GRADE 10 ESSENTIAL
UNIT G – TRANSFORMATIONS

UNIT NOTES

INTRODUCTION

Have you ever slid something across a coffee table? Have you ever rotated a steak on the BBQ?

Have you ever looked in a mirror? Have you ever played a video game?

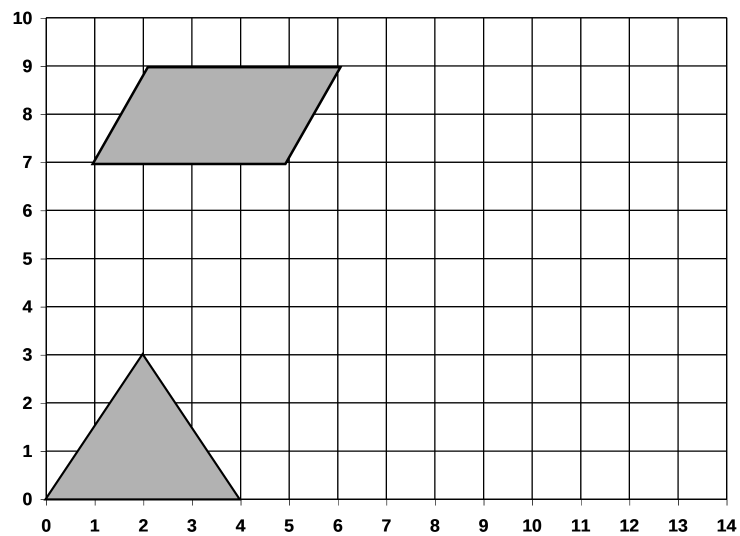
If you have then this unit will be fun and easy. You will learn how to shift shapes around, how to reflect them in a mirror, how to rotate them, and how to stretch or squish them.

Lined square graph paper and a ruler are required for this unit.

Copy the triangle five units to the right. (R5)

Copy the parallelogram one unit left and three units down. (L1, D3)

Just copy each corner point to move the entire shape.



The instruction (**L1**, **D3**) means Left one, down three. We always show the left or right **first**, the up or down **second**.

Translating a shape. Moving an entire shape, or sliding it, or more properly 'translating' it is easy. The instructions are easy too.

Translating a shape (transforming it left-right, up-down) is always given by an ordered pair of instructions (horizontal movement, vertical movement).

We make it even simpler by talking about moving in the x-direction, and the y-direction. (***x, y***) Moving right increases the x horizontally, moving up increases the y vertically.

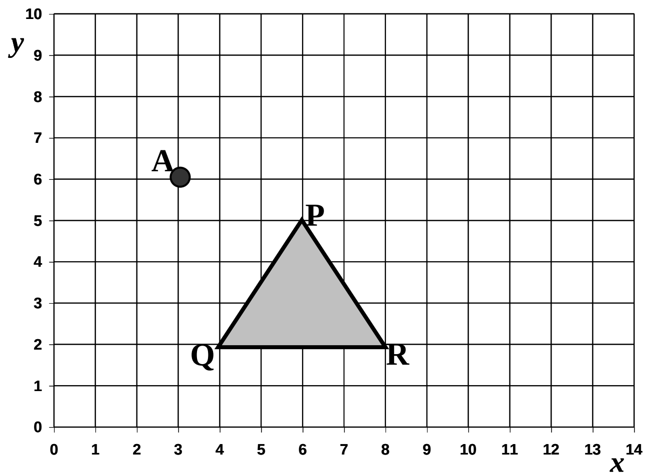
You might notice some references indicate the motion like this instead: $\begin{bmatrix} x \\ y \end{bmatrix}$ or $\begin{pmatrix} x \\ y \end{pmatrix}$ where of course the order still matters. The top number is the horizontal movement, the bottom is the vertical movement.

Of course, it is just as easy to count how many lines you move the shape as well.

LOCATING POINTS ON A GRID

Sometimes it is desirable to position a shape by numbers on a grid. We can use the (x, y) *Cartesian coordinate* system (invented by the mathematician DesCartes).

Notice the grid ***x*** axis and grid ***y*** axis. An axis means a directional number line.



The point '**A**' is at position (3, 6). The order of this pair of numbers matters: the ***x*** first, the ***y*** second.

Triangle **PQR** has corners at P(6, 5); Q(4, 2); and R(8, 2).

