GRADE 10 ESSENTIAL
UNIT E TRIGONOMETRY – CLASS NOTES

1. Triangles are everywhere! Almost every shape you see can be made of triangles. And every triangle can be made of two 'right' triangles. That is all we will study in this unit: 'right' triangles; triangles with a square corner.

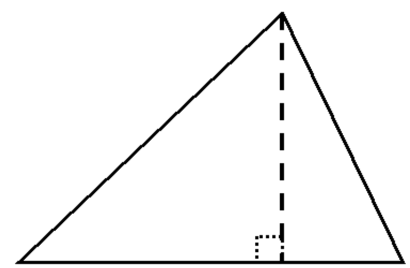
2. Even more curious, a triangle has six parts, three sides and three angles. But if you know any three of those things you can figure out the other three.

3. A triangle is a polygon with three angles. ['*tri*' + '*angle*']. Consequently it has three sides as well.

You might recall all the different kinds of triangles. →

Types of Triangles	
Based on angles	**Based on comparing sides**
right	equilateral
acute	isosceles
obtuse	scalene

4. Regardless of the types of triangles, the **most important triangle** is the **right triangle**. *Every* kind of triangle can be made of two right triangles and since every shape can be made of some triangles it is important to be rather familiar with a right triangle and all its magic.



THE ‘RIGHT’ (90 DEGREE) TRIANGLE

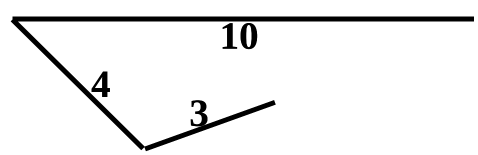
5. We will start to study the **right** triangle exclusively now. If you understand the right triangle you seriously understand every polygon shape in the universe. Every polygon can be made of triangles. Even circles can be made of triangles!

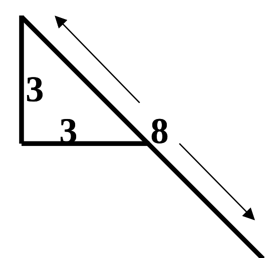
Pythagorean Theorem (The Law of Pythagoras)

This is very important! If there were 10 things you just had to know about math for the rest of your life this is one of them for sure.

6. **Experiment 1**. Take three different length slurpee straws or slips of paper and try to make a triangle of any kind with sides these or other different lengths. You might find there are some combinations of lengths that cannot make any triangle!

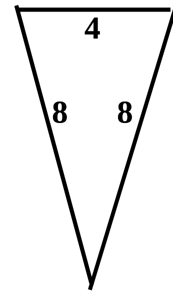
a. Try a **10**, a **4**, and a **3**. It doesn’t work.



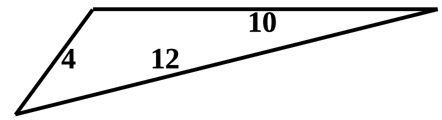


b. Try a **3**, a **3**, and a **8**. It doesn’t work.

c. But a **8**, a **8**, and an **4**, does work. What kind of triangle is it though?? _________



d. Try a **10** a **4** and a **12**, it works. What kind of triangle is that? _____________



7. **Experiment 2**. Take three different length slurpee straws or slips of paper and try to make a right triangle! This will be very difficult. How can it be so difficult if there are right triangles everywhere??!!

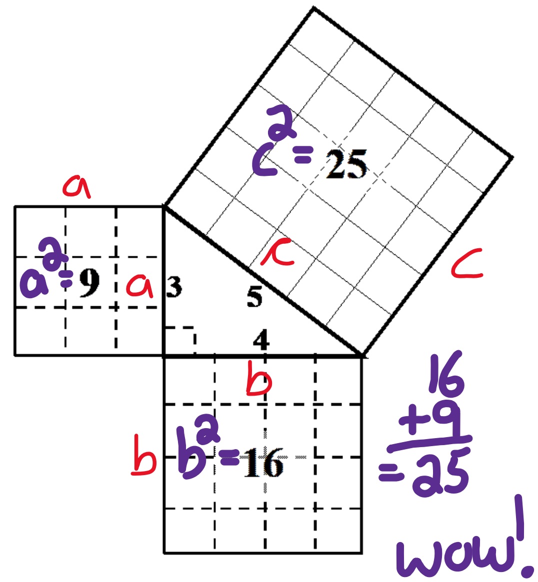
Obviously there is some **secret** to making right-triangles.

8. **Pythagorean Theorem**. One of the more ancient and critical concepts of all. Many cultures had discovered 2500 years ago that “***given any right-triangle, the square on the longest side equals the sum of the squares on the other two sides***”. The law has the Greek mathematician Pythagoras’s name because he proved it and recorded it. But ***ancient beadwork*** in some North American indigenous cultures suggests they knew it too.

This is one of those math concepts you really do need to know for the rest of your life! Honestly!

It's a law!

9. Given **any** right-triangle, the square on the longest side equals the **sum** of the squares on the other two [shorter] sides.



In 'algebraic' form:

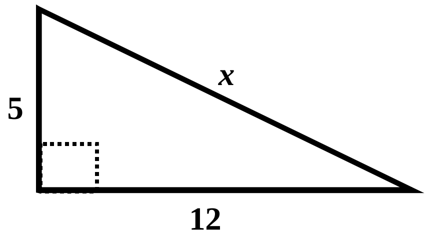
$c^2 = a^2 + b^2$; ******where *c* is the longest side****** ***c*** is <u>*alone by itself*</u> in the equation!!!!!! ****

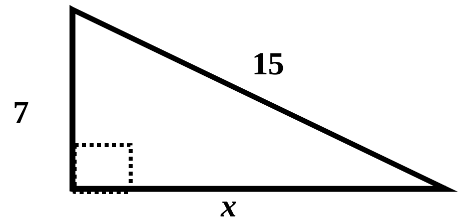
The longest side is called the ***hypotenuse*** and is **always *across*** from the **right angle**.

10. You most definitely need to add this idea to your two-page cheat sheet [reference notes]

Pythagoras Examples: *You try*!

11. Given the two right-angle triangles below; YOU find length ***x***. (The triangles are **not** '*drawn to scale*', so you can't just measure them with a ruler!)



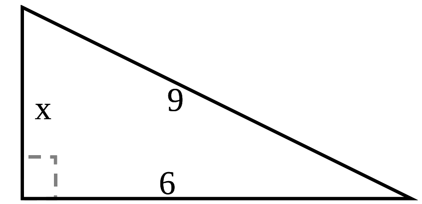


Ans: 13 exactly

Ans: ab*out* 13.27 if you round to two decimal places.

WORKED EXAMPLE – PYTHAGORAS

12. The more difficult problem is to find a short side when you are given the other short side and the hypotenuse.



Write down the formula (*you only have one!*)

$c^2 = a^2 + b^2$; now plug in the numbers in the correct place; (*'**c**' is the hypotenuse; **alone**; **by itself**!*)

$9^2 = 6^2 + b^2$ evaluate the squared numbers

So **$81 = 36 + b^2$** ; get the b^2 by itself

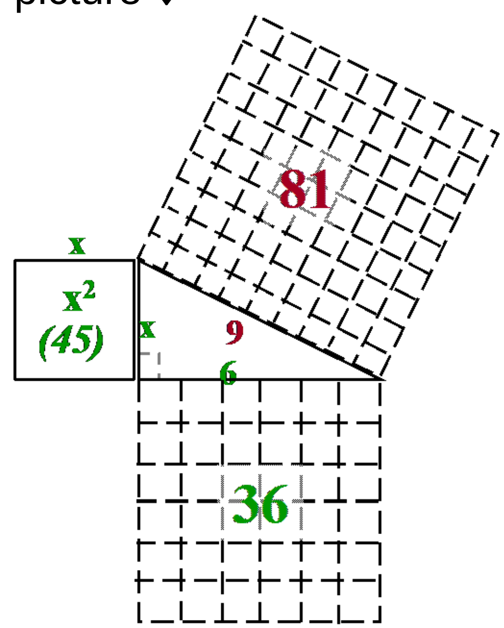
$81 - 36 = b^2$; evaluate (subtract)

$45 = b^2$; now calculate the '**b**' by 'unsquaring'

6.7082… = b

∴ **$b \doteqdot 6.71$** rounded to nearest hundredth

If you don't like all the fancy steps to the left (algebra) just sketch the picture ↓



Square and Square Roots of Numbers

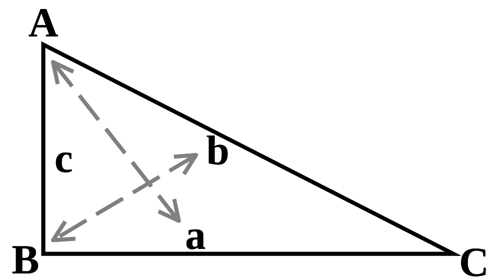
15. If you are not overly familiar with the idea of squares and square roots from previous studies try this quick lesson; complete the table here:

x →	**2**	**3**	**8**	**9**	**10**	**5**	**7**	$\leftarrow\sqrt{y}$
x^2 →	**4**	**9**	**64**	**81**	**100**	**25**	**49**	$\leftarrow y$

16. Evaluate the following squares and square roots (to nearest hundredth)

a. $\mathbf{2.5^2}$ 6.25	b. $\sqrt{6.25}$ 2.5	c. $\mathbf{7.07^2}$ **50**	d. $\sqrt{50}$ 7.07
e. $\mathbf{100^2}$ 10,000	f. $\mathbf{1000^2}$ 1,000,000	g. 7.1^2 50.41	h. $\sqrt{7.1^2}$ 7.1

Labelling Triangles (*special way*)



17. As usual CORNERS are labelled with CAPITAL LETTERS and sides are labelled with small letters.

18. But triangles have **special labelling**; the small letter across from a corner is the same as the CAPITAL letter at the corner across.

