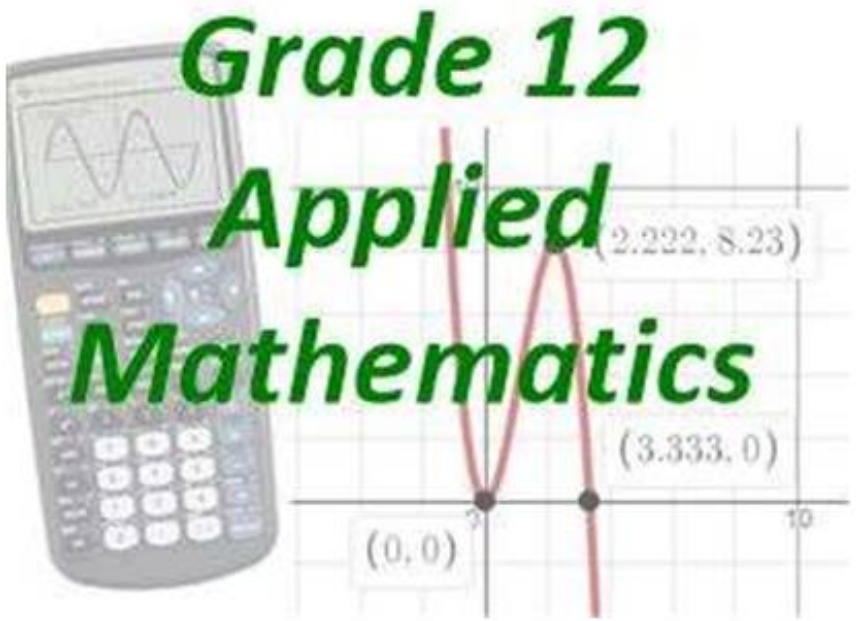


Quiz Debrief
Week 4

23-02-16

MrF

MY! GRADE 12 APPLIED COURSE REFERENCE NOTES *(copy them if you want)*

UNITS A & E- PROBABILITY, PERMUTATIONS & COMBINATIONS

Fundamental Counting Principle (FCP): If one event can occur in '**a**' ways, a second event in '**b**' ways, a third event in '**c**' ways, and so on, then the number of ways that all events can occur one after the other is the product **a*b*c**…*Eg*: number of license plates we can make 26*26*26*10*10*10. **Watch** if *repetitions* are allowed or not.

Permutations: ***Order does matter!*** 1st, 2nd, 3rd in a race of 12 runners. 12*11*10 possible choices. Or ${}_{12}P_3$.

Counting Non-Distinguishable Objects: **4 Red** balls, **2 Green** Balls. Number of *distinguishable* ways to arrange the 6 balls with 4 red and 2 green balls is $\frac{6!}{4!2!}$. Just like arranging YES and Nos. If only two different objects then = Combo.

Combinations. Arrangements of objects where ***order does not matter***. Selecting committees of people (no special positions or rewards, a group, bundle), Lotto 6/49, etc. *Eg*: how many ways can an **unorganized** committee of three people be formed from 12 people. ${}_{12}C_3 = \frac{12!}{(12-3)!3!} = \mathbf{220}$

$Prob(A) = P(A) = \frac{\#\ of\ Favoured\ Outcomes}{\#\ of\ Total\ Possible\ Outcomes}$; eg: **Prob(Draw a King) = 4/52**. //**Sample space**: the list of all possible outcomes. Use a tree or table. // **Outcome**: the result of one trial of an experiment (eg: flipping one coin has only H or T outcome)// **Event**: A set of outcomes. **Eg**: rolling two dice, an event might be the set of outcomes where doubles were rolled.

Complement. The probability of an event happening is "***1 – the probability it won't happen***". **Complement** of event A is $\bar{A}$. So $P(\bar{A}) = 1 - P(A)$. **At least once Probs**: Prob(**A's ≥ 1 time**) = $1 - P(no\ A's)$

ODDS. Odds in favour = favourable:not favourable; *eg*. wins : losses. Odds Against = not favourable : favourable

Formulas: **Factorial: 6! = 6*5*4*3*2*1; Permutation**: ${}_nP_r = \frac{n!}{(n-r)!}$; **Combination**: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Multiplying Probabilities: For successive events. *Keyword*: **AND**. Often one probability is *dependent* on the first. Often indicates whether something is drawn and with replacement or not.

