Grade 12 Essential

Quiz Debrief

Week 5

23-10-05

MrF

GRADE 12 ESSENTIAL
QUIZ - WEEK 5
GEOMETRY & TRIGONOMETRY

Name:_____
Date: _____

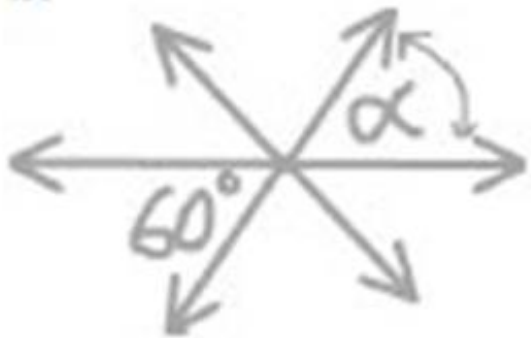
Time limit was 45 mins!

Show work where necessary (almost always).
Use your study sheet (cheat sheet) or mine
As always round decimal answers to the nearest 0.01 unless otherwise indicated.
We went 55
Diagrams here are not necessarily to scale, a ruler and protractor are no use.
Should take 15
Each individual question is worth 2 marks unless otherwise indicated.

1. Determine the measure of each angle, α, using the geometric relations regarding Triangle Sum Theorem, Supplementary Angles, Complementary Angles and the Opposite Angles (vertical angles), and Corresponding Angles in parallel lines 'theorems'. (1 mark each)

a.

b.



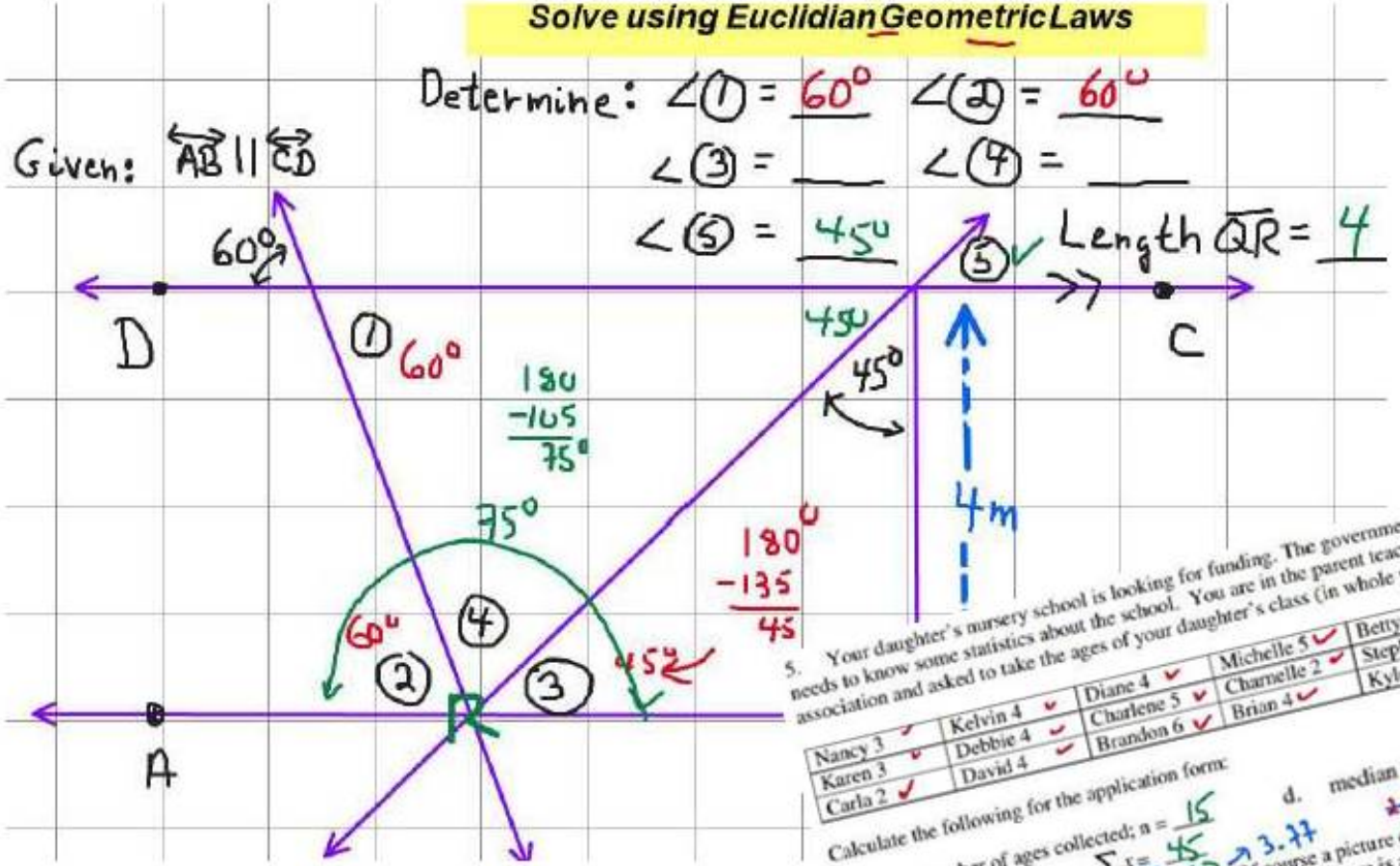
Solve using Euclidian Geometric Laws

Determine: $\angle 1 = 60^\circ$ $\angle 2 = 60^\circ$

$\angle 3 =$ ___ $\angle 4 =$ ___

$\angle 5 = 45^\circ$ Length $\overline{QR} = 4$

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



5. Your daughter's nursery school is looking for funding. The government needs to know some statistics about the school. You are in the parent teacher association and asked to take the ages of your daughter's class (in whole years)

