect the revenue. Examples include how well the team is playing, the opponent in the game, other activities going on in the city, etc.

2-2. b) $y = 100x - x2$ c) The dimensions of the rectangle that maximizes the area is 50 m by 50 m.

2-3. a) The quadratic regression equation is **$y = 0.0053 + 1.5119x - 0.0632x^2$**
b) The difference between the zeros will be the width of the tunnel
Zeros = -0.00351,23.92597
Width = 23.92597 - (-0.00351) = 23.92948 = 23.93 m
c) The maximum height of the tunnel is 9.05 ft at a distance of 11.96 feet to the right of the origin.

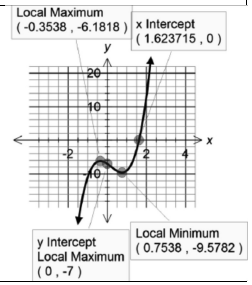
2-4.

Price ($)	10	15	18	25	30	40
# Tickets Sold	50	40	35	25	20	8
Revenue	500	600	630	625	600	320

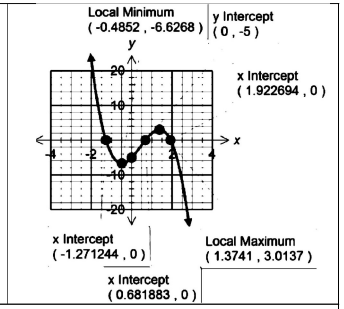
b) $y = 153.0281 + 44.7993x - 1.0136x^2$
c) The maximum point is (22.10, 648.04). The price that maximizes the revenue is $22.10. The maximum revenue is $648.04.
d) Weather is a big factor that would influence the number of customers for a boat cruise. Another factor would be time of year. A really nice fall day may not get many customers because many people might not be aware of the availability of a cruise.

2-5. a) The quadratic regression equation that fits the data is
$y = 13.3617 + 68.8419x - 15.4149x^2$
b) To find the height of the stuntman, plug in 1.5 for x. The height of the stuntman after 1.5 s, f(1.5), is 81.94 ft.
c) The stuntman is at a height of 50 feet at 0.62 and 3.85 seconds.
d) The maximum height of the stuntman is 90.22 ft at a time of 2.23 seconds.
e) The stuntman returns to the ground after 4.65 seconds.

3-1.
a) Relative Maximum: (-0.35, -6.18)
Relative Minimum: (0.75, -9.58)
b) x-intercept: (1.62, 0)
c) y-intercept: (0, -7)
d) For large negative values of x, large negative values of y are produced. For large positive values of x, large positive values of y are produced. The function is increasing to the right and decreasing to the left. [increasing from left to right with a couple humps in the middle!]



3-2. a) Relative Maximum: (1.37, 3.01)
Relative Minimum: (-0.49, -6.63)
b) x-intercepts: (-1.27, 0), (0.68, 0), (1.92, 0)
c) y-intercept: (0, -5)
d) For large negative values of x, large positive values of y are produced. For large positive values of x, large negative values of y are produced. The graph is decreasing right and increasing to the left.



3-3. The only zero is at x = 2.53.

3-4. $y = 0.14x^3 - 0.58x^2 - 0.44x + 1.74$
b) With a correlation coefficient of 0.9999…, the data very much follows the cubic model.
c) relative minimum: (3.10, -1.03)

FINAL REVIEW ANSWERS

Consult teacher for the final few review answers for the Polynomial Functions (or find them yourself!)