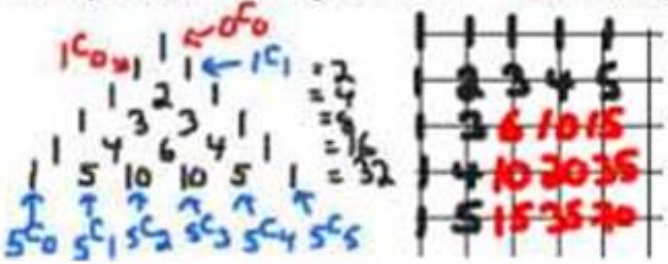
Independent example: Let **A** be event of being hit by bus. Let **B** be event of winning lottery. Both are independent events. Say P(A) = 0.01, and P(B) = 0.01. Prob of getting hit by a bus and winning the lottery = Prob(A **AND** B)= P(A)*P(B)=0.01*0.01=0.0001. Dependent example. Standard deck of cards. Probability of drawing a King **then** a Queen without replacing the first card. Let: K_1= event of drawing King first draw; Q_2 = event of drawing Queen second. **$P(K_1$ *and then* $Q_2) = P(K_1)*P(Q_2 \mid K_1) = \frac{4}{52} * \frac{4}{51} = \frac{4}{663} = 0.60\%$**

Adding Probabilities. For compound events, multiple events. *Keyword*: **OR**

Eg: Let **K = Set of all Kings**. Let **S = Set of all spades**.

Prob (**K OR S**) = P(**K**) + P(**S**) – Prob (**K AND S**). $= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} = 30.7\%$ We are subtracting out the common card to set K **AND** set S; that is the **K♠** so we *don't double count* it. Mutually exclusive Events. The two events or sets that share no common outcomes! ie: Kings AND Queens are mutually exclusive. ***OR = Add ; AND = Multiply ***

Pathways. As per usual PASCAL triangle method, or use the secret combination! ${}_nC_r$ where **n** is the total number of steps, **r** is *either* the downs or the rights. Doesn't matter what you count (downs or rights)! Just like selecting non-distinguishable objects.



All the Probability we learned!

Your cheat organized your way?

MrF

GRADE 12 APPLIED

Name:_____
Date: _____

WEEK 4 QUIZ 230216

CLOSED BOOK. Use mine and your Cheat Sheet

↳ tweak up your OWN?

Multiple Choice Select the best or closest answer:

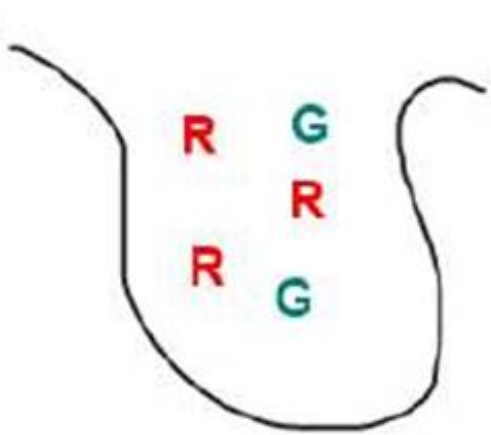
No need to show work (although one would wonder how you even got the answer then)! Guess if necessary. Use multiple choice techniques! Do not pick the obviously wrong answer(s).

There are eight regular questions (not including bonus). Do any six of the first eight. If you do all eight regular questions then the best six will be scored.
The bonus questions are separate.

