

# Grade 12 Essentials

Key

23

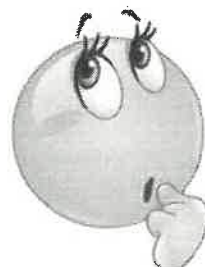
+ 14 BONUS

## Week 8 Quiz

### Solutions Debrief

MrF

24-05-23



MrF

**GRADE 12 ESSENTIAL  
WEEK 8 QUIZ 24-05-23**

Name: \_\_\_\_\_  
Date: \_\_\_\_\_

#### **OPEN BOOK; Take-Home.**

Round all decimal amounts to the nearest 0.01 unless otherwise indicated (standard).

Show work, show method. Generally, just *stating* an answer gets zero marks. Part marks possible

Time Limit: Due: Monday 10:00; absolutely no later than  
Collaborate with other students if you want! Late Penalty Applies

It is possible to get up to 150% on this quiz if you try the Bonus Questions!

Lucky for students this is a take home quiz due to the TV filming in the Classrooms. It is modified, made it a little more challenging, but five nights plus an at-home-study Friday, to complete it! Worth up to 150% again!

# Make sure your cheat sheet is up to snuff! Could be up to an extra 5% of course mark IF perfect

**Teacher's GRADE 12 ESSENTIAL - STUDY NOTES / CHEAT SHEET**

1. Study Notes (cheat sheet) Do your own, do not copy out of book or hand in. Evaluate content of each unit of course. Do not copy out of book or hand in.

**Algebra** - Unit 1: Linear Equations, Unit 2: Quadratics, Unit 3: Functions, Unit 4: Probability, Unit 5: Statistics

**Probability** - Odds in Favor (F) =  $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$   
Odds Against (A) =  $\frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}$   
E.g. 30% prob = 3:70 prob = 3:70 odds = 3:70 Odds in Favor = 3:70 Odds Against = 70:30

**Expected Value**  
EV = (Prin) \* (Net Gain) - (Prin) \* (Loss) If negative you lose that amt on average every day. Occasionally you may have multiple prizes.  
EV = (Prin) \* (Net Gain) + (Prin) \* (Net Gain) - (Prin) \* (Loss)

**VEHICLE FINANCE**  
Average Vehicle Price = Dealer price after tax, freight, license, etc. = Trade in for factor  
Car cost more than 20% of your gross income -> don't buy it. Max 10% max 40%  
Vehicle Finance: TDSP = Total Debt Service Ratio =  $\frac{\text{Total Monthly Payments}}{\text{Gross Monthly Income}}$   
Monthly Amount =  $\text{Prin} \cdot i \cdot \frac{1}{1 - (1+i)^{-n}}$  =  $\text{Monthly Amount} \cdot 25 \cdot 12$   
Final Value = Original Value \*  $(1 - \text{annual depreciation rate})^n$  Original Value does not include tax. E.g. \$20,000 \* 0.85 = \$17,000 for 15% depreciation after 12 yrs

**Overall Cost of Car** = Total Cash Payments + Down Payment  
Interest Paid = Total Cash Amount Paid - Amount Borrowed  
Cash cost = 32 weekly payments \* 26 to weekly payments  
Fuel Economy expressed as ratio. E.g. 20 km/L. Example:  $\frac{20 \text{ km}}{1 \text{ L}}$  where 20 is the number of km for 100 km. Should be approximately around 5 to 10 km/L for a normal family car. Fuel cost = 100 km \* 1.25 km/L = 80 L. 80 L \* \$1.20/L = \$96.00 + 3.33 tax

**STATISTICS**  
Mean:  $\bar{x} = \frac{\sum x_i}{n}$  sum up all the data and divide by the data set size, n  
Weighted Mean:  $\frac{\sum (x_i \cdot w_i)}{\sum w_i}$  where  $w_i$  is the weight of each data point  
Median: Line data up in ascending order, find the data value of the middle point.  
Mode: Most frequent value. E.g. in 12 data, 4 middle place is the 8th place. With 20 data, 4 middle place is the mean between the 10th and 11th place, which is 10 and a half place.  
Percentage Mark:  $\frac{\text{Number of correct answers}}{\text{Total number of questions}} \cdot 100$  round up, where B is the number of correct answers & Q is the total number of questions.  
Percentages are Quotient Fractions:  $\frac{P}{100} = \frac{A}{B} \Rightarrow P = \frac{A}{B} \cdot 100$   
Note: some references specify the tax rate = 10% if n is larger than 8 is small.