You Try. Mark and label the following **ordered pairs** on the Cartesian coordinate grid

Points: **B**(0, 2) ; **C**(10, 3); **D**(5, 5)

Trapezoid **FGHJ**: **F**(6, 7); **G**(7, 9); **H**(9, 9); **J**(10, 7).

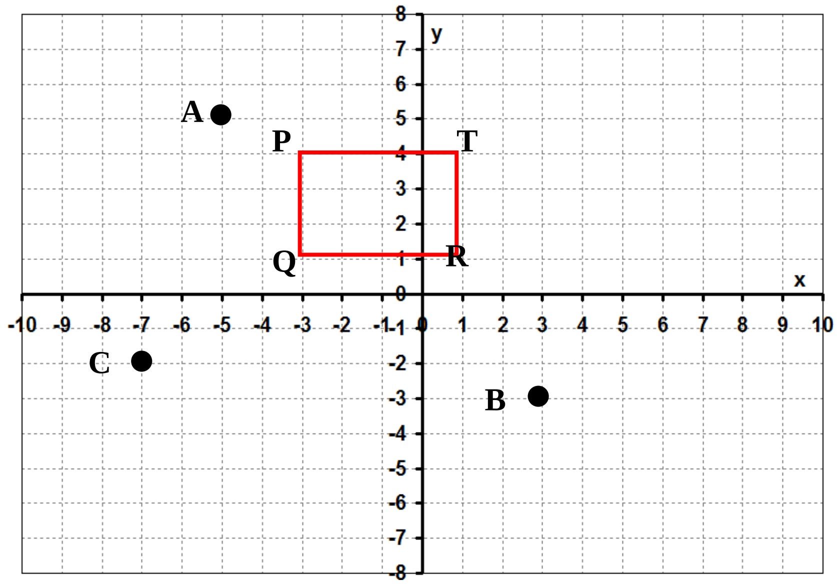
Translate the triangle **ΔPQR** by applying the translation (R2, U3) and state the position of its new related corner points: **P′**; **Q′**; and **R′**.

P′(____, ____); **Q′**(____, ____); **R′**(____, ____)

" P' " is pronounced '**P prime**'. That notation suggests that point P and P' correspond, they match up. It is useful to give the points related names since they are closely related.

THE FULL CARTESIAN GRID

Of course, you can sometimes be to the left of zero, or below zero too if you use the set of **integer numbers**. A full cartesian grid looks like this→



Observe the points the points:

A($^-$5, 5); **B**(**3;** $^-$**3**); and **C($^-$7, $^-$2)**.

Observe rectangle PQRT with corners: **P**($^-$3, 4); **Q**($^-$3, 1); **R**(1, 1); and **T**(1, 4).

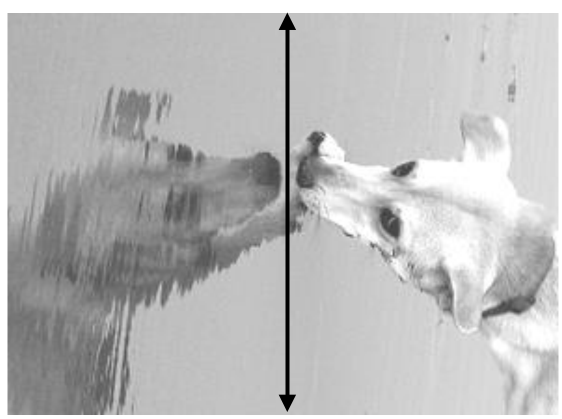
You try. Mark a trapezoid with points: **K**($^-$2, $^-$2); **M**(2, $^-$2); **N**(2, $^-$5); and **P**($^-$4, $^-$5). Notice also from Grade 9 symmetry studies that this trapezoid is not symmetrical.

Now translate your trapezoid KMNP by applying the translation (2, $^-$3) or (Right2, Down3) or (R2, D3) or $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$. Lots of different ways you will see in references to say the same thing for translating and transforming a shape!

REFLECTING A SHAPE

Reflection is making a mirror image of a shape or a point. Of course, you need to know where your mirror and your actual image meet. Normally the mirror is along an *x*-axis or a *y*-axis on the Cartesian grid.

At the right is a reflection **across** or **in** or **through** the y-axis.