1. The probability of drawing a green marble in one draw [P(G)] is:

a. 100% b. 60%

c. 40% d. 3 : 2



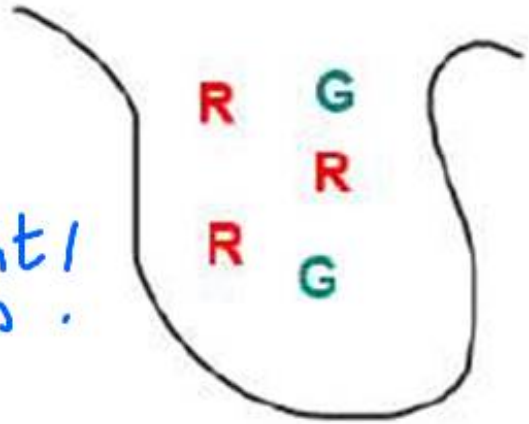
$$P(G) = \frac{\#\text{ of Green}}{\text{Total \# possible}} = \frac{2 \text{ green}}{5 \text{ marbles}} = \frac{2}{5} = 40\%$$

2. The probability of drawing a Green marble on the first draw then a Green marble on the second draw $P(G_1, G_2)$, **without replacement**, is:

a. 1/10 b. 20%

c. 0% d. 100%

↳ dependent / prob.



$$P(G_1, G_2) = P(G_1) \cdot P(G_2 | G_1)$$

one less Green since no replacement

$$= \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

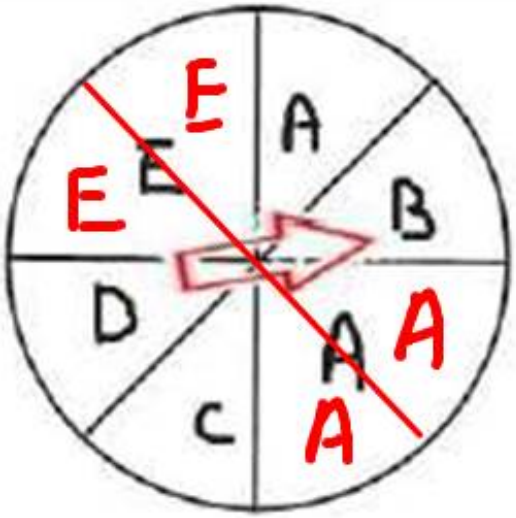
one less marble since did not replace first

The probability of drawing a Green on the first draw and then, without replacing it, drawing another Green GIVEN THAT you already took out a green marble and stuck it in your pocket

3. A fair spinner at the right. The probability of spinning an 'A' outcome is:

a. 1/6 b. 6%

c. 3/8 d. 25%



$$P(\text{Spin "A"}) = \frac{\#\text{ of "A" outcomes}}{\#\text{Total Possible Outcomes}}$$

$$= \frac{3}{8}$$

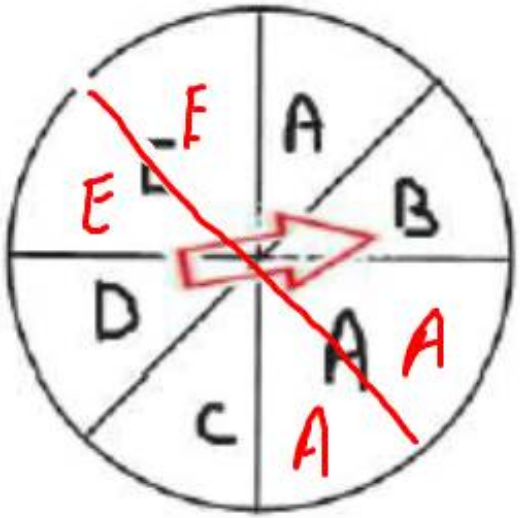
Sectors must be same size!

Outcomes must be equally likely!

4. A fair spinner at the right. If you spin the spinner 40 times, how many times should you ***expect*** it to have an outcome of 'D'?

a. ~~1/8~~ 5 b. ~~6%~~ 10

c. ~~3/8~~ 15 d. ~~25%~~ 50



$P(\text{spin D}) = 1/8$ expect one D, every 8 spins

Proportions method ⇒ $\frac{1}{8} = \frac{x}{40}$; $x = 40/8 =$ 5 times

we should expect a "D" usually, on average.

Solve equation method

$40 \cdot \frac{1}{8} = \frac{\#\text{ of "D"}}{40 \text{ tries}} \cdot 40$

$5 = 40 \cdot \frac{1}{8} = \#\text{ of "D"}$

5. Mr.F has 27 students. He has three Burger King coupons. One for a Burger, one for a Milkshake, and one for French Fries. He puts all the student names in a hat. How many different ways can Mr.F randomly hand out the three coupons? Assume a student is only allowed to get one coupon (not putting names back in the hat!).

a. mega lots!