**HOUSE FINANCE**  
Check Debt Service Ratio: Monthly Mortgage Payment / Monthly Gross Income  
CDR =  $\frac{\text{Monthly Mortgage Payment}}{\text{Monthly Gross Income}}$  10% max 32%  
Cannot spend more than 32% of your gross income on a house, whether

**Property Tax**  
Property Assessment = Property Assessment \* (Portion Percentage)  
Property Tax = Portion Assessment \* Mill Rate = (\$20000 \* 0.005) = \$1000  
Mill Rate =  $\frac{\text{Property Tax}}{\text{Assessed Value}}$  =  $\frac{1000}{20000} = 0.005$  = 5 mills = 5/1000 = 0.5%  
Property Municipal Mill Rate for Police, Fire and Paramedics, snow removal, park, library, etc.  
School Mill Rate is applicable on same tax base by school board, same calculation method.  
- \$200 to \$1,000 sometimes applied.

**PROBABILITY**

**GEOMETRY AND TRIGONOMETRY**

Sum of Interior Angles of a Polygon:  $S = (n - 2) \cdot 180^\circ$  where n is the number of sides of the polygon.  
Central Angle of Regular Polygon Sector:  $C = \frac{360^\circ}{n}$  where n is the number of sides.  
Number of Diagonals in a Polygon:  $D = \frac{n(n-3)}{2}$  where n is the number of sides.

**SOH CAH TOA for Right Angle Triangles**  
Sin A =  $\frac{\text{Opposite}}{\text{Hypotenuse}}$  Cos A =  $\frac{\text{Adjacent}}{\text{Hypotenuse}}$  Tan A =  $\frac{\text{Opposite}}{\text{Adjacent}}$   
Pythagoras:  $a^2 + b^2 = c^2$   
Cosine Law for side a given two sides & angle B:  $a^2 = b^2 + c^2 - 2bc \cos(A)$  etc.  
Cosine Law for angle A given three sides:  $\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$  etc.

**Trigonometry Side-Loss**  
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Geometric Formulas**: Circumference of a circle =  $2\pi r$   
Area: Rectangle and Parallelogram both:  $A = \text{base} \cdot \text{height}$   
Sum of four sides areas: SA square =  $2a^2$ , SA rect =  $2ab$   
Volume: Volume = Area of Base \* Height =  $a^2 h$

**Tax Rates**  
15% 8% (Mortgage) 17% GST 5% PST  
Pay taxes every time you spend money.

## Cheat Sheet Marking Rubric As discussed weekly since September!

Item	Weight Factor of Item
Covers each unit studied	20%
Lists formulae used for each unit, some with example usage	20%
[not necessary to write out Unit Conversions, or Geometric Shapes formulae such as cones, prisms. Do not copy out Loan Tables, etc)	20%
Uses diagrams where appropriate, graphic organizers, etc. Pictures are worth a 1,000 words.	20%
Is accurate and correct	20%
Is neat and readable	20%

1. Determine the measure of length a.

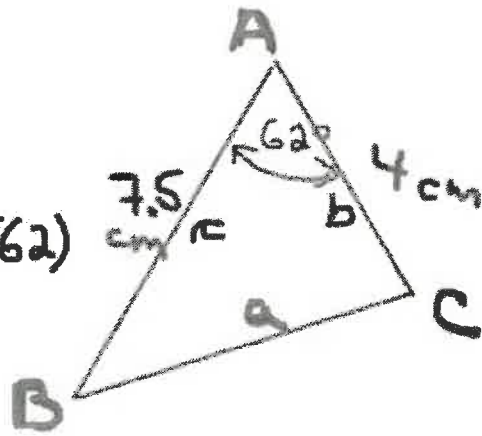
4

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$$

$$a^2 = 4^2 + 7.5^2 - 2 \cdot 4 \cdot 7.5 \cdot \cos(62)$$

$$a^2 = 44.0817 \dots$$

$$a = \sqrt{\text{ANS}} \approx 6.64 \text{ cm}$$



show units!