Nancy 3	Kelvin 4	Diane 4	Michelle 5	Betty 3
Karen 3	Debbie 4	Charlene 5	Charnelle 2	Steph 2
Carla 2	David 4	Brandon 6	Brian 4	Kyle 4

19
16
20 / 55

Calculate the following for the application form:

a. number of ages collected; $n =$ 15

b. sum of all the ages, $\sum x =$ 45

c. mean age, $\bar{x} = \frac{\sum x}{n} = \frac{45}{15} = 3$

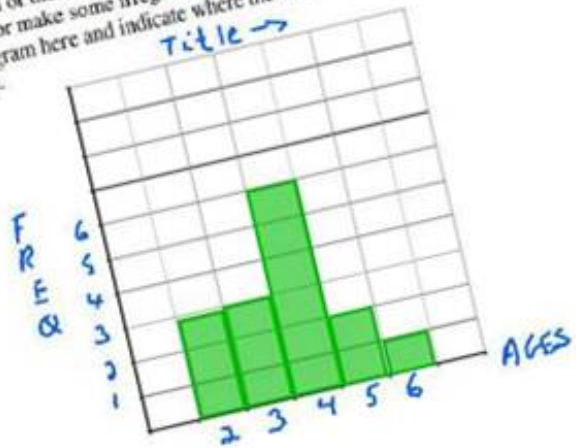
e. age mode(s) = 4

d. median age, $\tilde{x} =$ 4

2, 2, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 6

Of course a picture of your data is always important to see how data is distributed or if there is any 'lopsided-ness' (also known as 'skewed' data) that favours one end or the other of the range of data and that might throw off the central values or make some irregular 'clumping' of data.

Graph a histogram here and indicate where the statistics would be displayed on the histogram.



a.

We had done a warm up just before

STATISTICS

Mean. $\bar{x} = \frac{\sum x_i}{n}$; sum up all the data and divide by the data set size, **n**

Weighted Mean: $\frac{\sum(x_1 \cdot wf_1 + x_2 \cdot wf_2 + x_3 \cdot wf_3 + \cdots)}{(wf_1 + wf_2 + wf_3 + \cdots)} = \frac{\sum x_i f_i}{\sum wf_i}$

Median, $\tilde{x}$. Line data up in ascending order, find the data value at the middle place.

Middle place = $\frac{(n+1)}{2}$. Eg: n= 17 data → middle place is the 9th place. With 20 data → middle place is the mean between the 10th and 11th place, value in 10 and a 'halfth' place.

Percentile Rank. $PR = \frac{B + {}^1/_2 E}{N} * 100$;round up!; where **B** is the number of scores below, E is the number equal; and **N** is the total number.

Percentiles and Quartile Ranks. P_{25} ■ Q_1; P_{50} ■ Q_2 ■ **Median;** P_{75} ■ Q_3.

Note: some references simplify this too $\frac{B}{N} * 100$ ...

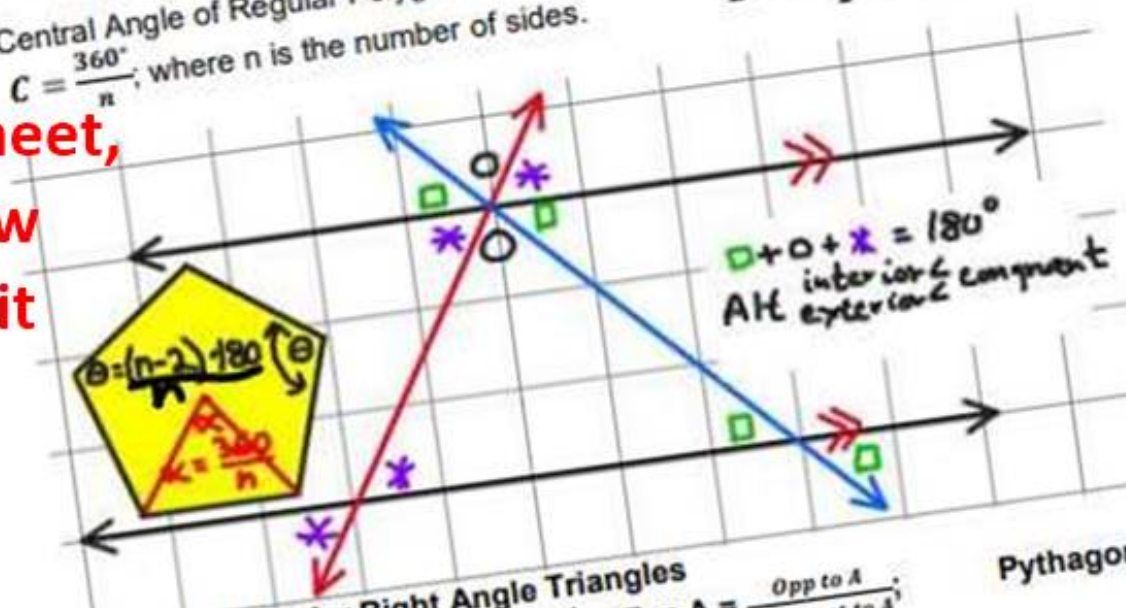
GEOMETRY AND TRIGONOMETRY

Sum of Interior Angles of a Polygon; **S = (n – 2) * 180°**; where n is the number of sides of the polygon

Central Angle of Regular Polygon Sector; **C** $C = \frac{360°}{n}$; where n is the number of sides.