So, side '**a'** can also be called side $\overline{BC}$
Some folks label the sides and some the corners that make the sides!

19. We label triangles that way because of the three bears! You know: papa bear, momma bear, baby bear. What??

20. You may have casually noticed in your experience the 'Papa bear, Momma bear, Baby bear' rule (that is *my* name for it!)

My 'Papa bear, Momma bear, Baby bear' rule just says that the biggest angle (the 90° papa bear in a right triangle) has the biggest side **across** from it (the hypotenuse) , the **baby bear angle** has the smallest side **across** from it, and the **momma bear** has the middle-sized side **across** from it.

21. Go ahead, sketch out some triangles see if I am wrong. See if you can find a 'counter-example' that shows I am wrong. And it works for any triangle, even if it is not a right triangle. This is a pretty clear **relationship** between the corner angles of a triangle and the corresponding side across from that corner angle!

(*btw: if you are not doodling on scrap paper when studying math then you definitely are not thinking or interacting with your thoughts*)

Triangles are getting more magic all the time. There is more!

'Un-natural' Numbers (Irrational Numbers)

22. Often when doing a square root you will end up with a crazy number on your calculator like:

$\sqrt{5} = 2.236067977$. It actually turns out to be closer to this really $\rightarrow$:

2.2360679774997896964091736687313……………; *(it goes forever with no pattern)*

but your calculator display is too small to show an infinite string of numbers!

23. So normally you would just properly round the answer to 2 or 3 decimal places as directed in an assignment.

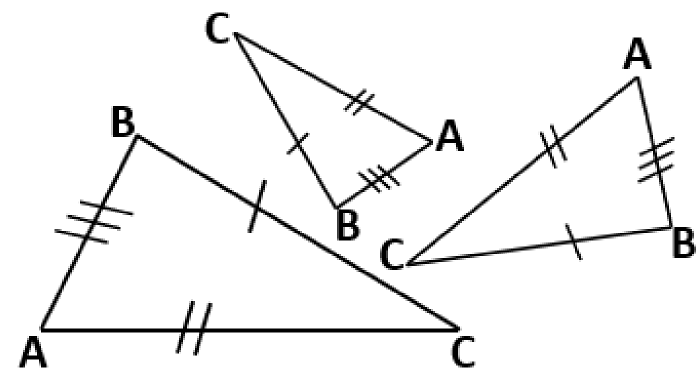
So now you know how to find a missing side when you know the other two sides of a right-angled triangle using the Pythagorean Theorem.

SIMILAR TRIANGLES

24. There is another way to find missing sides of a triangle if you know one side and you know that it is similar to another triangle with some known sides.

Similar Triangles

25. Triangles (or any shape) are called '*similar*' if they have the same shape. The three triangles in the diagram to the right are similar triangles. They have the same shape (the same corner angles) and corresponding sides are *proportional* in length.



'*Corresponding*' sides of similar triangles are marked with similar number of 'tick' marks. Corresponding corners have the same letters here.

26. Further, if you know at least three parts of a triangle including at least one side then you know the shape **and the size** of that triangle.

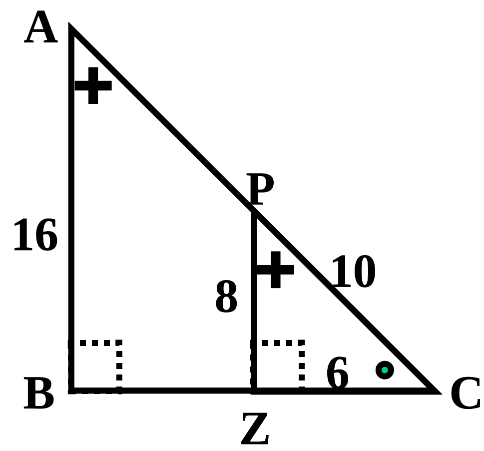
Similar Triangles Calculation

27. Triangles **ABC** and **PZC** are '*similar*'. They have the same corner angles at **C**. They both have a **90º corner** and therefore the left-over angles at **A** and **P** are the same. Three angles the same makes them similar. Consequently the *corresponding sides* of the triangles are *proportional*.

$$\frac{\overline{AB}}{\overline{PZ}} = 2; \quad \frac{\overline{AC}}{\overline{CP}} = 2; \quad \frac{\overline{BC}}{\overline{CZ}} = 2$$

So side $\overline{AC}$ must be **20** and side $\overline{BC}$ must be **12**.

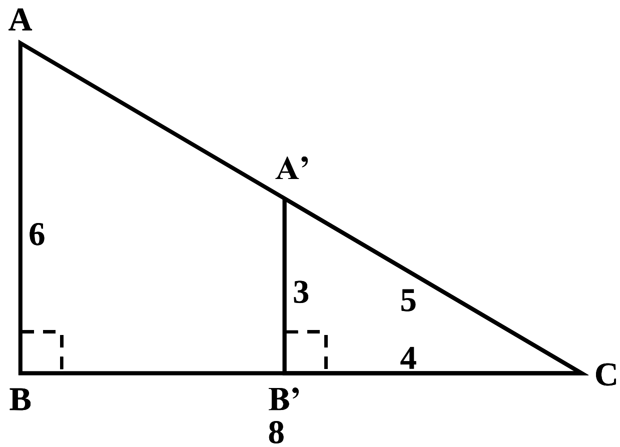
Triangles ABC and **PZC** are '*similar*' can be written as '**ΔABC ~ ΔPZC'** in mathematical notation.



$\frac{16}{8} = \frac{20}{10} = \frac{12}{6}$ **=2.** All lengths of the '**mommy**' triangle are just twice the '**baby**' triangle's corresponding lengths.

28. **You try a few:**

a. Determine all the missing line segments in this figure.



This could be rafters in a house or girders in a bridge!

$\overline{AC}$: ________________

$\overline{BC}$: ________________

$\overline{AA'}$: ________________

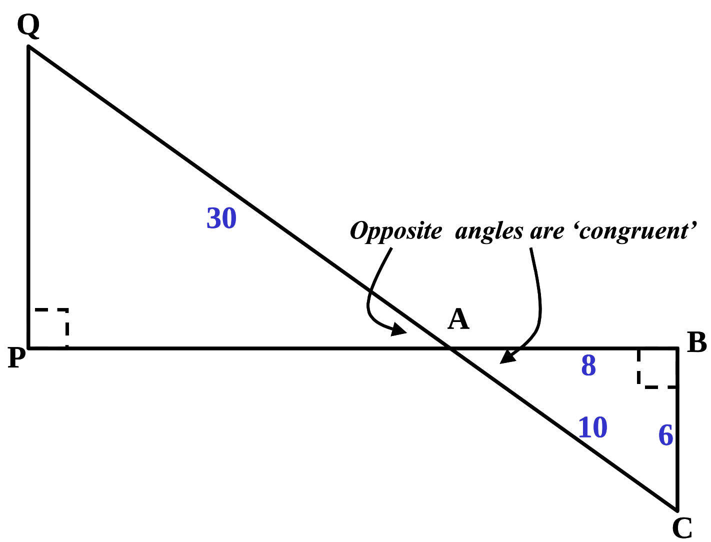
$\overline{BB'}$: ________________

Our Solution:

*(notice that we sometimes use **B** and **B’** to mark **corresponding** [matching] **corners**. **B’** is pronounced “**B prime**”)*

b. Find Length $\overline{PQ}$ and Length $\overline{AP}$ of these similar triangles.

Our Solution:



ΔABC and ΔAPQ are similar (ie: ΔABC ~ ΔAPQ) since they have all three of the same angles.