- Label Diagram
- Select Formula
- Write Down Formula
- Plug in Numbers
- Solve
- Check

$$4^2 + 7.5^2 - 2 \cdot 4 \cdot 7.5 \cdot \cos(62)$$

$$= 44.08170628$$

$$\sqrt{44.08170628}$$

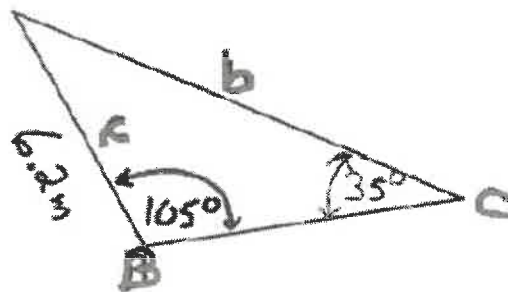
$$= 6.639405563$$

Trigonometry Cosine Law:

Cosine Law for side a across from angle A:  $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(A)$ ; etc

2. Determine the measure of side b.

Two angles on a side:  
SINE LAW  
angle & side pairs



4

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\cancel{\sin(105)} \cdot \frac{b}{\cancel{\sin(105)}} = \frac{6.2}{\sin(35)} \cdot \sin(105)$$

$$b = \frac{6.2 \cdot \sin(105)}{\sin(35)}$$

$$\frac{b}{\sin(105)} \rightarrow \frac{6.2}{\sin(35)}$$

$(b = 10.44 \text{ m})$  TLAR.

show units!

$$6.2 \cdot \frac{(\sin 105)}{\sin(35)}$$

$$= 10.4410$$

- Label Diagram ✓
- Select Formula ✓
- Write Down Formula ✓
- Plug in Numbers ✓
- Solve ✓
- Check ✓

MRF

Show work 2

3. Statistics. Determine the mean, median, and mode of the data set. (4 marks)

Mean,  $\bar{x} = \frac{\sum x}{n} = \frac{36}{7} = 5.14$  {1, 3, 4, 4, 8, 4, 12}

Median,  $\tilde{x} = 4$  {1, 3, 4, 4, 8, 4, 12}  $\frac{(7+1)}{2} = 4^{th}$  element

Mode = 4 is the most frequent

Range =  $x_{max} - x_{min} = 12 - 1 = 11$

4

STATISTICS

Mean,  $\bar{x} = \frac{\sum x_i}{n}$ ; sum up all the data and divide by the data set size, n

Weighted Mean:  $\frac{\sum(x_1 \cdot wf_1 + x_2 \cdot wf_2 + x_3 \cdot wf_3 + \dots)}{\sum wf_i} = \frac{\sum x_i f_i}{\sum f_i}$

Median,  $\tilde{x}$ . Line data up in ascending order, find the data value at the middle place.

Middle place =  $\frac{(n+1)}{2}$ . Eg: n = 17 (data) -> middle place is the 9<sup>th</sup> place. With 20 data -> place is the mean between the 10<sup>th</sup> and 11<sup>th</sup> place, value in 10 and a 'halfth' place.

my cheat sheet

4. Statistics- Percentile Rank. Josh was writing a qualification exam for a desirable government job. He got 45 marks out of a possible 78 marks on the exam. 345 applicants wrote the exam, 288 got a worse exam score than Josh, and six others had the same score as Josh.

a. Determine Josh's mark on the qualification exam as a percent.

b. Determine Josh' Percentile Rank on the examination.

c. Explain (using proper grammar) whether you think the exam was likely easy or difficult.

a)  $45/78 = x/100$ ;  $x = 57.69$ ;  $\frac{57.69}{100} = 57.69\%$

b)  $PR = \frac{B + \frac{1}{2}(E)}{N} \cdot 100 = \frac{288 + \frac{1}{2}(6)}{345} \cdot 100 \uparrow = 85$   $P_{85}$

c) Josh had a fairly low mark on the exam, but 85% of student were the same or worse than him, so the exam must have been pretty difficult! Josh was in the top 15% of those who wrote, despite the poor exam mark.