Number of Diagonals in a Polygon; **D** $D = \frac{n(n-3)}{2}$; where n is the number of sides.

You have MY cheat sheet, and perhaps by now you have adapted it to your own!

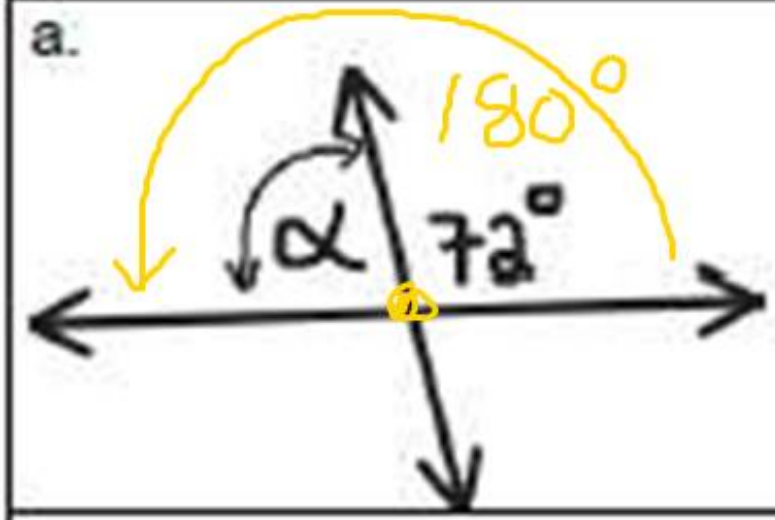


SOH CAH TOA for Right Angle Triangles

Sin A = $\frac{Opp\ to\ A}{Hypotenuse}$; cos A = $\frac{Adj\ to\ A}{Hypotenuse}$; Tan A = $\frac{Opp\ to\ A}{Adjacent\ to\ A}$;

Pythagoras: $c^2 = a^2 + b^2$

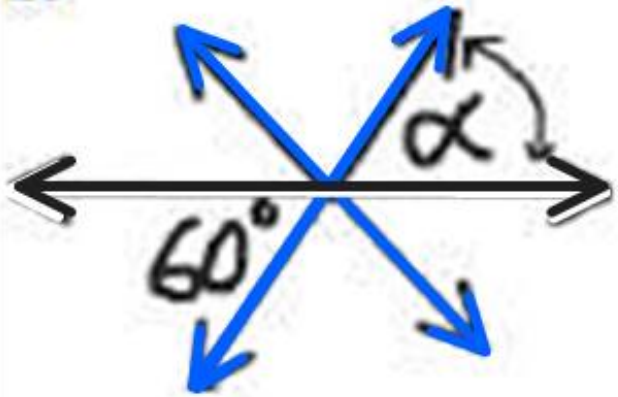
a.



$\alpha = 108^\circ$

since it is "supplementary" to the 72° angle. They make a "linear pair"

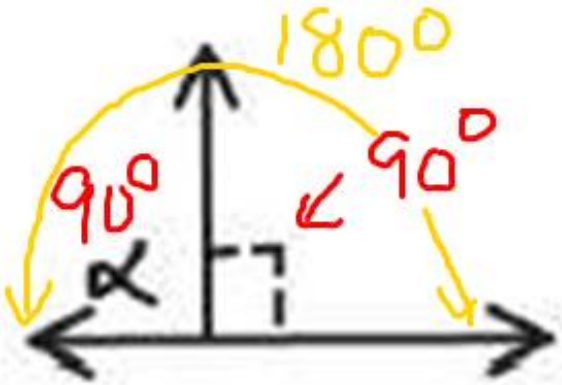
b.



$\alpha = 60^\circ$

since it is "opposite" the 60° of the intersecting lines!

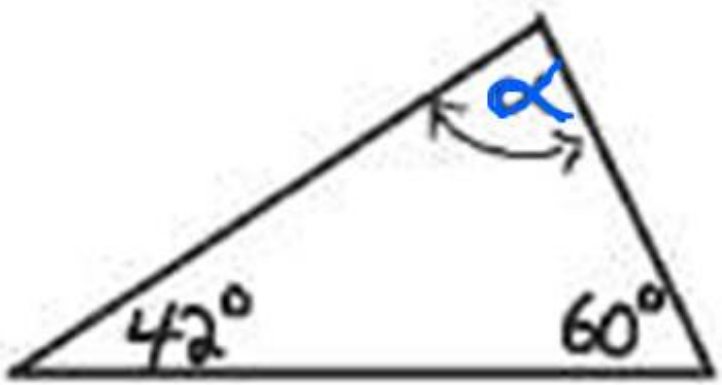
c.



$\alpha = 90^\circ$

Since forms a linear pair, supplementary

d.