INDIRECT MEASUREMENT USING SIMILAR TRIANGLES

29. Using similar triangles is critical to performing **indirect measurement**.

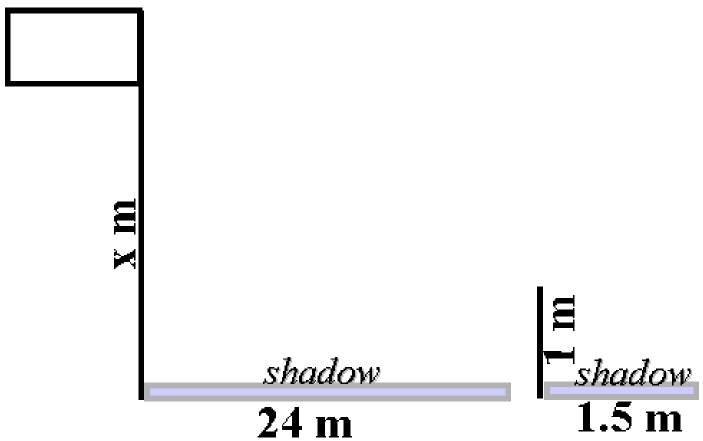
How tall is a certain flag pole? Do you climb it with a tape measure and measure it ***directly***? Do you chop it down and measure it directly?

Let us calculate the height of a flag pole ***indirectly*** without having to climb it.

How high is the flag pole?

30. You have been paid $1300 to repair a flag pole. You need to measure its height, ***x***.

You stick a stick in the ground (vertically) and measure its length and the length of its shadow. That makes a similar triangle to the flagpole and its shadow.



$\therefore$ the flag pole has a height of: ___________

measure the shadows the same time of day!

31. Curious thought! What could you do if it wasn't a sunny day?

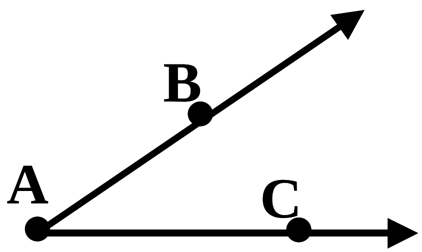
ANGLES AND USING A PROTRACTOR

32. A triangle has six parts; it has three sides and it has three corner angles. We have not paid much attention to the corners and angles yet.

It works out that if you know three parts of a triangle you know all the other parts too!

We have been measuring and comparing the lengths of sides of a triangle but how do you name angles and then measure angles at the corners.

Naming Angles



33. An angle is named by the two *rays* that form it.

Ray $\overrightarrow{AB}$ and ray $\overrightarrow{AC}$ share the same end point at point **A** and make **angle BAC**; we write it in simpler symbols as: ∠**BAC**.

The '**vertex**' or corner where the rays meet is common to both rays so it is the **central letter** in the name of the angle.

35. You *could* call this angle ∠**CAB** as well, but then you have ambiguity (confusion) since there are two different names for the same thing. Consequently, we tend to ensure that we put the **B** and **C** in alphabetic order to avoid confusion. It is more properly called ∠**BAC.**

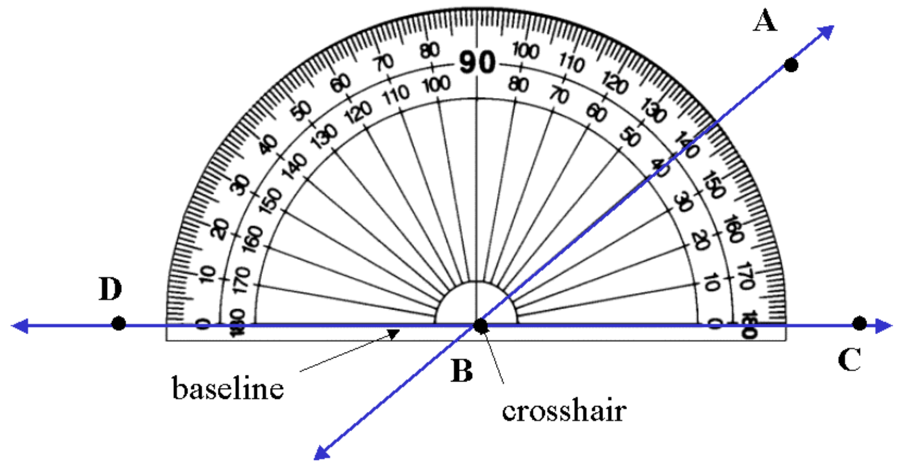
USING A PROTRACTOR

Using a protractor

40. Put the cross hair of the protractor on the vertex of the angle with one of the lines along the baseline of the protractor. Count, **starting from zero**, the number of degrees on the appropriate inner or outer ring

The '*measure*' of **angle ABD = 140**°.

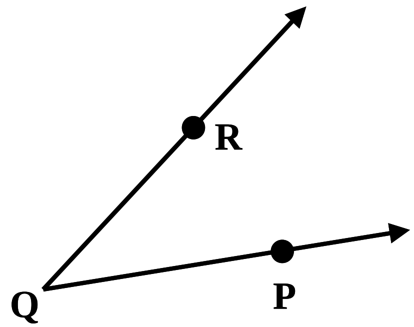
We also say: **m∠ABD = 140°**.



Determine m∠**ABC** ______________°

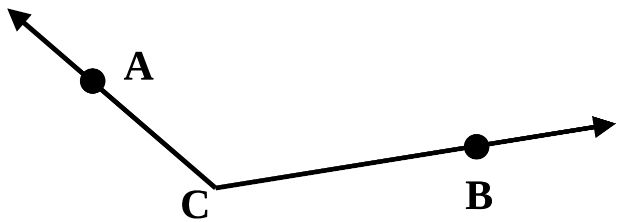
41. Name the following angles and measure them to the nearest **whole** degree:
(be aware that your answer may differ from your classmate's by one or two tick marks depending on how good you are at lining up the protractor and what eye you squint through, etc.)

a.



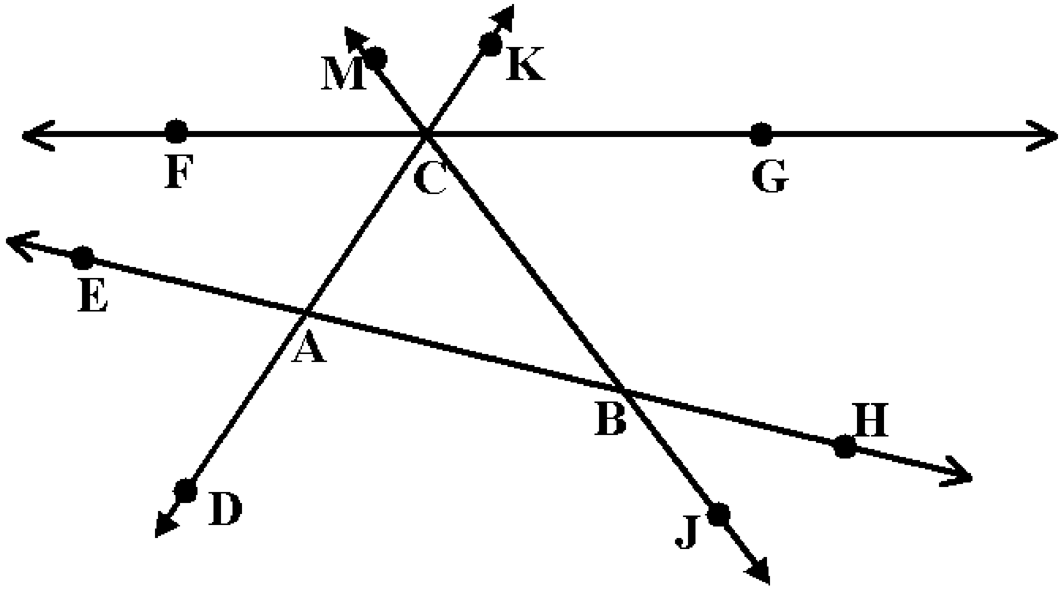
Name: _______
Measure: _______

b.



Name: _______ Measure: _______

42. Measure and record below the indicated angles in units of degrees:

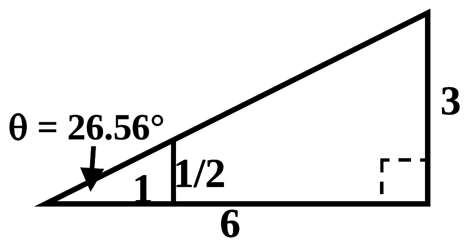


you may need to 'produce' a line; extend it.