85<sup>th</sup> place

7

Percentile Rank.  $PR = \frac{B + \frac{1}{2}E}{N} \cdot 100$  round up, where B is the number of scores below number equal; and N is the total number. Percentiles and Quartile Ranks.  $P_{25} = Q_1$ ;  $P_{50} = Q_2 = \text{Median}$ ;  $P_{75} = Q_3$ . my cheat sheet

5. **Problem Solve – Use a Table – Follow a Pattern.** A frog is on a lily pad, it eats one fly on the first lily pad. It hops to a second lily pad and eats three more flies than eaten on the first lily pad, then it jumps to a third lily pad and eats three more than the previous lily pad, and so on, so that at every lily pad it eats three more flies than the previous lily pad.

Complete the table:



Lily Pad	1	2	3	4	5	6	7	8	9	10
Flies	1+3	4+3	7+3	10+3	13+3	16+3	19+3	22+3	25	28
Total Eaten	1	5	12	22	35	51	70	92	117	145

State the answers to the following: (1 mark each)

a. How many flies total will the frog have eaten when he has eaten the flies on the 8<sup>th</sup> lily pad? Answer: 92

b. On which lily pad will it have eaten its 50<sup>th</sup> fly? Answer: 6<sup>th</sup> lily pad

Follow the pattern  
Step-by-step

6. **Probability.** The probability of a dog successfully performing a trick is 75%. Determine the **odds against** the dog performing the trick successfully.

$$P(\text{success}) = \frac{\# \text{ of outcomes that are success}}{\# \text{ of total outcomes}} = \frac{\# \text{ of successes}}{\# \text{ of tries}}$$

$$P(\text{success}) = \frac{75}{100} = \frac{3}{4}$$

$$\frac{3 \text{ successes}}{4 \text{ tries}}$$

Odds against  
 $1 : 3$   
 Not Success : Success

Just like in sports betting, if it is a sure thing it will not have a good payout!

**PROBABILITY**

$$\text{Prob of Event A} = \frac{\text{favoured outcomes}}{\text{total possible outcomes}}$$

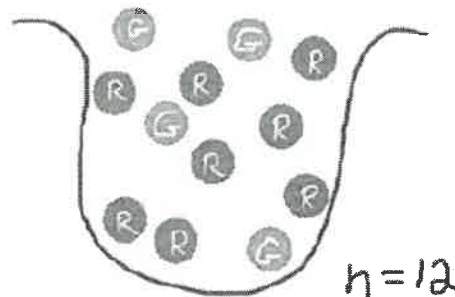
Odds in Favour (For). Favoured : Unfavoured ; (wins : no wins); (success : failure)

Odds Against. unfavoured : favoured

Eg 30% prob = 3/10 prob = 0.3 prob → 3:7 Odds in Favour → 7:3 Odds Against

$$\frac{75}{100} = \frac{3}{4}$$

7. **Probability.** A bag contains 4 Green marbles and 8 Red marbles. Determine the probability of drawing a Red Marble. [Prob(Draw Red)]. Express the answer as a reduced fraction and as a %.