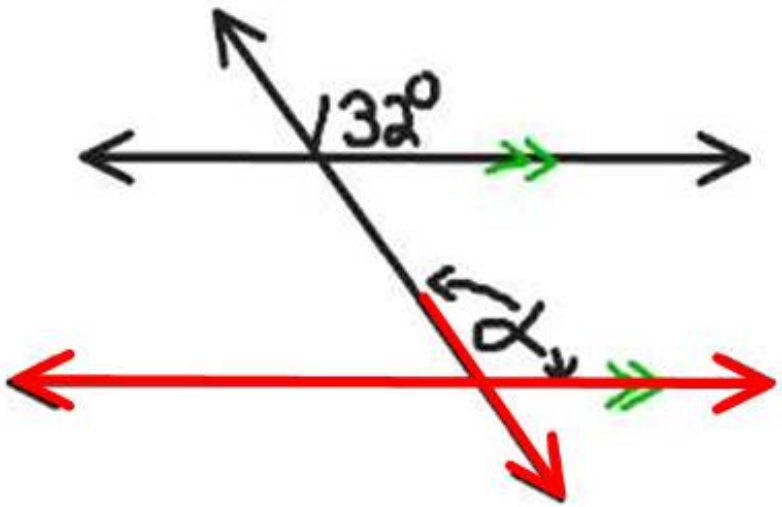
$\alpha = 78^\circ$

SInce Triangle Sum Theorem

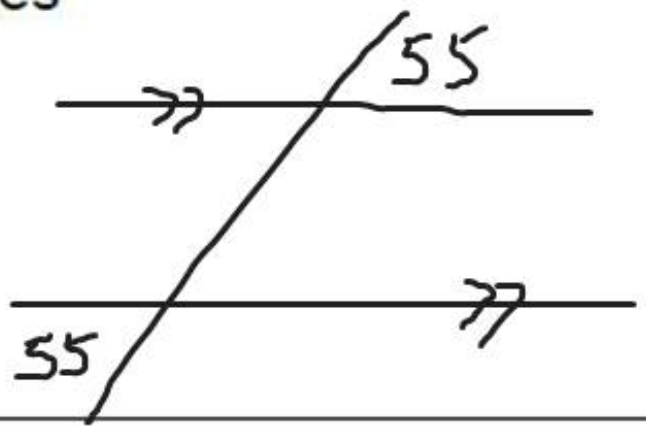
Sum of the inside angles of a triangle equals 180°

$180 - (42+60) = 78^\circ$

e.



f. draw your own interesting geometry relationship that demonstrates one of the rules