Anytime you use a measuring device there is an associated error, so if you are within one or two degrees of the given answer that is ok.

a. m∠BAC 70°	b. m∠DAE 70°	c. m∠HBJ 38°	d. m∠ABC 38°
e. m∠ABJ 142°	f. m∠BAC 70°	g. m∠ABH 180°	h. m∠DAH 110°
h. m∠KCM 72°	h. m∠MCK 72°	h. m∠GCM 127°	h. m∠FCG 180°

TRIGONOMETRIC RATIOS OF RIGHT-ANGLE TRIANGLES

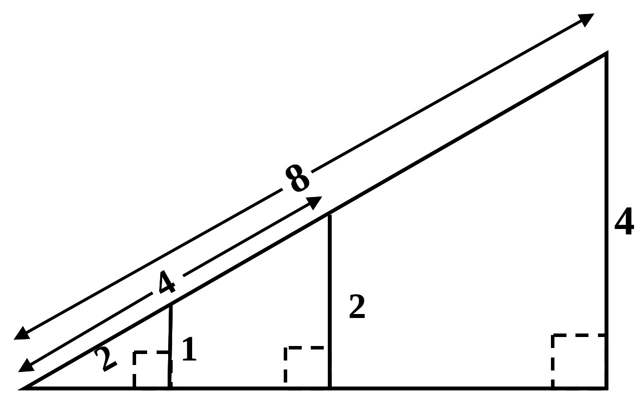


43. You know the exact shape of any right-angle triangle as soon as you are told one of its *trigonometric ratios*. So for example, if your Aunt says she wants you to build her a ramp up to her house with a '***tangent of 0.5***' she wants you to build a ramp that **rises** half a foot for every foot it **runs** along the ground. This works out to be pretty much **26.56º.**

44. Whether you measure the ramp's bottom corner angle by a **rise** per run of $\frac{3}{6}\ or\ \frac{1/2}{1}\ or\ \frac{6}{12}$ it would still be the same triangle ***shape***, the same **angle**, the same **slope**.

45. Or maybe your Aunt changes her mind, (*it is the well-deserved prerogative of all women to change their mind*) she says she wants a ramp that rises ***two feet for every four feet*** she rolls ***along*** the slope of the ramp instead!

46. So you could buy a 4 foot board and slant it 2 feet above the ground, **or** a **2** foot board and slant it **1** foot above the ground, **or** maybe an **8** foot board and slant it **4** feet above the ground.



How about a six foot board? How many feet above the ground is its height? :____________.

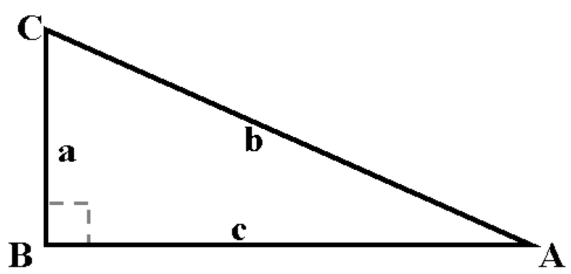
47. They all make the same angle with the ground! It works out that the measure of the bottom left corner, ie: the angle with the ground, is **exactly 30°**.

All these similar ***triangles*** have exactly the same corresponding angles.

It works out if you measure it that ***every*** right-triangle that rises one foot up for every two feet along its *hypotenuse* that the corner opposite is exactly a measure of 30 degrees (**30°**). Theother one is therefore **60°.**

DEFINITION OF TRIGONOMETRIC RATIOS OF A TRIANGLE

48. Look at a right triangle. There is one to the right.



Recall my Momma Bear and Baby Bear rule. What happens to the measure of corner **A** if we make side **a** smaller but leave the hypotenuse the same?

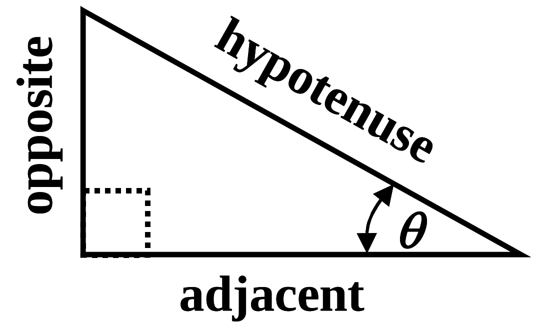
____

What happens to the measure of corner **A** if we make side **a** bigger and leave the hypotenuse the same?

____

Explore this *yourself* some more changing **side c** to see how corner A changes.

49. You can measure the '*pointiness*' of an angle 'θ' of a right-angle triangle by three different measures:



Tangent of θ = $\dfrac{opposite\ length}{adjacent\ length}$

50. We tend to use Greek letters like '*Theta*', θ, to represent angles.

Sine of θ = $\dfrac{opposite\ length}{hypotenuse\ length}$

Cosine of θ = $\dfrac{adjacent\ length}{hypotenuse\ length}$

MNEMONIC TO REMEMBER THE TRIGONOMETRIC RATIOS

51. A Mnemonic is a sentence or series of letters that helps you remember something.

SOH CAH TOA. Chant it to yourself 10 times now!

'SOH' reminds you that **S**ine is **O**pp / **H**yp
'CAH' reminds you that **C**osine is **A**dj / **H**yp
'TOA' reminds you that **T**angent is **O**pp / **A**dj

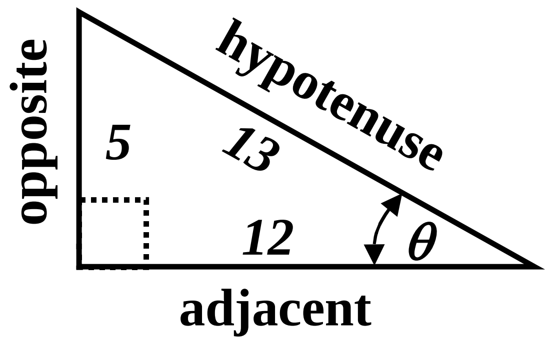
Write it at the top of every page of every trigonometry question! **Seriously!** If it not there I will not help you! It is a formula!

Think and scribble! A trig ratio is really just a way to compare any triangle with another similar triangle having one of the sides equal to one.

PRACTICE: TRIGONOMETRIC RATIO – SECTION REVIEW

52. Find the sine, cosine, and tangent ratios of the triangles below (express as exact values and as a decimals to three places). (*Caution: The triangles are not drawn to scale*).

a.



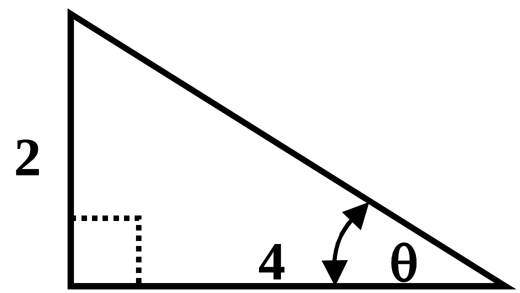
$\sin(\theta) = \frac{O}{H} = \underline{\qquad}$ = **0.**_________ decimal

$\cos(\theta) = \frac{A}{H} = \underline{\qquad}$ = **0.**_________ decimal

$\tan(\theta) = \frac{O}{A} = \underline{\qquad}$ = __________

Ans: 5/13 or 0.385 ; 12/13 or 0.923, 5/12 or 0.417

b.



$\sin(\theta) = \frac{O}{H} = \underline{\qquad}$ = **0.**__________

$\cos(\theta) = \frac{A}{H} = \underline{\qquad}$ = **0.**__________

$\tan(\theta) = \frac{O}{A} = \underline{\qquad}$ = __________

Ans: $^{2}/_{\sqrt{18}}\ or\ 0.471;\ ^{4}/_{\sqrt{18}}\ or\ 0.943;$
$^{2}/_{4}\ or\ ^{1}/_{2}\ or\ 0.500$

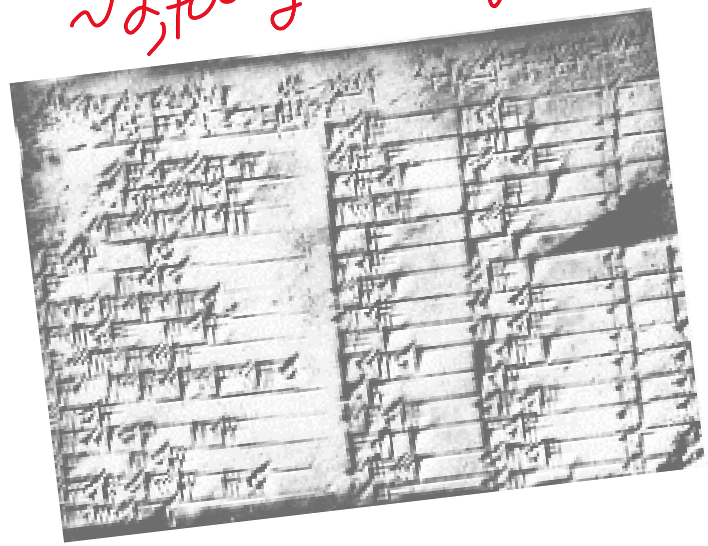
TRIGONOMETRY TABLES

53. In the ***old days*** (early ***70's***) we had ***no*** calculators, certainly none that did trigonometry. If you wanted a value of a *trigonometric ratio* for any given angle of a right angle triangle you had to look it up in a table. See the abbreviated table below:

degrees	sin	cos	tan
0	0	1	0
10	0.174	0.985	0.176
20	0.342	0.940	0.364
30	0.500	0.866	0.577
40	0.643	0.766	0.839
50	0.766	0.643	1.192
60	0.866	0.500	1.732
70	0.940	0.342	2.747
80	0.985	0.174	5.671
90	1.000	0.000	undefined

Trigonometric Ratios Table
Rounded to three decimal places

Here is what they used ~2,700 years ago



A more full table of values is included at the end of these notes as an Appendix.