$$P(\text{Draw Red}) = \frac{\# \text{ of Red}}{\# \text{ Total marbles}}$$

$$= \frac{8}{12} = \left(\frac{2}{3}\right) = \left(66.67\%\right)$$

7

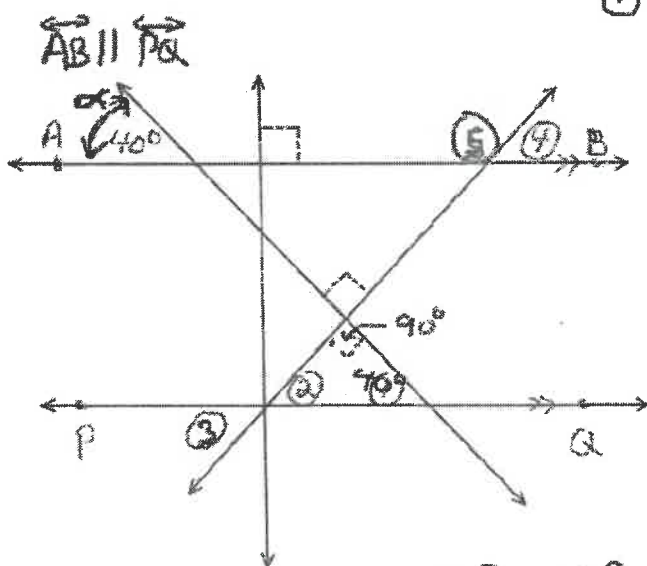
Prob of Event A =  $\frac{\text{favoured outcomes}}{\text{total possible outcomes}}$

$\frac{8}{12}$	
	$= \frac{2}{3}$
	$= 0.6666666$

**MORE BONUS QUESTIONS**

(Try them! It is possible to get 150% on this quiz)

**BONUS: Euclidian Geometry (1 mark each)**



(1) =  $40^\circ$  since corresponds with  $\angle A$

(2) =  $50^\circ$  Triangle Sum Theorem

(3) =  $50^\circ$  since a vertical (opposite) angle to (2)

(4) = (2) "corresponds" =  $50^\circ$  with angle (2)

(5) =  $180^\circ - (4)$   
 $= 180^\circ - 50^\circ = \underline{130^\circ}$

STATE the measure of angles (1) =  $40^\circ$  (2) =  $50^\circ$

(3) =  $50^\circ$  (4) =  $50^\circ$  (5) =  $130^\circ$

Since (4) + (5) are supplementary and make a linear pair

5

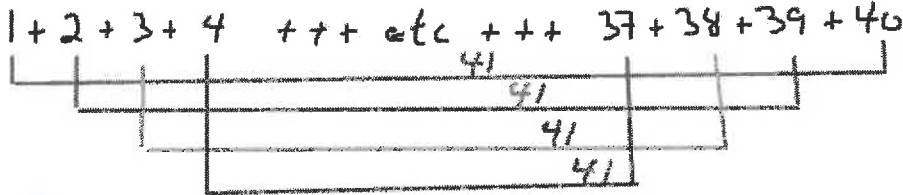
2 + 5  
~~5~~

Bonus: (1 Mark)

Show work

Determine the sum of the counting numbers from 1 to 39

Strategy: Easy to find the sum from 1 to 40, then just subtract the 40!



See a pattern?

Pairing up the 'bookends', how many pairs?

$$20 \cdot 41 = 820$$

Sum of the numbers from

$$1 \text{ to } 40 \text{ is } 820$$

$$\begin{array}{r} 820 \\ - 40 \\ \hline 780 \end{array}$$

Not there

Several other logical solutions have also been demonstrated

```

1+2+3+4+5+6+7+8+
9+10+11+12+13+14
+15+16+17+18+19+
20+21+22+23+24+2
5+26+27+28+29+30
+31+32+33+34+35+
36+37+38+39
780

```

Bonus. Conversion (2 Marks)

Convert two weeks into seconds. 2 wk = \_\_\_\_\_ sec

$$2 \text{ wk} \cdot \frac{7 \text{ days}}{1 \text{ wk}} \cdot \frac{24 \text{ hr}}{1 \text{ days}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$$

$$= 1,209,600 \text{ sec}$$

One million, two hundred nine thousand, six hundred seconds

$$2 \text{ wk} \cdot \frac{7 \text{ day}}{\text{wk}} = 14 \text{ days}$$

easier?