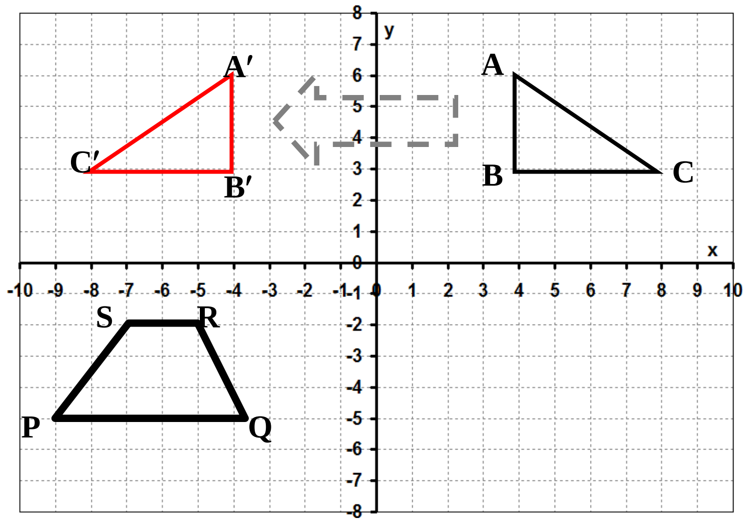
Reflection ***across*** or ***in*** or ***through*** the y-axis.

REFLECTING A SHAPE ON THE CARTESIAN GRID

Reflect the triangle ABC across the y-axis.

All points on the right of the y-axis move to the same place on the left of the vertical y-axis. The corner point(s) x values become their opposite; the corner point y values remain unchanged.

You reflect trapezoid PQRS across the y-axis



Reflection through y-axis
[Left become right, right become left!]

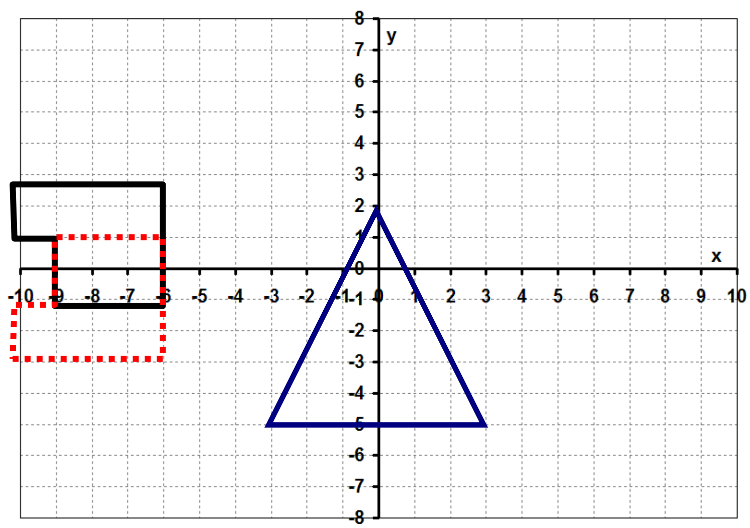
To *reflect* a shape across the y-axis all you really need to do is geometrically copy the points to the opposite side of the y-axis. So, an x of 6 right (+6) becomes an ***x*** of 6 left (–6), etc.

An alternate method and more advanced method to reflect the shape through the y-axis is just to use an analytic method where all the x values become their opposite and all the y values remain unchanged. Some advanced references show this as **(x, y) ↔ ($^-$x, y)**. So, for example the point **(3, 5)** would become **($^-$3, 5)** and the point **($^-$8, $^-$5)** would become **($^+$8, $^-$5)**.

Reflection across the x-axis

Identical to the refection across the y-axis, except a vertical *flip* instead of a horizontal flip.

Here an irregular rectilinear shape has been reflected across (or 'through') the x-axis.



Now you try→

You reflect the triangle across (through) the ***x***-axis. Points above the x-axis go below, points below the x-axis go above. The y-coordinates become their opposites. Up becomes down, down becomes up. The *x*-coordinate does not change; it is 'invariant'.

An '*analytic'* way to show that that there was a reflection across the x-axis is to show that all the *y* values in the coordinate became their opposite; ie: the transformation **(x, y) ↔ (x, ⁻y)**. Or one may want to just relabel the grid and flip or rotate it appropriately.