$\alpha = 132$

"Corresponding" angles formed by a transversal and parallel lines

Alternate Exterior angles are congruent

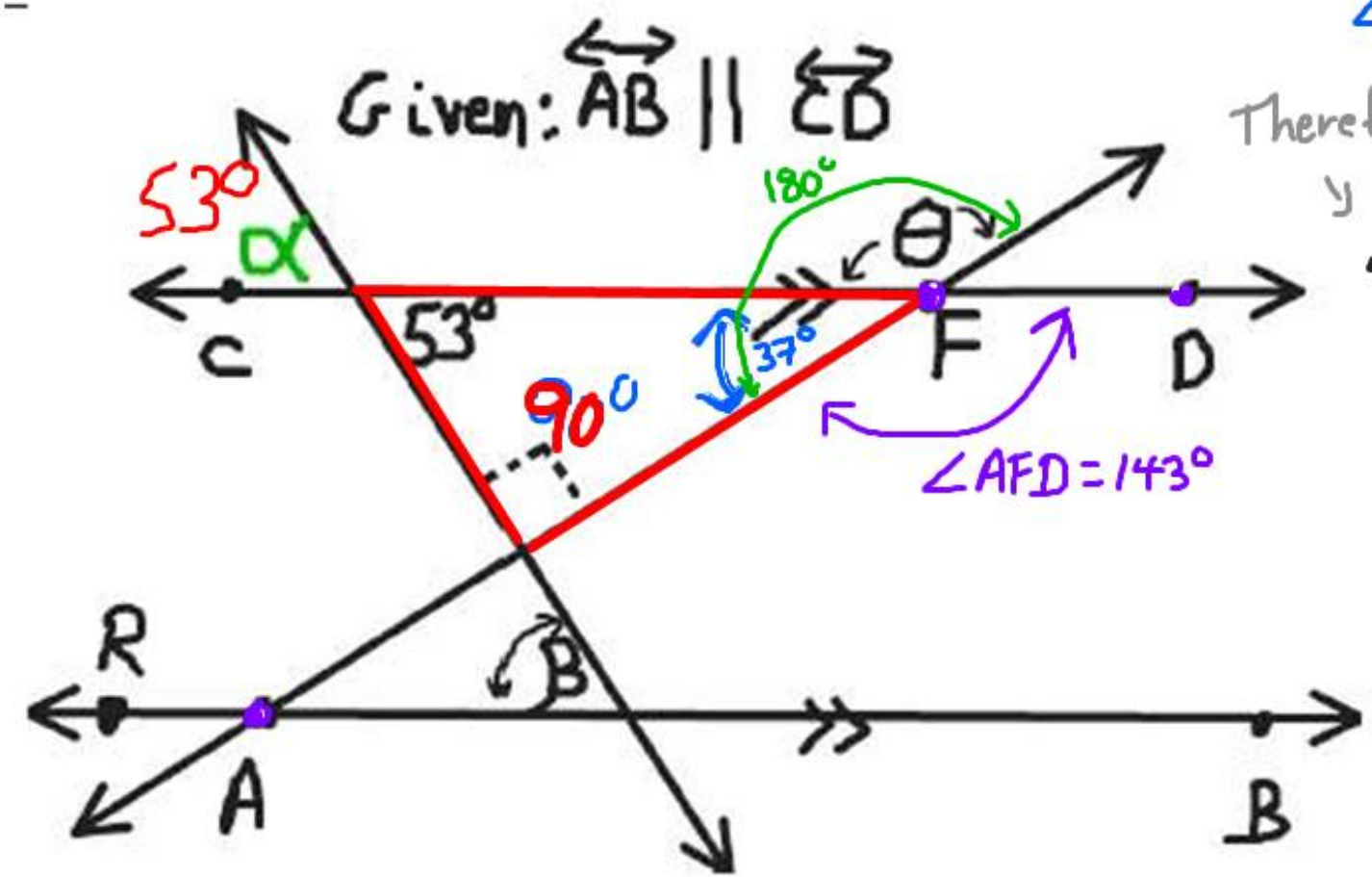
is my example

2. Given $\overline{AB} \parallel \overline{CD}$ (diagram not to scale!)

State the measure of angles: Alternate Interior

α: 53° ; β: 53° θ: 143° ; m∠**AFD**: measure of angle AFD = θ = 143° opposite angle rule

(1 mark each)



∠AFC = 180° - (90° + 53°)
= 180° - 143°
= 37°

Therefore

∴ θ = 180 - 37°
= 143° since it forms a "linear pair with ∠AFC

$$2\times2 \cdot \tfrac{3}{4}\sqrt{126}\cdot\cos 5\beta - \tfrac{527}{15} + \left(\sqrt{\beta\alpha+4}\right)^2 + 618\Pi +$$
$$\sqrt{728} + \alpha\beta x + x^2 y\left[\left(22^3 + 68\right)\cdot xz\right] + \cos\beta + \operatorname{tg} 682$$
$$\gamma^2\sqrt[6]{992} + 4\left(62\gamma\beta z\right) - 85\sqrt[6]{297}\cdot\beta + xyz^2\cdot(6x)^2 +$$
$$\alpha\beta\gamma\cdot 123\cdot\sqrt[5]{\Pi} + \left(462:\sqrt{82}\right) + \left(455 + 5x + 16z\right) +$$
$$\left[225f : 24\left|zy\beta + 2\sqrt{6}\right) + 2\right] + AC^2 + \sqrt{421} +$$
$$-\tfrac{31}{24}\cdot 8^{125} + \operatorname{ctg} 869 - \tfrac{23}{32} + \left(\sqrt{12981} : 12\right) +$$
$$\left(\cos\alpha^3 \cdot \operatorname{tg}\beta\right)\cdot\left(698x + 2224\right) + \operatorname{tg} 441^2$$
$$+ \left[\left(615^{120} - x\right)\cdot\left(5yz + x\right)\right] + 162\alpha + \beta -$$
$$Q^2\cdot 64\cos\alpha + 667^3 + \sqrt{0{,}1} + x = 4$$

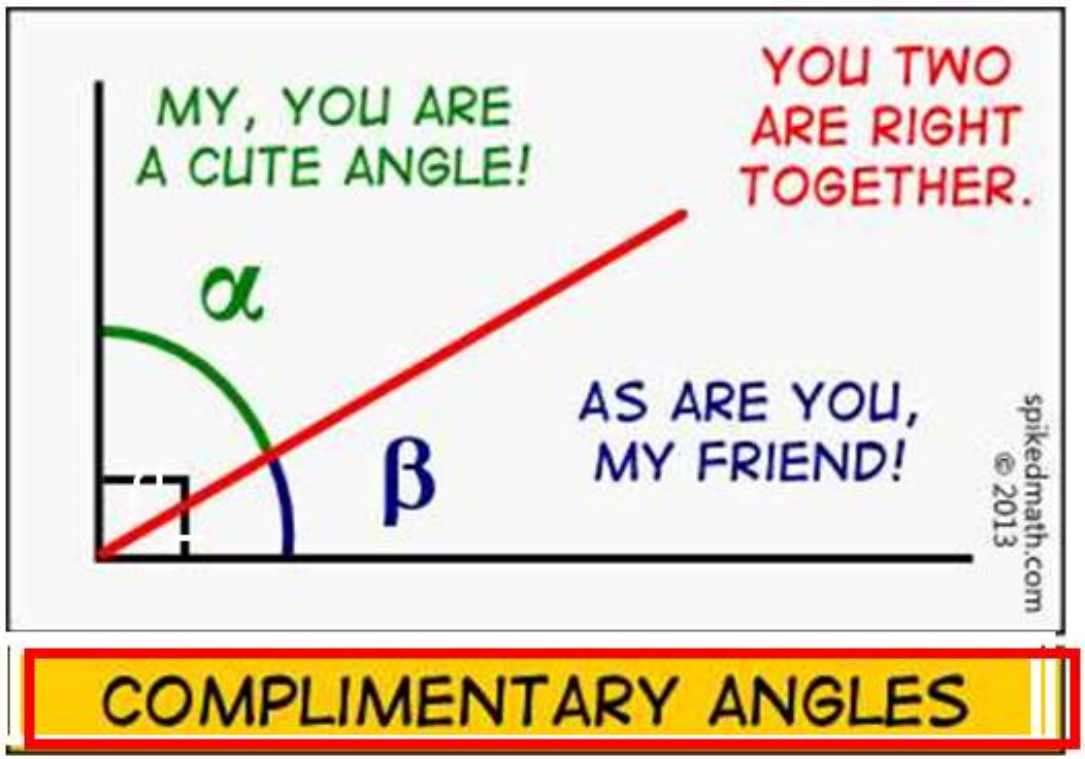
3. Explain in words what a complementary angle is. A picture would certainly help too

Complementary angles form a 90° angle

~~compliment~~

$\alpha + \beta = 90^\circ$

"alpha" "beta"



Have you been casually reading glossary at back of notes??