$$14 \text{ days} \cdot \frac{24 \text{ hr}}{\text{day}} = 336 \text{ hr}$$

$$336 \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 20,160 \text{ min}$$

$$20,160 \text{ min} \cdot \frac{60 \text{ (sec)}}{1 \text{ min}} = 1,209,600 \text{ Sec}$$

F3

**Using a Formula.** (2 Marks) The formula to convert degrees Celsius [ $^{\circ}\text{C}$ ] to the American degrees Fahrenheit [ $^{\circ}\text{F}$ ] is given by the formula:

$$^{\circ}\text{F} = \frac{9}{5} \cdot ^{\circ}\text{C} + 32;$$

Back in Grade 11

Convert 25 degrees Celsius ( $25^{\circ}\text{C}$ ) into degrees Fahrenheit  $^{\circ}\text{F}$ :

$$25^{\circ}\text{C} = \boxed{77}^{\circ}\text{F}$$

$$^{\circ}\text{F} = \frac{9}{5} \cdot ^{\circ}\text{C} + 32$$

$$^{\circ}\text{F} = \frac{9}{5} \cdot 25 + 32 = 45 + 32 = \boxed{77^{\circ}\text{F}}$$

2

M/F

6

**Bonus - Using a Formula.** The [Pre-Calculus] formula to count the total number of flies eaten in the question above is given by:

$S_n = \frac{n}{2}[2a_1 + (n-1) \cdot d]$ , where  $S_n$  is the total sum of flies eaten,  $a_1$  is the number of flies on the first lily pad,  $n$  is the number of the lily pad, and  $d$  is the amount by which each successive lily pad's fly count increases;  $d = 3$  in this case.

*This is Grade 11 stuff if you ever do the Pre-Calculus or the 'Pure' Math as they call it in other provinces. But Essential Math students know how to blindly plug into a formula!*

In our case:  $S_n = \frac{n}{2} \cdot [2 \cdot 1 + (n-1) \cdot 3]$

evaluate that formula (plug-in) for  $n = 15$  to calculate how many flies were eaten total [ $S_n$ ] after the 15<sup>th</sup> lily pad. [2 marks]

2

After 15 Lilly pads the frog has eaten a total sum of  $\boxed{330}$  flies.

$$S_{15} = \frac{15}{2} \cdot [2 \cdot 1 + (15-1) \cdot 3] = 7.5 \cdot [2 + 42] = 330 \text{ flies}$$

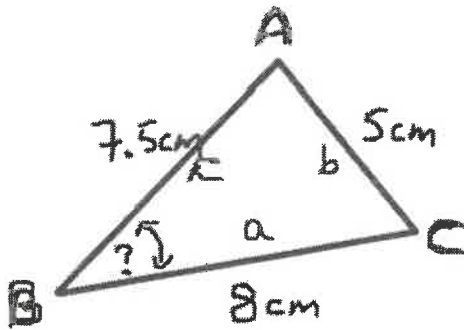
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9	10	11	12	13	14	15
25	28	31	34	37	40	43
117	145	176	210	247	287	<b>330</b>



Bonus: Determine angle B.

Cosine Law  
Since Given 3 sides



$$\angle B = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c} \right)$$

$$\angle B = \cos^{-1} \left( \frac{8^2 + 7.5^2 - 5^2}{[2 \cdot 8 \cdot 7.5]} \right)$$

$$\angle B = \cos^{-1} \left( \frac{95.25}{120} \right)$$

$$\angle B = 37.46^\circ \quad \text{T.L.A.R.}$$

Smallest angle  
has smallest  
side across ✓

- Label Diagram
- Select Formula
- Write Down Formula
- Plug in Numbers
- Solve
- Check

Cheat sheet  
 $\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$ ; etc

$$\cos^{-1} \left( \frac{95.25}{120} \right) = 37.46265$$

**Five nights, with a  
long weekend and at-  
home-study Friday,  
to complete a 60  
minutes quiz**

**Easy 150%**

*Handwritten signature*

**On time, on target**

**BEST BAR NONE**

**LOAD CLEAR !**



**Determined to  
Deliver**