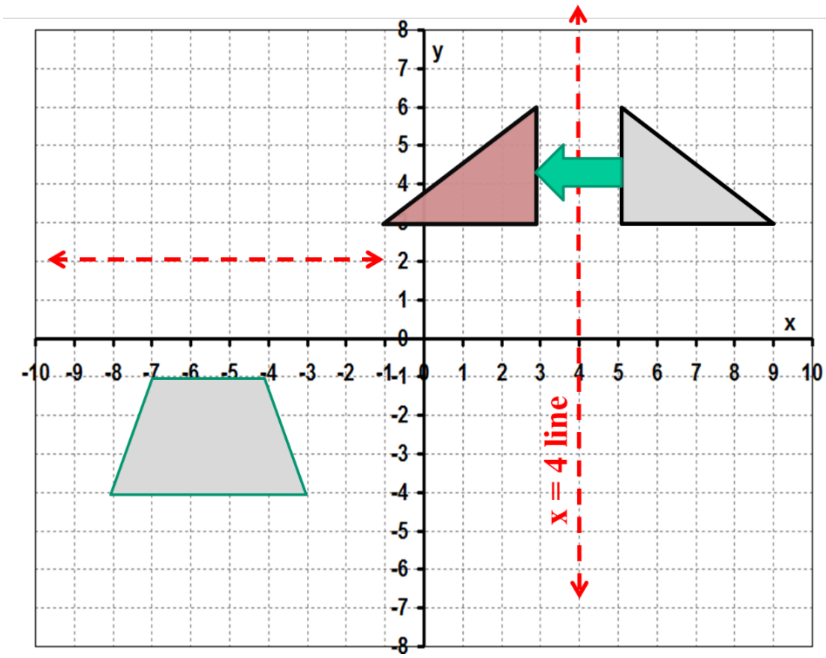
REFLECTION ACROSS *ANY* VERTICAL OR HORIZONTAL LINE

Sometimes you may want to reflect across a line other than the zero lines (the x and y axis lines)

Notice the triangle is reflected across the '**x** = **4**' line.

You reflect the trapezoid across the '**y = 2'** line

Can you come up with an 'analytic way' to do the mapping of the corners? (Very advanced)



The '***x = 4***' line is called that because every point on the line has an *x* value of 4. **You Draw** an ***x* = *9*** line! Make sure you use a straight edge and put arrows on both ends!

ROTATIONAL TRANSFORMATION

Sometimes we rotate a shape. Of course, when you rotate something it has to pivot around a certain point. And it must rotate in a direction either clockwise or anti-clockwise. The easiest rotation is around the **origin** point at **(0, 0)**. In high school we tend to limit rotations to multiples of **90°**; so, rotation angles of **90°**; **180°**; **270°** clockwise or counter clockwise.

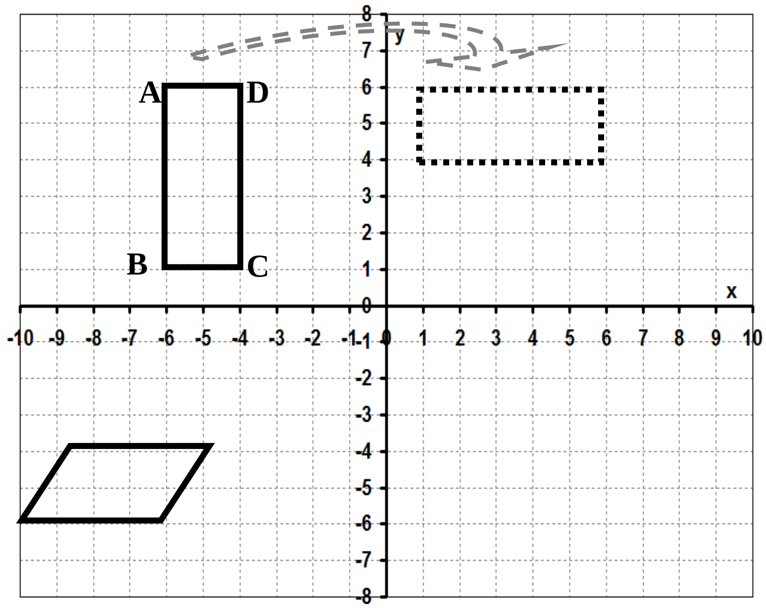
The rectangle ABCD has been rotated about the origin **(0, 0)** by **90°** *clockwise*.

You rotate the original rectangle 180° anti-clockwise.

Now *you* rotate the original rectangle 180° clockwise! ie: two 90s

Now *you* rotate the parallelogram 90° ***anti-clockwise***.

Marking the shape on a piece of clear plastic and rotating that might be the best method.



Fun with rotations about the origin

But can you see the analytic transformation for a clockwise rotation?
y becomes x'; x becomes negative y'.