4. **Multiple Choice**. Select the one best answer. A Scalene Triangle is one that (1 mark)

a. has no sides the same length

b. has four sides

c. has 5 sides

d. has six sides

SIlly question, but trying to show you that once you eliminate the silly answers, the one left over must be correct.

Also, the obvious answer to this question will help you with the last bonus question!

Test hint! Sometimes one question will help answer another!

4. **Statistics.** 225 Students wrote a University Entrance Exam. Karen scored 39 marks out of a possible 58 on the Entrance Exam. One other student had the same score. 120 students had a lower score.

a. Determine Karen's mark as a percent on the exam.

b. Determine Karen's percentile rank

↳ "per hundred"

a. Exam Score: $39/58 = \frac{x}{100}$; $x = 67.24$

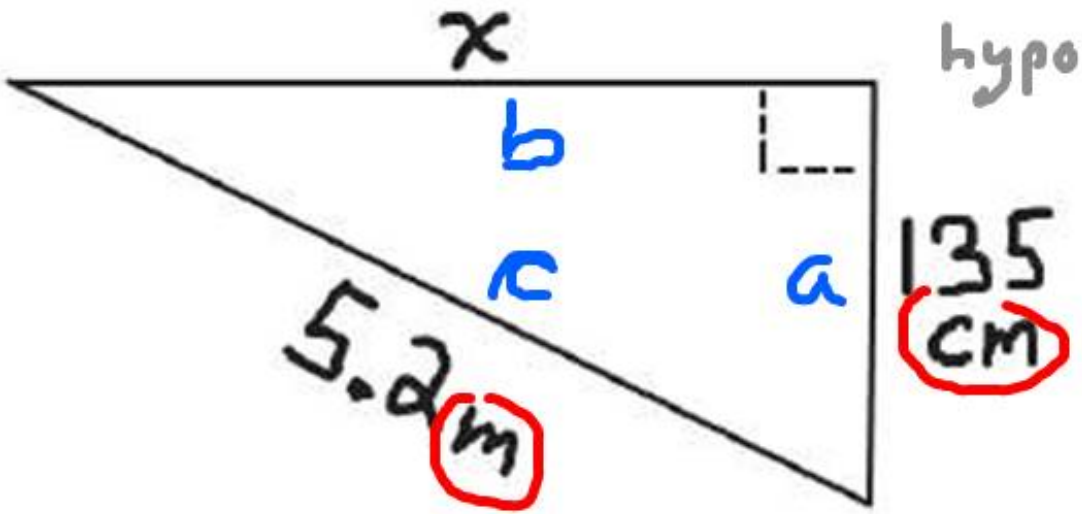
∴ Karen got $67.24/100$ or 67.24%

b) $PR = \frac{[B + \frac{1}{2}(E)]}{n} \cdot 100$ ↗ Round up

$= \frac{[120 + \frac{1}{2}(2)]}{225} \cdot 100 = 53.78$ ↗ (54)

P_{54}, 54th place, ...

5. Determine length x. (two marks)



$c^2 = a^2 + b^2$

hypotenuse — Shorter legs

$5.2^2 = 1.35^2 + x^2$

$27.04 = 1.8225 + x^2$

$-1.8225 \quad -1.8225$

$25.2175 = x^2$

$x = \sqrt{25.2175} \approx 5.02\text{m}$

Careful!

135 cm = 1.35m

Check?

$5.2^2 = 1.35^2 + (5.02)^2$

5.2^2

$= 27.04$

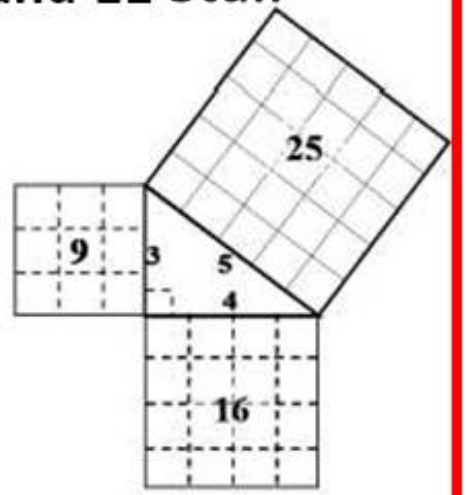
$1.35^2 + 5.02^2$

$= 27.0229$

Grades 10 and 11 Stuff

$c^2 = a^2 + b^2$

*where **c** is the length of the hypotenuse and **a** and **b** are the lengths of the shorter two sides*



$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

or

$25 - 9 = 16$

6. **Problem Solve**. Five years ago my teacher was twice my age. I am 26 now, how old is the teacher now? (two marks)

Guess & Check?

	Teacher now ↓	Teacher 5 years ago	me 5 years ago	me today	
	✗ 40	35	2·_ = 35 17.5 +5	22.5	WRONG!
Try again	✗ 45 ?	40	40÷2 20 +5	25	WRONG! Closer?
	✗ 50 ? -5	45	45÷2 22.5 +5	27.5	WRONG! Too High
	47? -5 =	42	42÷2 21	21+5 26 ✓	Yes

✓ ↖ Teacher is 47 today

Luv these?

You can see this method can be cumbersome!
But it works

6. **Problem Solve**. Five years ago my teacher was twice my age. I am 26 now, how old is the teacher now? (two marks)

Logic? Work backwards?

Luv these?