True
is there a better answer?

b. $_{27}P_3$ ↑

~~c~~ $_3C_{27}$ ↑ l.o.l

~~d~~ ~1.7 way

? Really

An obvious perm?

$_{27}P_3$

nPr(27,3)

= 17550

27 nPr 3

17550

The order matters!

Select 3

27 students

$\frac{27}{B} \cdot \frac{26}{m} \cdot \frac{25}{FF}$

↑ Can't be same

= 17,550 ways to award the three different coupons to different students

6. In a normal deck of 52 cards (see sample space at back), what is the probability of drawing a Queen **OR** a Heart?

a. $\frac{4}{13}$ b. 25% c. $\frac{13}{52}$ d. $\frac{1}{4}$

lol these are the same

OR = Add

Count the sample space outcomes!

Subtract the Q of ♡

$$P(Q \text{ OR } ♡) = \frac{4Q + 13H - 1}{\text{TOTAL Cards}} = \frac{16}{52}$$

$$= \frac{4}{13}$$

Formula Method

$$P(Q \text{ OR } ♡) = P(Q) + P(♡) - P(\text{It is both a Queen and a } ♡)$$

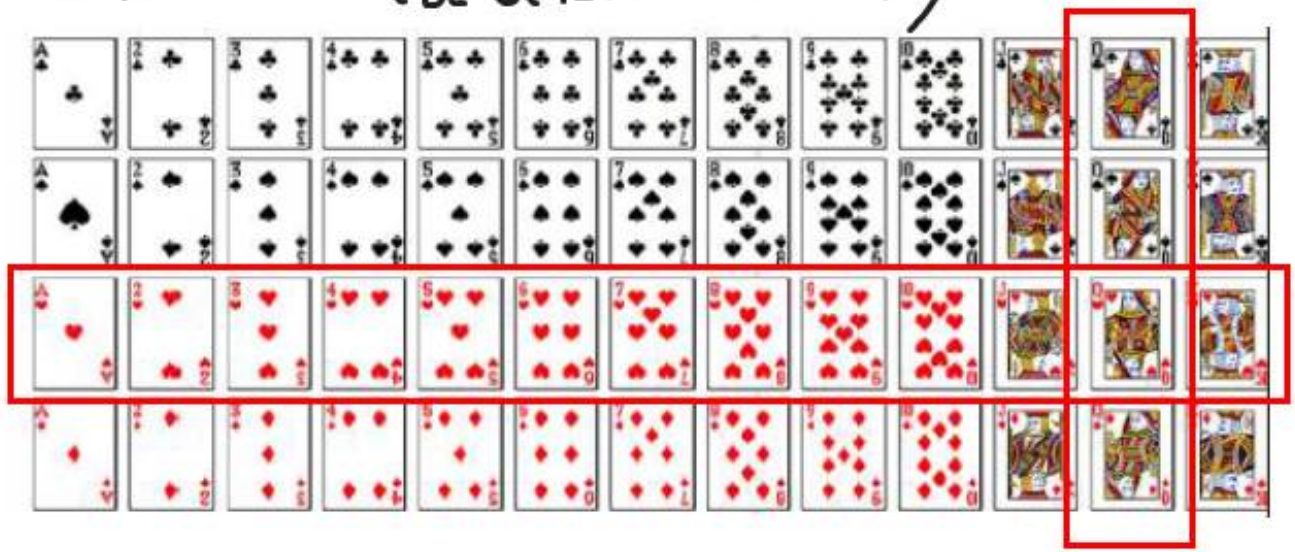
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Hearts →

Queens ↓



7. There are nine people at a math addiction support group. If everyone shakes hands with everyone else, how many handshakes will there be?

Combo! Combo!