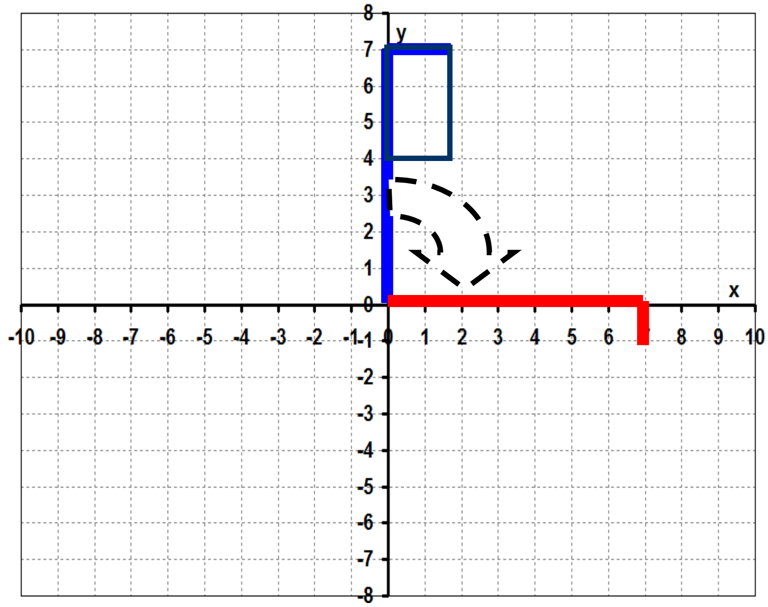
Advanced thinking. For those looking for an analytic method (just using numbers or mathematical expressions) to rotate a shape around the origin by 90° clockwise x's become –y's and y's become x's;
ie: (x, y) ↔ (⁻y, x). Pretty advanced thinking usually reserved for calculus but think about it.

Rotations are difficult.

Maybe using a simple shape to do a rotation transformation on each of the points would be easier.

Rotate the **L** shape through 90° clockwise.

Now pretend the **L** shape is attached to every point on the shape you are rotating.



I call this my 'Hockey Stick' method. Just hook a hockey stick to one point and then rotate the hockey stick!

Easier??

CURIOUS! Think about this if you dare! If we do a reflection about the y-axis *and then* a reflection about the x-axis (or vice versa), that is identically the same as a 180° rotation (either direction). Try it, you might like it! If you plan on any serious future mathematics studies this is an important idea.

ROTATING ABOUT A POINT ON THE SHAPE

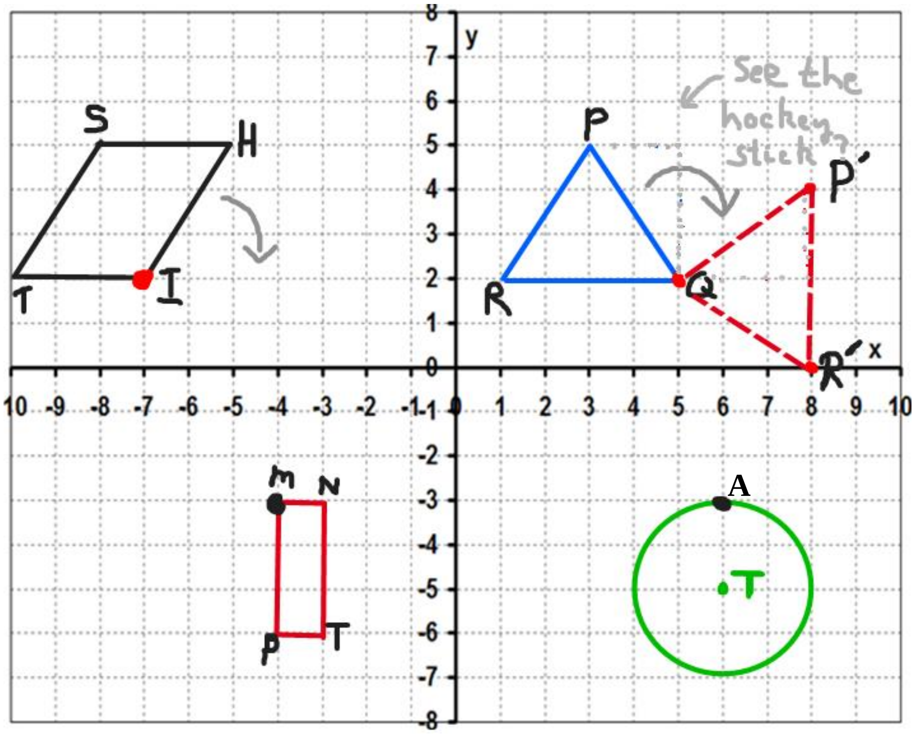
We have rotated only about the origin (0, 0) so far. But what if we want to rotate the shape about itself?