I am 26 now, therefore five years ago I was 21.
If I was 21 five years ago AND the teacher was twice my age 5 years ago then the teacher must have been 42 five years ago.
Therefore I deduce that today the teacher must be 47 years old

Algebra? **Starting to get this Algebra through 'osmosis'?** → slowly seeping in

Let x = teacher age now. Then $(x-5)$ is teacher age 5 years ago
But my age 5 years ago was $(26-5)$ AND teacher back then was twice my age

$x-5 = 2\cdot(26-5)$

$x-5 = 2\cdot 21$

$x-5 = 42$

$+5 \quad +5$

$x = 47$; Teacher age now!!

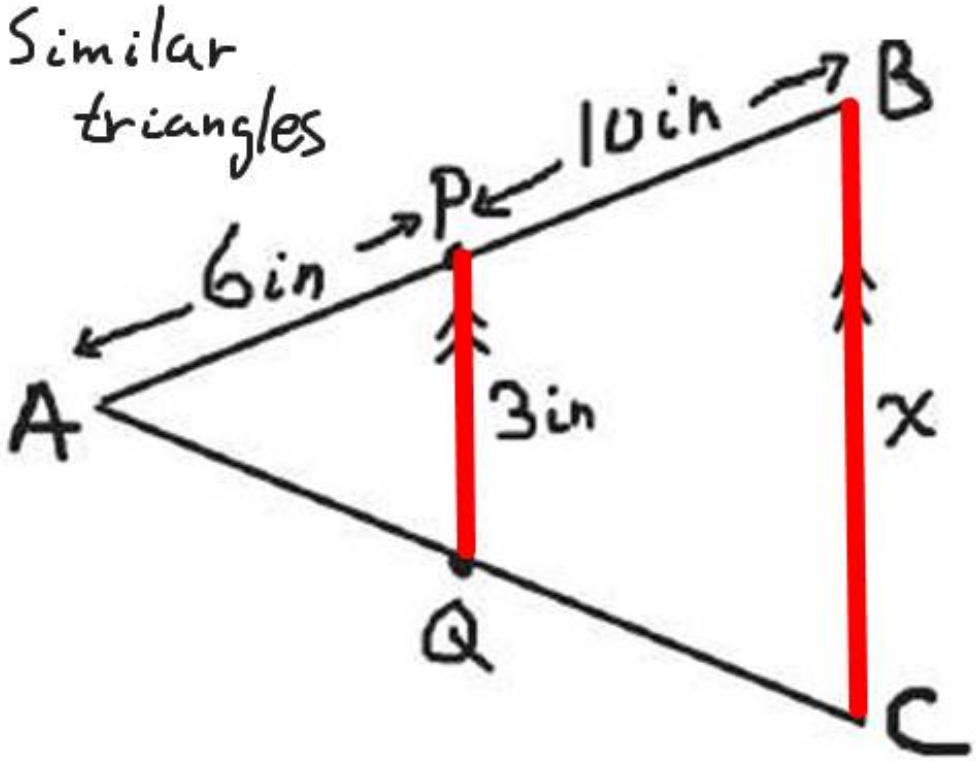
$x-5 = 2\cdot y$

Fairly Advanced?

7. Given that $\Delta ABC \sim \Delta APQ$. (ie: ΔABC is similar to ΔAPQ)

a. State the length of $\overline{AB}$.

b. Determine length x.



Length $\overline{AB} = \overline{AP} + \overline{PB}$

a) 6 in + 10 in = 16 in

b) $\frac{\text{mom} \Rightarrow x}{\text{Baby } 3} = \frac{16}{6}$ ← mom, ← Baby