54. Make sure you know how to find the **Sin**, **Cos**, and **Tan** ratios of any angle on your calculator to get the above results. Be careful to ensure your calculator is operating in 'degree' mode. Make sure you know how to do it on a variety of calculators too; some calculators you press the **sin** , **cos** , **tan** button after the angle, some you press it before!

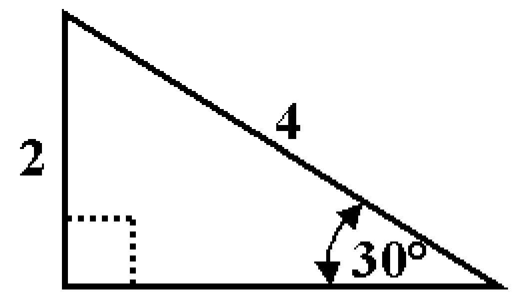
55. Complete this table for selected sine, cosine and tangent functions below using a calculator: (make sure it makes sense by estimating from the table). Usually, we like to be a bit more accurate with trig ratios so go to the nearest thousandths, 0.001, rounding.

sin(60°) = 0.866	tan(60°) = 1.732	cos(45°) = 0.707	sin(45°) = 0.707
Tan(25°) = 0.466	tan76° = 4.011	cos(73°) = 0.292	tan(90°) = infinity??

56. **Angular Mode on a calculator**. Be careful when working with angles on a calculator or any computer application. Angles can be measured in several units, *not just degrees*. For now you only know degrees [**°**] so make sure your calculator shows a little '**DEG**' in the display when using trigonometry formulas. Note if you are in EXCEL that EXCEL does not understand degrees. See me if you want to know how to make EXCEL work for angles.

INVERSE TRIGONOMETRIC FUNCTION

60. The three trigonometric functions studied so far ***give a numerical value*** for ***any given angle*** measure. The value that they give is the ratio of two sides of a right-angle triangle. For example, every right-angle triangle that has a corner whose sine is **0.5** has an opposite side that is half as long as its hypotenuse and it is a **30°** corner. Notice the final corner must be **60°**.



sin(30º) = O / H = 2/4 = 0.5

You would know this too if you ever studied geometry

61. Every angle has a ratio of a sine, a cosine, and a tangent function that goes with it, as evident from the tables.

But, ***what if you are told the ratio*** (the numerical value) and want to find the angle [°] that goes with it? That is called performing an '***inverse trigonometric function'***. All you do is look up '***backwards***' in the tables. You are *undoing* the 'trig function'.

degrees	sin	cos	tan
0	0	1	0
10	0.174	0.985	0.176
20		0.940	0.364
30	0.500	0.866	0.577
40	0.643	0.766	0.839
50	0.766	0.643	1.192
60	0.866	0.500	1.732
70			2.747
80	0.985	0.174	5.671
90	1.000	0.000	undefined

Rounded to three places. So naturally a more accurate calculator will give slightly more accurate answers.
*Trig values are generally **irrational**, they cannot be represented exactly by a decimal*. *Consequently, inverse trig functions will seldom find the exact angle.*

To use a table backwards, or to '*undo*' an operation or function, is called an '**inverse function**'.

The '***inverse cosine***' of the ratio **0.940** is very near **20°**.

The '***inverse tangent***' of **2.747** is very near **70°**.

Inverse trig functions are written like this:
$\mathbf{sin^{-1}(x)}$, $\mathbf{cos^{-1}(x)}$, $\mathbf{tan^{-1}(x)}$.

Inverse trig functions take a **number** and **find the angle** that made that number (ratio).

The inverse trig functions are also *sometimes called* 'arc-sine', 'arc-cosine', and 'arc-tangent' or simply '**arcsin**', '**arccos**', and '**arctan**'.

INVERSE TRIG FUNCTIONS ON YOUR CALCULATOR.

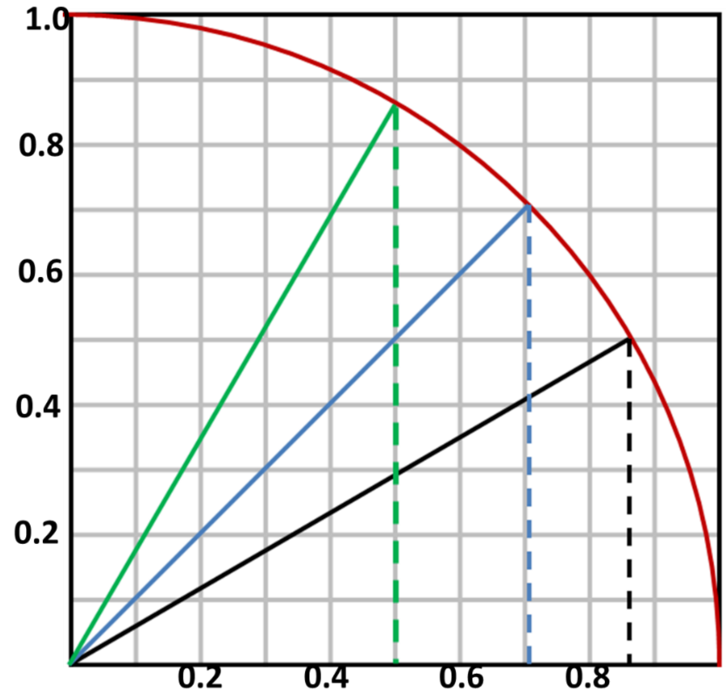
62. Complete the table below using *your* calculator. Check that it is sensible from the table. Round to the nearest 0.1 degree. Try them on a different calculator too so you learn to use a variety of calculators.

a. $\tan^{-1}(1.732)$=	b. $\cos^{-1}(0.577)$=	c. $\sin^{-1}(0.500)$=	d. $\tan^{-1}(1.000)$=
e. $arcsin\left(\frac{\sqrt{3}}{2}\right)$=	f. $tan^{-1}(\sqrt{3})$=	g. $cos^{-1}(1.6)$ *(trick!)*	h. $sin^{-1}(1)$

63. Notice that all you really need to find all the parts of a right-triangle are three parts (one of which must be a side) then you can figure out all the rest for the exact size and shape of the triangle! So being given two sides of a triangle and there being told it has a 90º corner you can figure out the entire triangle.

TRIGONOMETRIC RATIOS AS CENTRAL ANGLES ON THE ARC OF A CIRCLE

64. If you did not like studying triangles, maybe this sector of a circle will help! Get your teacher to explain this. This is actually much easier to understand and to estimate the trigonometric ratios. It turns out that the **sine** of an angle is just how **high** up you are on the circle (or a ferris wheel), the **cosine** is just how far to the **right** you are on the circle, and the tangent is just how high up you are if you extended out the very right edge of the diagram! omg.