Triangle PQR has been rotated 90 clockwise about corner point Q.

You rotate parallelogram SHIT clockwise about invariant Point I.

You rotate rectangle MNTP ***anti*-clockwise** about the corner M.

You rotate the circle named '**Circle T**' 180° about the invariant point A.
Hint: rotate the centre and then redraw.



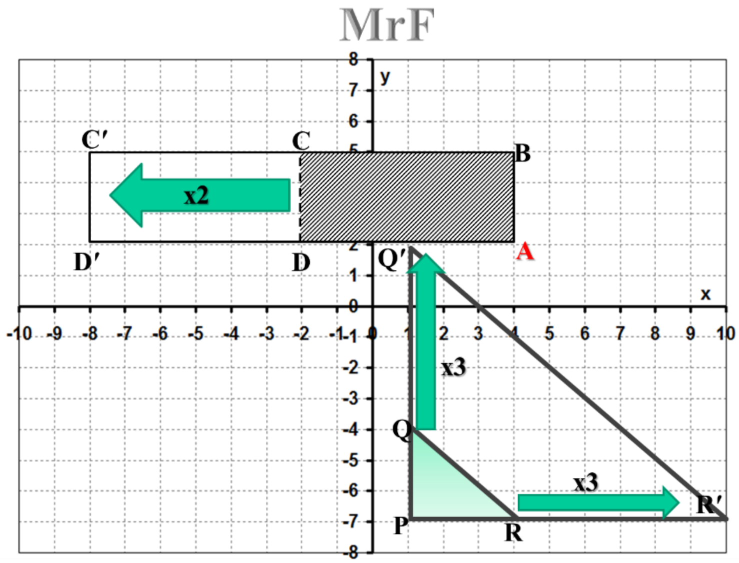
SCALING OF A SHAPE

We will do lots of Scaling when we study scale models in Grade 11. This section is included just for completeness of the subject.

Changing the size of a shape is another form of transformation. We can make the scale half as long so the edges are all half as long, or we could maybe triple all the edges (a scale factor of 3).

We have translated (slid or shifted) shapes, we have reflected shapes, we have spun shapes around (rotation). Let's blow them up now! And shrink them! And crush them! Just like your XBox does when it moves a shape across the screen then flips it and blows it up.

'Scaling' a shape means to change its size. We can make it smaller (compress or dilate) or larger (stretching or expanding). We can scale about a shape's centre or about any corner.