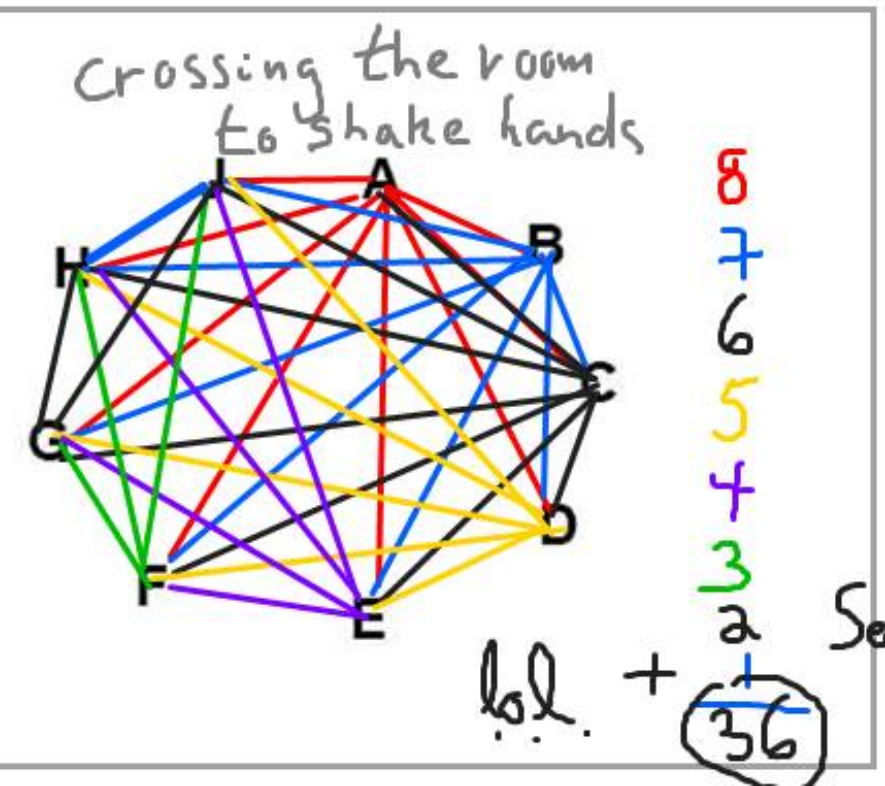
OMG! Have done this several times

a. 81 b. 9! c. 21 d. 36

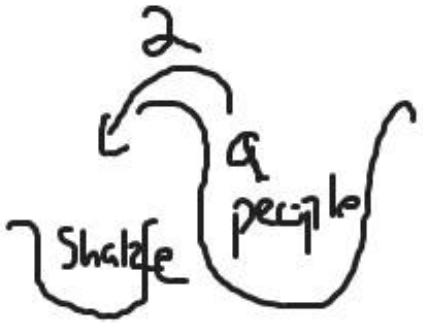
$_9C_2$

An <u>un-ordered</u> arrangement of some objects selected from other objects

$_9C_2 = 36$



See a pattern?



9 people! Select two at a time to shake hands!

Selecting AB is same as BA order doesn't matter!

8. Josh invests some money in a savings device that pays 9.5% **compounded quarterly**. At the end of 8 years he cashes it in for $4,874.53.
What was the initial Principal amount he had deposited? [Recommended you use an App!]. His initial investment was:

a. $2,000

~~b.~~ $10,075

~~c.~~ almost double due to the rule of 72

d. $2,300

Just test the sensible answers see which one works!

$$2000 \cdot \left(1 + \frac{0.095}{4}\right)^{(8 \cdot 4)} = 4238.72526$$

$$2300 \cdot \left(1 + \frac{0.095}{4}\right)^{(8 \cdot 4)} = 4874.53405$$

works ✓

use App?

Present Value	-2,300.00
Payments	0
Future Value	4,874.53
Annual Rate (%)	9.5
Periods	32
Compounding	Quarterly

Solve manually?