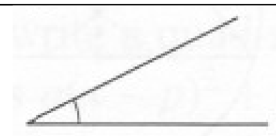
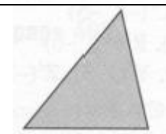
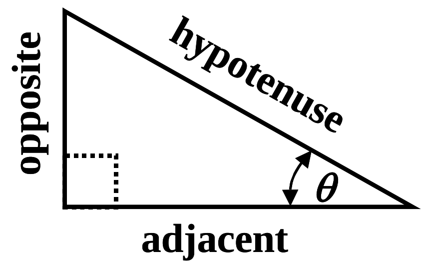
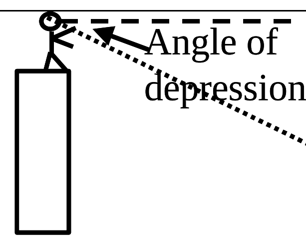
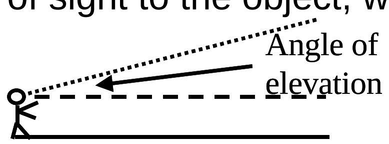


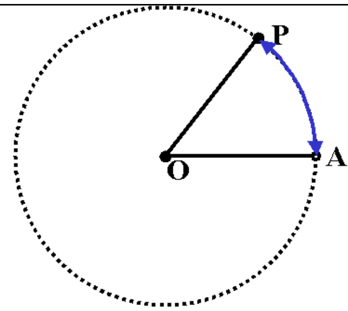
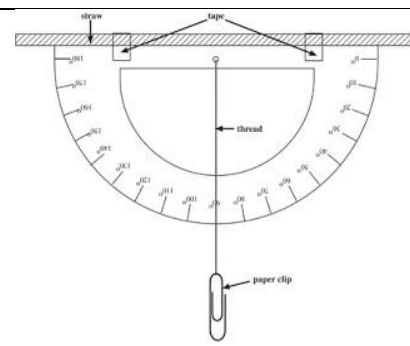
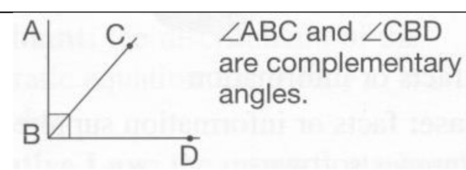
CONCLUSION

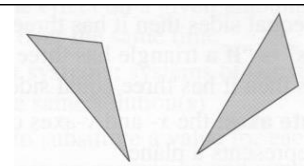
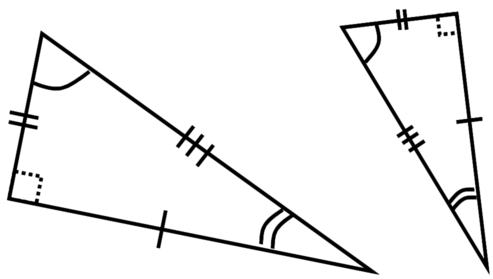
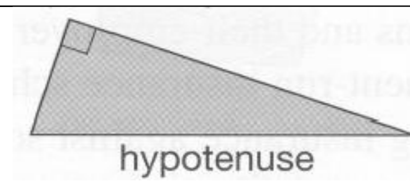
65. Congratulations for completing this Grade 10 Essential Trigonometry. 40% of it should have been review from previous studies. So, it should have been a good refresher.

In Grade 11 you will learn how to handle triangles that are not right-angle.

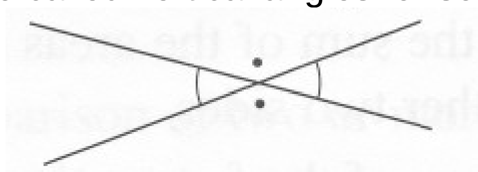
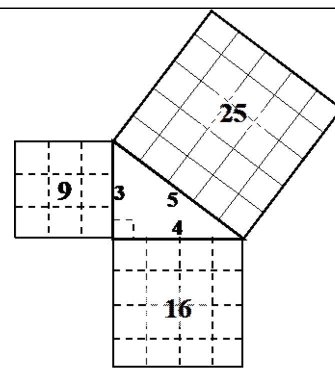
APPENDIX A - GLOSSARY

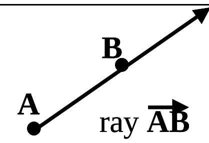
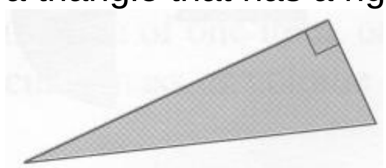
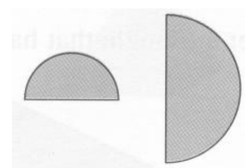
acute angle	an angle measuring less than 90°	
acute triangle	a triangle with three acute angles	
adjacent	‘next to’. In a triangle the side that is closest to the angle being measured, that is not the hypotenuse, is the adjacent side. The remaining side is called the opposite side. ***Caution***, the opposite and adjacent sides switch depending upon which corner you are working with.	
Angle of depression:	the angle formed between the horizon line and the line of sight to the object, which is below the observer's eye.	
Angle of elevation	the angle formed between the horizontal plane and the line of sight to the object, which is above the observer's eye.  *Notice if you are looking down at someone and they are looking up at you that your angle of depression to them is their angle of elevation to you!*	
algebra	the manipulation of values, variables, and expressions in a formula to solve a problem using very basic and logical rules.	

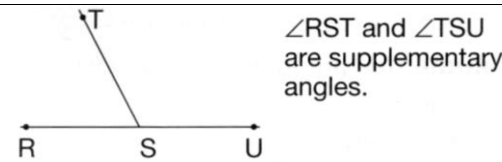
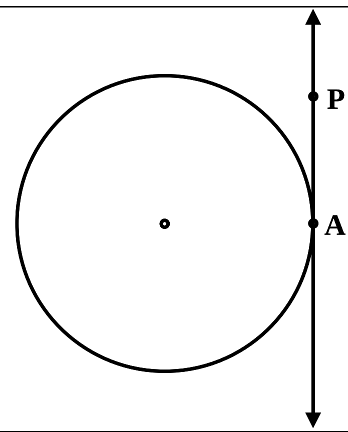
altitude	the perpendicular distance from the base of a figure to the highest point of the figure. Also the height of an object above the earth's surface.
approximation	a number close to the exact value of a measurement or quantity. Symbols such as ≈ or ≅ or are used to represent approximate values.
central angle	the angle formed ('*the angle subtended*') by the arc of a circle measured from the centre of the circle. Angle AOP is a central angle. There is a central measure of 360° around a full circle. 
clinometer	a simple device to measure angles of elevation and depression. The vertical is determined by gravity. Once used in surveying and navigation. A slurpee straw and protractor make a great clinometer. 
complementary angles	two angles whose sum is 90° ∠ABC and ∠CBD are complementary angles. 

congruent	figures that have the same size and shape, but not necessarily the same orientation 
cosine	for an acute $\angle A$ in a right triangle, the ratio of the length of the side adjacent to $\angle A$, to the length of the hypotenuse: $$cos(\angle A) = \frac{length\ of\ side\ adjacent\ to\ \angle A}{length\ of\ hypotenuse\ side} = \frac{adj}{hyp}$$
corresponding	corresponding angles and corresponding sides of two similar figures explains which side and which angle of each figure 'marries up' with the matching side and angle of the other similar figure.  *Similar Triangles with Corresponding parts indicated*
degrees	a unit of measure of an angle. Other units of angle measure include radians and mils.
hypotenuse	the longest side of a right-angle triangle. The side opposite the right angle in a right triangle. 
indirect measurement	making a measurement of an object without actually directly measuring the object. Made possible by measuring a scale model of the object.

Inverse trig function	The inverse trigonometric functions of $\mathbf{sin^{-1}}$, $\mathbf{cos^{-1}}$, and $\mathbf{tan^{-1}}$ 'undo' their respective functions. They tell you the angle that goes with any particular trigonometric ratio. **Example**: if you want to know what the angle θ for a corner that has a sine of 0.5 you use: The angle θ you want is = $\mathbf{sin^{-1}(0.5)}$; and it tells you the angle is **30** degrees. Also called **arcsin**, **arccos**, and **arctan**.
irrational number	a number that is **not rational**. (*think: irresponsible means NOT responsible*). It cannot be calculated to perfect accuracy, it cannot be fully written down. It has **no** rational pattern of repetition in its decimal places. Examples of irrational numbers are π and $\sqrt{5}$, etc. It exists in your head but cannot be written down.
isosceles acute triangle:	a triangle with two equal sides and all angles less than 90°
isosceles obtuse triangle:	a triangle with two equal sides and one angle greater than 90°
isosceles right triangle:	a triangle with two equal sides and a 90° angle
isosceles triangle:	a triangle with [at least] two equal sides
legs	the shorter sides of a right triangle that form the right angle

opposite angles	the equal angles that are formed by two intersecting lines. (Also called vertical angles for some reason) 
perpendicular	two lines are perpendicular if the angle between them is 90 degrees. We use the math symbol: ' ⊥ ' or use a little square in the corner of shapes.
polygon	the union of three or more line segments that are joined together so as to completely enclose an area.
proportion	a statement that two ratios are equal. Usually expressed as a fraction. Eg: if one muffin takes 6 raisins, then 10 muffins takes 60 raisins. $\frac{1}{6} = \frac{10}{60}$ or 1:6 = 10:60
Pythagorean Theorem	for any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides or symbolically: $\mathbf{c^2 = a^2 + b^2}$; **provided** that **c** is the hypotenuse length. 
Pythagorean triple	Three natural numbers that satisfy the Pythagorean theorem. One example is: 3, 4 and 5. 5, 12, 13 also works.
repeating decimal	a decimal in which the decimal digits endlessly repeat in a pattern. A repeating decimal can *always* be re-written as a fraction. Eg: **3.566566566566**… is **3563/999** as a fraction**.**

ray	a half line. It has one end point and continues in a fixed direction without any end. It is named by labelling its end point and another point on the ray. 
right angle	an angle whose measure is 90 degrees.
right triangle	a triangle that has a right angle. 
rhombus	a quadrilateral with four equal sides. Really just a square that is leaning. A rhombus is a special case of a parallelogram.
similar figures	figures with the same shape, but not necessarily the same size 
sine	for an acute $\angle A$ in a right triangle, the ratio of the length of the side opposite $\angle A$, to the length of the hypotenuse $$sin(\angle A) = \frac{length\ of\ side\ opposite\ to\ \angle A}{length\ of\ hypotenuse\ side} = \frac{opp}{hyp}$$
square of a number	To multiply two of the same number together. example: **3 * 3 = 9**; **6 * 6 = 36**; **25 * 25 = 625**. We often use a little 'two exponent' to write a square, eg: $\mathbf{6^2 = 36}$ or $\mathbf{9^2 = 81}$, etc. You have a button on your calculator that does a square, the $\mathbf{x^2}$ button.