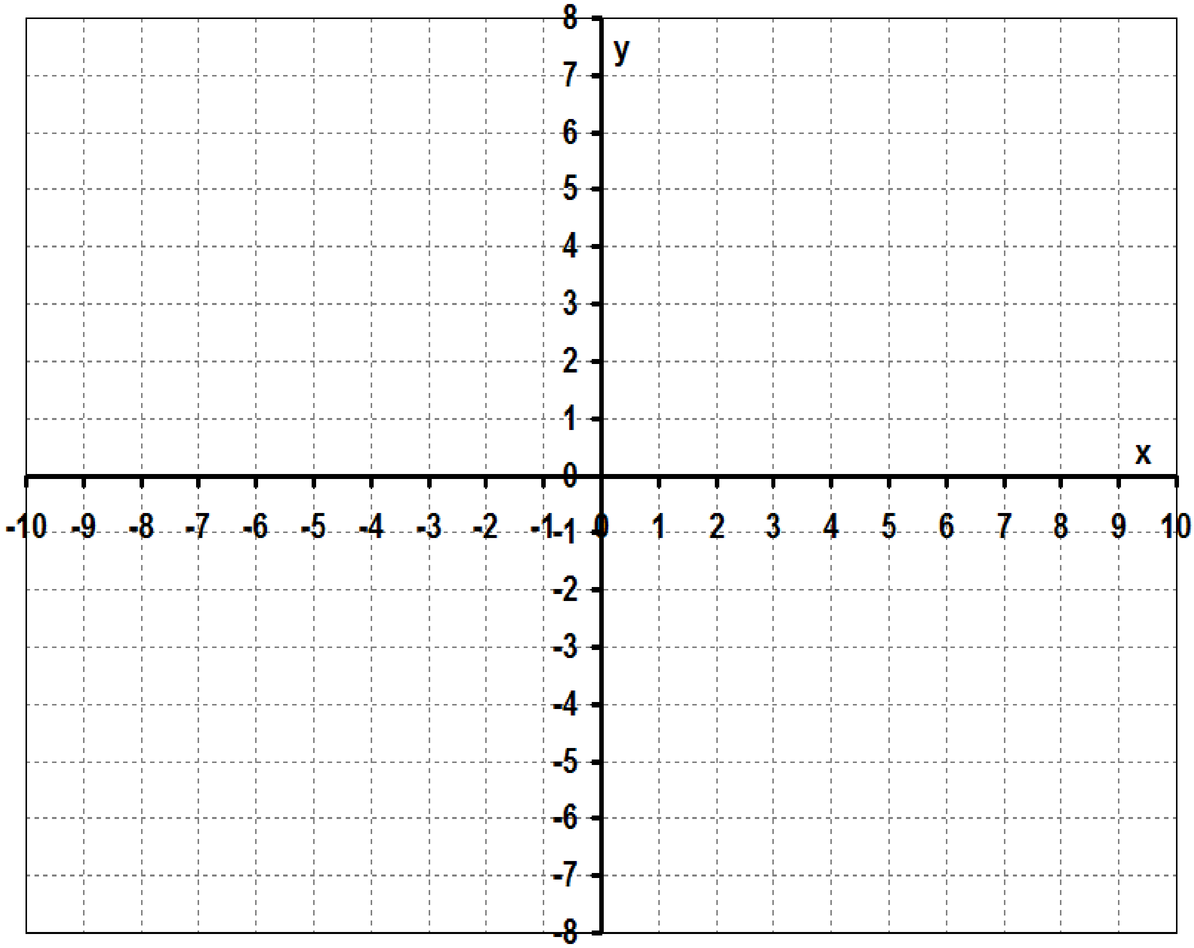


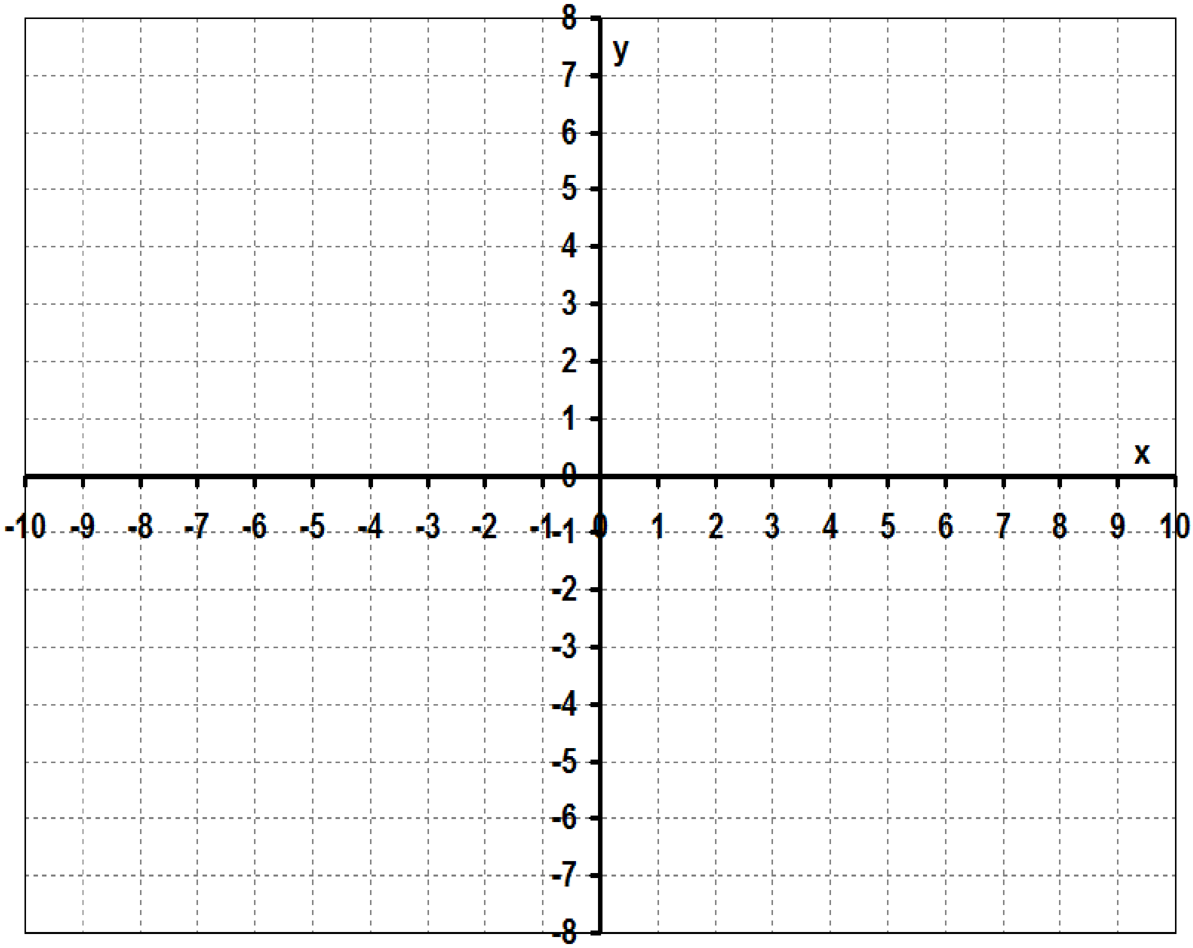
Rectangle ABCD has been stretched ***horizontally*** by a 'factor of two' from the point A.