$$4874.53 = P_0 \left(1 + \frac{0.095}{4}\right)^{(8 \cdot 4)}$$

$$\frac{4874.53}{2.119362631} = \frac{P \cdot 2.119362631}{2.119362631}$$

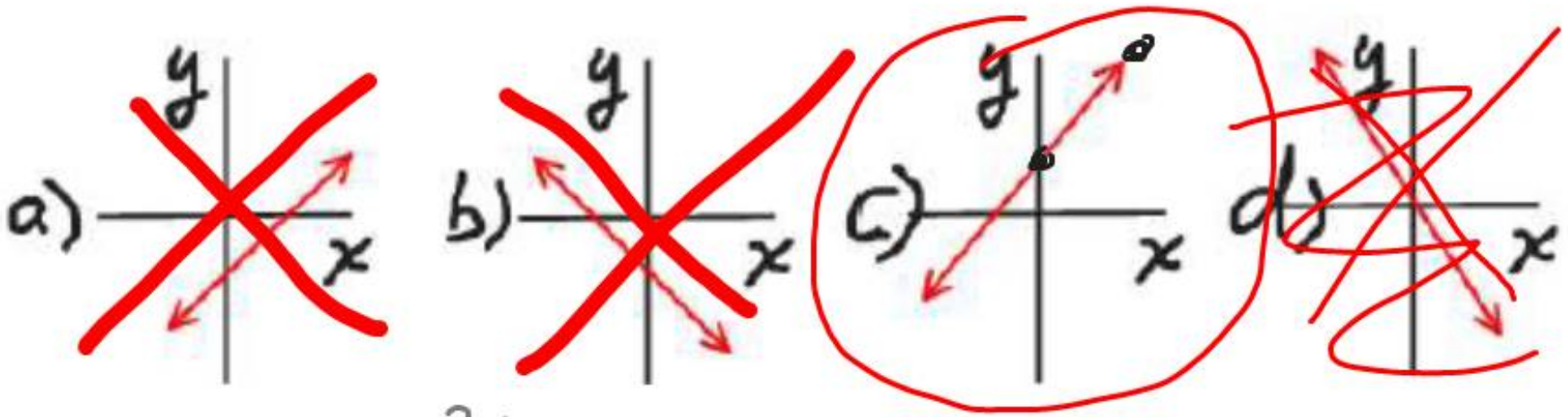
$$P = \$2299.998\ldots = \$2300$$

BONUSES (Extra questions for 1 extra mark each)

(Grade 11 Stuff you should know!)

Grade 11

Bonus 1. Which of these graphs best represents the equation y = 3*x + 2
(circle best letter choice)



$y = \frac{3x}{1} + 2$ → $\frac{\Delta y}{\Delta x} = \frac{3}{1}$ → Slope: 3 up, 1 right

x	y
0	2
4	14

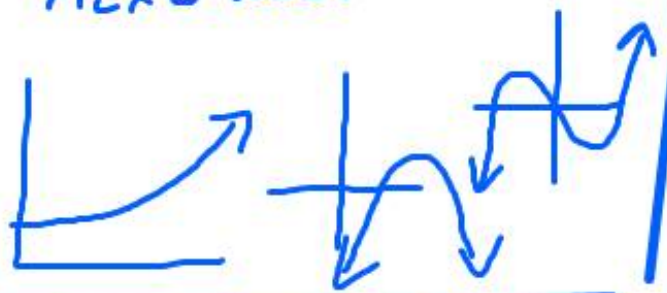
$3(0)+2=2$

$3(4)+2=14$

Wait for the fun stuff

$3x^2 + x - 5 = y$

Lots more of this next week



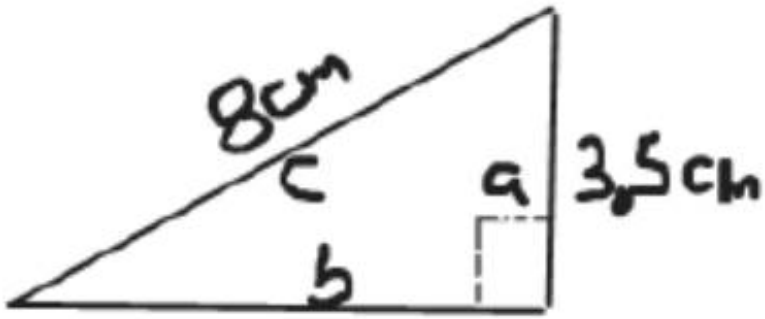
Bonus 2. The length of side '**b**' is: (best or closest answer)

a. 7.2 cm

b. 76.25 cm

c. 8.73 cm

d. ${}_8C_{3.5}$



$c^2 = a^2 + b^2$

$8^2 = 3.5^2 + b^2$

$64 = 12.25 + b^2$

$51.75 = b^2$

$b = \sqrt{51.75} = 7.19$

7.2 is closest

This real bread and butter

Pythagoras

Good Work Team!