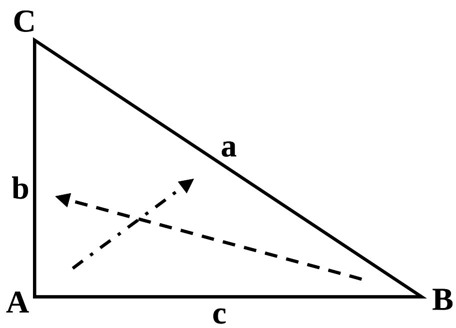
square root	Square root of some number, **x**, is the desired number that, when multiplied by itself gives that number, **x.** for example, $\sqrt{9}=3$ because **$3^2 = 9$**. To do a square root is really just to 'un-square' a number. Just doing a square backwards! The $\sqrt{\ }$ is called the radical sign. The number inside it is called the radicand. In $\sqrt{10}$, 10 is the radicand. Find that button on your calculator too.
Sum of angles in a triangle	One of the important properties of a triangle. The sum of the angles inside a triangle is 180°.
supplementary angles	two angles whose sum is 180° 
tangent of a triangle	for an acute $\angle A$ in a right triangle, the ratio of the length of the side opposite $\angle A$, to the length of the side adjacent to $\angle A$ $$tan(\angle A) = \frac{length\ of\ side\ opposite\ to\ \angle A}{length\ of\ side\ adjacent\ to\ \angle A} = \frac{opp}{adj}$$
tangent to a curve	a line that touches a curve or shape in just one place. Where it touches is called the point of tangency. At the right, line AP is tangent to the circle. 
theorem	a statement that has been proven.
theta	a greek letter. The lower case *theta* looks like this: θ. Often used as a symbol to represent an angle. Other greek letters that mathematicians tend to use to indicate an angle include especially Alpha, α, and Beta, β. Or heck, you could just use '***x***' if you want.

triangle types	Triangles have certain named characteristics depending on their lengths and also their angles. **Length Characteristic**: all three sides the same : **equilateral** at least two sides the same: **Isosceles** no two sides the same : **scalene** **Angle Characteristic**: all acute angles: **acute triangle** a 90 degree square corner: **right triangle** on obtuse angle (>90°): **obtuse triangle**

APPENDIX B: EXTRA NOTES AND EXAMPLES

LABELING A TRIANGLE

1. You may find that many text books and other study units label individual triangles like this:



2. We always name the corners with CAPITAL LETTERS, and the sides opposite those respective corner angles with the *same but small* 'lower case' letters.

3. So in the case of this type of labelling you could say that:

$$\boldsymbol{sin}(\boldsymbol{B}) = \frac{\boldsymbol{Opp}}{\boldsymbol{Hyp}} = \frac{\boldsymbol{b}}{\boldsymbol{a}} \qquad \boldsymbol{sin}(\boldsymbol{C}) = \frac{\boldsymbol{Opp}}{\boldsymbol{Hyp}} = \frac{\boldsymbol{c}}{\boldsymbol{a}}$$
$$\boldsymbol{cos}(\boldsymbol{B}) = \frac{\boldsymbol{Adj}}{\boldsymbol{Hyp}} = \frac{\boldsymbol{c}}{\boldsymbol{a}} \qquad \boldsymbol{cos}(\boldsymbol{C}) = \frac{\boldsymbol{Adj}}{\boldsymbol{Hyp}} = \frac{\boldsymbol{b}}{\boldsymbol{a}}$$
$$\boldsymbol{tan}(\boldsymbol{B}) = \frac{\boldsymbol{Opp}}{\boldsymbol{Adj}} = \frac{\boldsymbol{b}}{\boldsymbol{c}} \qquad \boldsymbol{tan}(\boldsymbol{C}) = \frac{\boldsymbol{Opp}}{\boldsymbol{Adj}} = \frac{\boldsymbol{c}}{\boldsymbol{b}}$$

you might see some interesting patterns when it is presented that way!

THE GREEK LETTERS *(you will not be tested on this, enquiring minds only)*

4. Often we use Greek letters to show the measure of an angle. It just makes formulas a little easier to follow if Greek letters are used for angles, and English (ie: Latin) letters for lengths.

5. Below is a table of Greek letters. Of course they have 'lower case' and 'upper case' versions just like us. The Greek letters we tend to use most often though to measure angles are: **θ, α, and β** or **'Theta', 'Alpha',** and **'Beta'**.

Alpha α Α	**Beta** β Β	**Gamma** γ Γ	**Delta** δ Δ	**Epsilon** ε Ε	**Zeta** ζ Ζ
Eta η Η	**Theta** θ Θ	**Iota** ι Ι	**Kappa** κ Κ	**Lambda** λ Λ	**Mu** μ Μ
Nu ν N	**Xi** ξ Ξ	**Omicron** ο Ο	**Pi** π Π	**Rho** ρ P	**Sigma** σ Σ
Tau τ Τ	**Upsilon** υ Υ	**Phi** φ Φ	**Chi** χ Χ	**Psi** ψ Ψ	**Omega** ω Ω

6. In **Grade 10** math you will generally use only θ, α, β, and of course π. Later in your math studies you will use σ and Σ for statistics.

If you are starting any high school science classes you might see: 'Δ' for '*change*' in motion, 'Ω' for resistance in electricity, 'ρ' for density of a fluid, 'λ' for wavelength of sound waves or light waves, and perhaps 'ω' for rotation rate in physics.

You will get to use the remainder if you go to university!

PRACTICE PROBLEMS

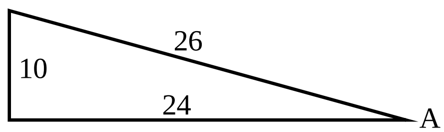
1. In addition to all your assignments you may want to have a good set of notes with these basic problems that you solve here.

2. Calculate the sin, cos, tan, and the value of the angle in degrees of the following indicated angles of the given ***right-angle*** triangles.

a. Calculate sin, cos, and tan, to the nearest three decimal places. (Two decimal places is not sufficiently accurate for finding angles that are close to zero or one)

b. Calculate angles to the nearest 0.1 degrees

a.



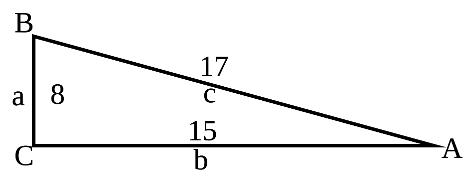
sin(A) = $\frac{Opp}{Hyp} = \frac{10}{26} = 0.385$

cos(A) = $\frac{Adj}{Hyp} = \frac{24}{26} = 0.923$

tanA = $\frac{Opp}{Adj} = \frac{10}{24} = 0.417$

Angle A (ie: $m\angle A$) = <u>22.6</u>°

b.