↑ matching pieces; ↑ matching pieces

$\frac{x}{3} = \frac{16}{6}$

$x = 3 \cdot 16/6 = 8$ in

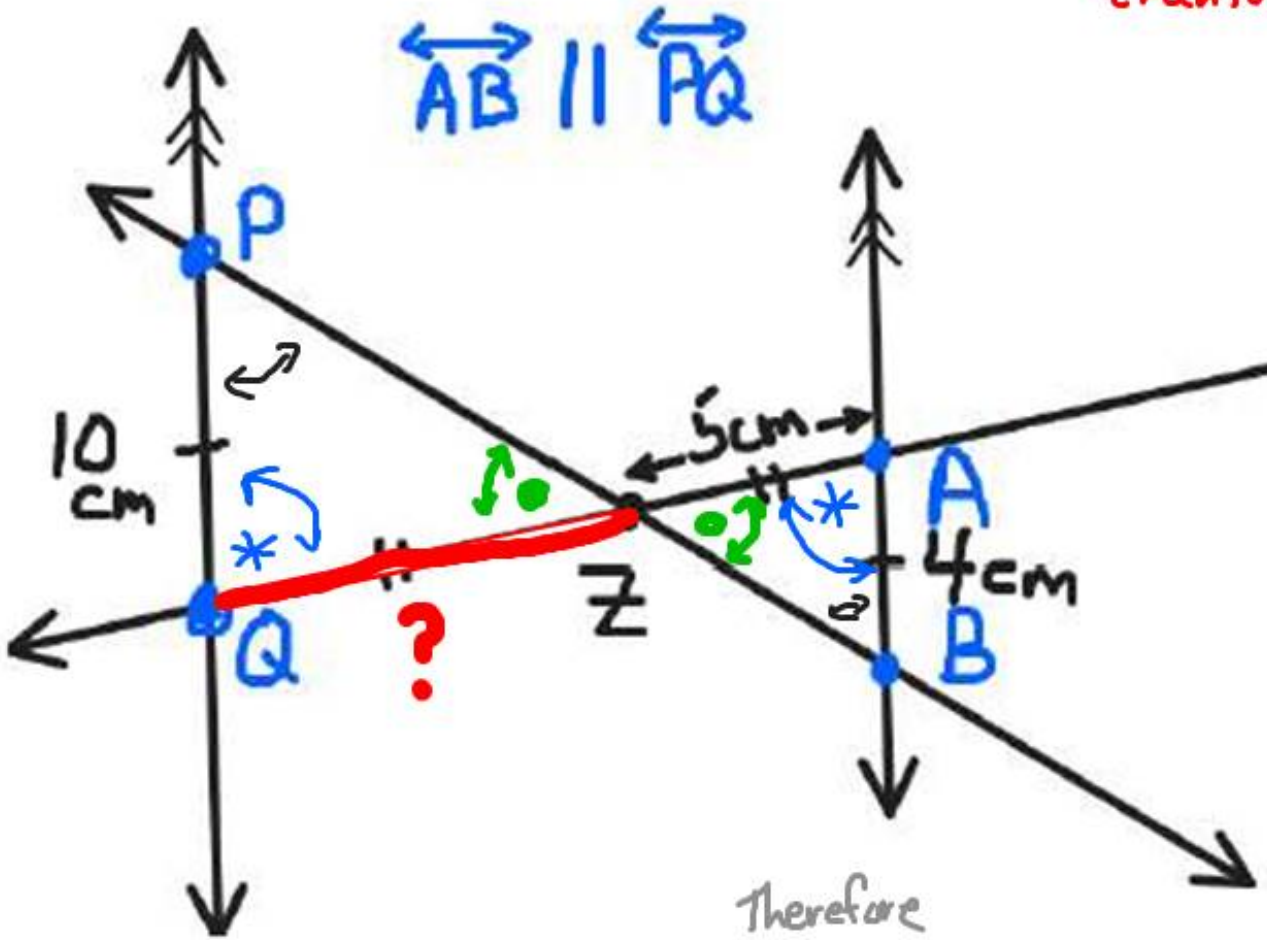
Fancy: $\frac{BC}{PQ} = \frac{AB}{AP}$

TLAR? if it were drawn more accurately?

8. **BONUS.** Given $\overline{AB} \parallel \overline{PQ}$., Lines $\overleftrightarrow{BP}$ and $\overleftrightarrow{AQ}$, determine length $\overline{QZ}$

transversals

$\overline{QZ}$



OMG! Logic!

① $\angle AZB = \angle PZQ$ ✓ (opposite angles)

② $\angle BAZ = \angle PQZ$ (alternate interior angles)

③ $\therefore \angle QPZ = \angle ABZ$ (triangle sum theorem)

④ $\therefore \triangle ABZ \sim \triangle QPZ$ They are similar

Therefore

③ $\therefore \frac{PQ}{AB} = \frac{QZ}{AZ}$

$\frac{10}{4} = \frac{QZ}{5}$; $QZ = 12.5\,cm$

Ok, a little intense!

But beautiful

You probably got the answer more instinctively

9. **Bonus.** Determine the sum of the counting (integer) numbers from 1 to 39. (two marks)

$1+2+3+++\text{etc}+++37+38+39 = ?$

Lol! Isn't the sum from 1 to 40 = 820? $41 \cdot 20 = 820$

So take away the 40 gives 780

A Problem solve technique:
Solve a simpler version first

OR

a middle number

$1+2+3 \quad +++ \text{etc} +++ \quad 37+38+39$

← ODD Number of values

$1+39=40$
$2+38=40$
$3+37=40$

How many 40's pairing up the ends!??

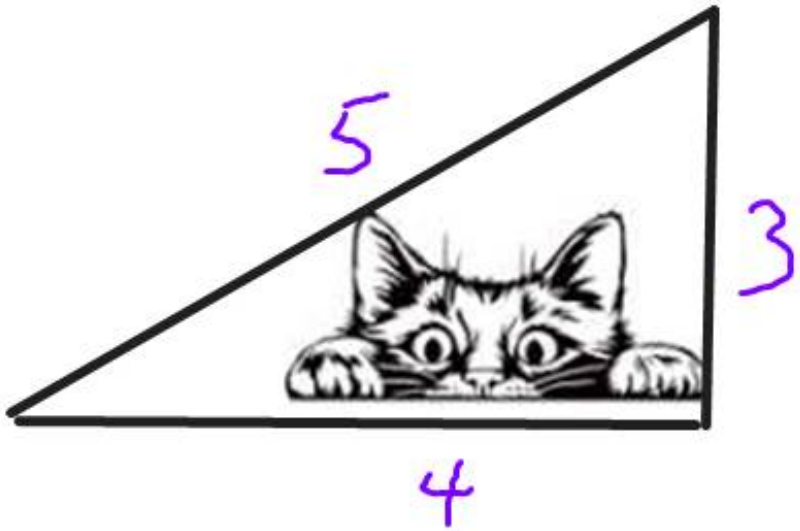
ONLY 19 forties plus the one in the middle ("20") that never paired up!

$40 \cdot 19 = 760$
$+20$
780

There are formulae too if you do pre-cal or use spreadsheet

10. **Bonus.** Draw me a cute little kitty cat cuddled up inside a Scalene Triangle.

Been flipping through glossaries?
Scalene is a triangle that has no sides (or angles) same
Which was obvious from a previous question in this quiz!



ON TIME, ON TARGET

Determined to Deliver

Carry the load!