Triangle PQR has been stretched *vertically* **and** *horizontally* by a 'factor of three' from point P.

You draw a trapezoid using **F**(0, 0); **G**(-6, 0); **H**(-10, -6); **J**(0,-6) and compress it horizontally *and* vertically by a factor of ½ about point **H**.

BLANK CARTESIAN GRID PAPER




y
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8
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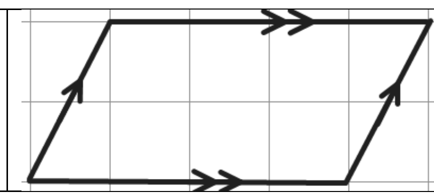
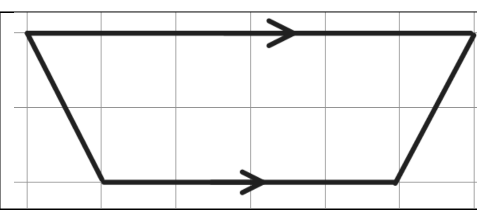
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

GLOSSARY

Analytic	To explain something with a formula or mathematical expression. SO rather than say move the x coordinate 2 units right the analytic expression would be $x' = x + 2$.
Cartesian Grid	A grid system invented by Descartes, a famous mathematician. A coordinate system.
Dilate	To shrink or compress. A form of scaling; changing size
Integer numbers	Counting numbers and negatives of them
Invariant	An 'invariant point' is a point that doesn't move
Line of symmetry	A line or lines about which a figure is symmetrical
Ordered Pair	A pair of numbers in which the order is important, for example (x, y) of a Cartesian grid position. Three blocks East and five North is ***not*** the same as five east and three north! The **order is important** in an **ordered** pair.
Parallelogram	A geometric quadrilateral shape having two pairs of sides parallel 
Prime mark	A mark on a symbol to show it is closely related to another symbol. P and P' would be examples of points that correspond to each other, they match up, they are related.
Reflect	To make a mirror image; across or through, or in a reference line.
Rotate	To change position in an angular sense about some point.
Transformation	To change shape or position
Translate	To shift, or slide, or move.
Trapezoid	A quadrilateral with only one pair of parallel sides. ** UK and American definitions conflict ** 

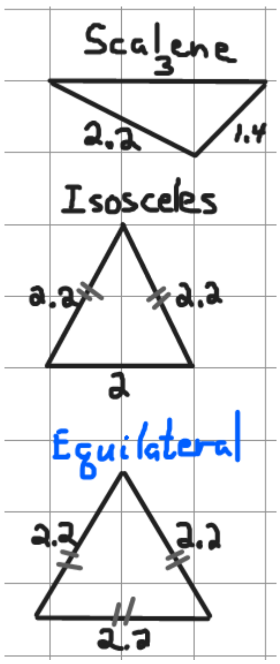
Triangles	A polygon having 3 sides. They can be described two ways: • by their angles, or. • by their lengths of sides	

Naming Triangles by Lengths of sides

Scalene: all sides different lengths

Isosceles: Two (*or more*) sides the same length.

Equilateral: All sides the same length.



Naming Triangles by corner Angles

Right: Having a 90° corner

Acute: Having all sharp corners (all less than 90°; all angles < 90°)

Obtuse: Having one angle more than 90° (an angle > 90°)