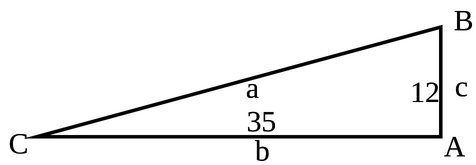
sin(B) = $\frac{Opp}{Hyp}$ =

cosB = $\frac{Adj}{Hyp}$ =

tan(B) = $\frac{Opp}{Adj}$ =

Angle B (ie: $m\angle B$) = ________°

c.



sin(C) = $\frac{Opp}{Hyp}$ =

cos(C) = $\frac{Adj}{Hyp}$ =

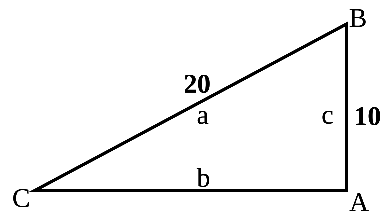
tanC = $\frac{Opp}{Adj}$ =

Angle C (ie: $m\angle C$) =

______________°

Hint: You will need Pythagoras to help answer completely

d.



sin(B) = $\frac{Opp}{Hyp}$ =

cos(B) = $\frac{Adj}{Hyp}$ =

tanB = $\frac{Opp}{Adj}$ =

Angle B (ie: $m\angle B$) =

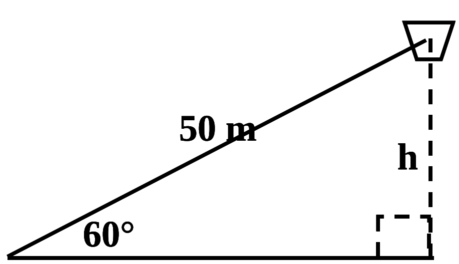
______________°

WORD PROBLEMS INVOLVING TRIGONOMETRY

3. Remember, if you know any three parts of a triangle you can solve all the other three parts of the triangle! (*well …. except for one type of case*)

*Make sure your calculator is in **degree mode**! Degrees is the ways earth people measure angles. There is a much better unit you will learn if you take calculus some day.*

4. **Example find a side**: Carol and her son are flying a kite. The kite makes an angle of **60°** to the ground, the string is **50** metres long. What is the height, ***h***, (*ie*: altitude) of the kite above the ground?



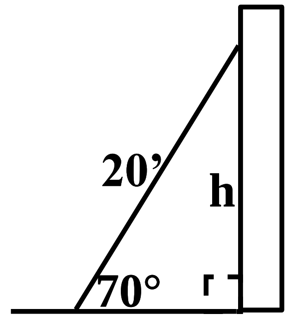
We have three things given (two angles and a length of a side) so we can find all the rest.

$sin(60) = \frac{h}{50}$; $\therefore 0.866 = \frac{h}{50}$; mult bs by 50

$\therefore 50 * 0.866 = h$ So **h = 43.30** m .

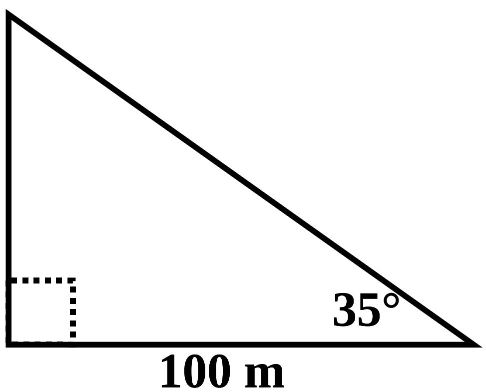
You Try:

5. Gabriel is leaning a 20 foot ladder against a wall. The ladder says on it should not be more than a 70° angle with the ground for safety reasons. How high up the wall, height '***h***', will the ladder reach?



You try: *Make sure your calculator is in degree mode!*

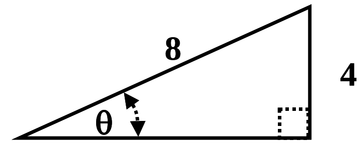
6. Florence wants to calculate the height of a building. She stands 100 metres away from the base of the building and measures a 35° 'angle of elevation' to the top of the building. What is the height of the building**?**
Hint: *Use the tan function.*



Example find an angle.

7. Your Aunt wants to make a ramp up to her doorstep. The step is **4 ft** above the ground. The board she gives you is **8 feet** long. *What angle* will the ramp make with the ground?

a. We want the bottom left corner angle. We are given that it is a right angle, the opposite side to the angle is 4 and the hypotenuse is 8. Easy!



$$sin\,\theta = \frac{Opp}{Hyp} = \frac{4}{8} = 0.5$$

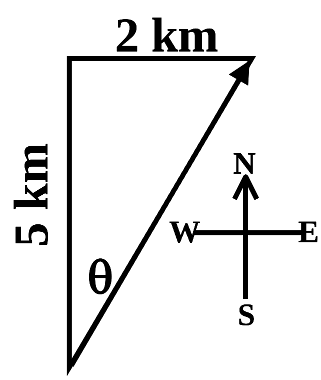
b. Now find what angle has a sine of **0.5**! Use the tables or better yet the sin^{-1} button on the calculator.

$\theta = sin^{-1}(0.5) = 30°$ so, angle θ = 30°

Make sure your calculator is in degree mode!

You Try

8. A police helicopter is over a certain building in the city. The crew is told to go five km north and two km east. What angle from north must it fly? (*Hint: inverse tan, ie:* ***tan^{-1}***)



If you like this sort of question we do lots in the Grade 12 Applied Vectors Unit and in Physics.

APPENDIX C: TRIGONOMETRIC VALUES FOR ANGLES 0° TO 90°

Angle	Sin	Cos	Tan	Angle	Sin	Cos	Tan
0	0	1	0	46	0.7193	0.6947	1.036
1	0.0175	0.9998	0.0175	47	0.7314	0.6820	1.072
2	0.0349	0.9994	0.0349	48	0.7431	0.6691	1.111
3	0.0523	0.9986	0.0524	49	0.7547	0.6561	1.150
4	0.0698	0.9976	0.0699	50	0.7660	0.6428	1.192
5	0.0872	0.9962	0.0875	51	0.7771	0.6293	1.235
6	0.1045	0.9945	0.1051	52	0.7880	0.6157	1.280
7	0.1219	0.9925	0.1228	53	0.7986	0.6018	1.327
8	0.1392	0.9903	0.1405	54	0.8090	0.5878	1.376
9	0.1564	0.9877	0.1584	55	0.8192	0.5736	1.428
10	0.1736	0.9848	0.1763	56	0.8290	0.5592	1.483
11	0.1908	0.9816	0.1944	57	0.8387	0.5446	1.540
12	0.2079	0.9781	0.2126	58	0.8480	0.5299	1.600
13	0.2250	0.9744	0.2309	59	0.8572	0.5150	1.664
14	0.2419	0.9703	0.2493	60	0.8660	0.5000	1.732
15	0.2588	0.9659	0.2679	61	0.8746	0.4848	1.804
16	0.2756	0.9613	0.2867	62	0.8829	0.4695	1.881
17	0.2924	0.9563	0.3057	63	0.8910	0.4540	1.963
18	0.3090	0.9511	0.3249	64	0.8988	0.4384	2.050
19	0.3256	0.9455	0.3443	65	0.9063	0.4226	2.145
20	0.3420	0.9397	0.3640	66	0.9135	0.4067	2.246
21	0.3584	0.9336	0.3839	67	0.9205	0.3907	2.356
22	0.3746	0.9272	0.4040	68	0.9272	0.3746	2.475
23	0.3907	0.9205	0.4245	69	0.9336	0.3584	2.605
24	0.4067	0.9135	0.4452	70	0.9397	0.3420	2.747
25	0.4226	0.9063	0.4663	71	0.9455	0.3256	2.904
26	0.4384	0.8988	0.4877	72	0.9511	0.3090	3.078
27	0.4540	0.8910	0.5095	73	0.9563	0.2924	3.271
28	0.4695	0.8829	0.5317	74	0.9613	0.2756	3.487
29	0.4848	0.8746	0.5543	75	0.9659	0.2588	3.732
30	0.5000	0.8660	0.5774	76	0.9703	0.2419	4.011
31	0.5150	0.8572	0.6009	77	0.9744	0.2250	4.331
32	0.5299	0.8480	0.6249	78	0.9781	0.2079	4.705
33	0.5446	0.8387	0.6494	79	0.9816	0.1908	5.145
34	0.5592	0.8290	0.6745	80	0.9848	0.1736	5.671
35	0.5736	0.8192	0.7002	81	0.9877	0.1564	6.314
36	0.5878	0.8090	0.7265	82	0.9903	0.1392	7.115
37	0.6018	0.7986	0.7536	83	0.9925	0.1219	8.144
38	0.6157	0.7880	0.7813	84	0.9945	0.1045	9.514
39	0.6293	0.7771	0.8098	85	0.9962	0.0872	11.43
40	0.6428	0.7660	0.8391	86	0.9976	0.0698	14.30
41	0.6561	0.7547	0.8693	87	0.9986	0.0523	19.08
42	0.6691	0.7431	0.9004	88	0.9994	0.0349	28.64
43	0.6820	0.7314	0.9325	89	0.9998	0.0175	57.290
44	0.6947	0.7193	0.9657	89.5	1.0000	0.0087	114.589
45	0.7071	0.7071	1.0000	90	1	0	Infinity