

**MY! GRADE 12 APPLIED COURSE REFERENCE NOTES (copy them if you want)**

**UNITS A & E- PROBABILITY, PERMUTATIONS & COMBINATIONS**

**Fundamental Counting Principle (FCP):** If one event can occur in 'a' ways, a second event in 'b' ways, a third event in 'c' ways, and so on, then the number of ways that all events can occur one after the other is the product  $a*b*c...$ . Eg: number of license plates we can make  $26*26*26*10*10*10$ . **Watch** if repetitions are allowed or not.

**Permutations:** **Order does matter!** 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> in a race of 12 runners.  $12*11*10$  possible choices. Or  ${}_{12}P_3$ .

**Counting Non-Distinguishable Objects:** **4 Red balls, 2 Green Balls.** Number of distinguishable ways to arrange the 6 balls with 4 red and 2 green balls is  $\frac{6!}{4!2!}$ . Just like arranging YES and Nos. If only two different objects then = Combo.

**Combinations.** Arrangements of objects where **order does not matter.** Selecting committees of people (no special positions or rewards, a group, bundle), Lotto 6/49, etc. Eg: how many ways can an **unorganized** committee of three people be formed from 12 people.  ${}_{12}C_3 = \frac{12!}{(12-3)!3!} = 220$

$Prob(A) = P(A) = \frac{\# \text{ of Favoured Outcomes}}{\# \text{ of Total Possible Outcomes}}$ ; eg: **Prob(Draw a King) = 4/52.** // **Sample space:** the list of all possible outcomes. Use a tree or table. // **Outcome:** the result of one trial of an experiment (eg: flipping one coin has only H or T outcome) // **Event:** A set of outcomes. Eg: rolling two dice, an event might be the set of outcomes where doubles were rolled.

**Complement.** The probability of an event happening is " **$1 - \text{the probability it won't happen}$** ". **Complement** of event A is  $\bar{A}$ . So  $P(\bar{A}) = 1 - P(A)$ . **At least once Probs:**  $Prob(A's \geq 1 \text{ time}) = 1 - P(\text{no } A's)$

**ODDS.** Odds in favour = favourable: not favourable; eg. wins : losses. **Odds Against = not favourable : favourable**

**Formulas:** Factorial:  $6! \equiv 6*5*4*3*2*1$ ; Permutation:  ${}_nP_r = \frac{n!}{(n-r)!}$ ; Combination:  ${}_nC_r = \frac{n!}{(n-r)!r!}$

**Multiplying Probabilities:** For successive events. **Keyword: AND.** Often one probability is dependent on the first. Often indicates whether something is drawn and with replacement or not.

**Independent example:** Let A be event of being hit by bus. Let B be event of winning lottery. Both are independent events. Say  $P(A) = 0.01$ , and  $P(B) = 0.01$ . Prob of getting hit by a bus and winning the lottery =  $Prob(A \text{ AND } B) = P(A)*P(B) = 0.01*0.01 = 0.0001$ .

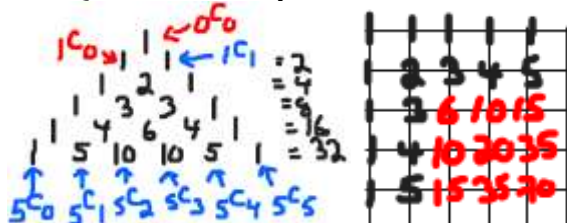
**Dependent example.** Standard deck of cards. Probability of drawing a King then a Queen without replacing the first card. Let:  $K_1$  = event of drawing King first draw;  $Q_2$  = event of drawing Queen second.  **$P(K_1 \text{ and then } Q_2) = P(K_1)*P(Q_2 | K_1) = \frac{4}{52} * \frac{4}{51} = \frac{4}{663} = 0.60\%$**

**Adding Probabilities.** For compound events, multiple events. **Keyword: OR**

Eg: Let **K = Set of all Kings.** Let **S = Set of all spades.**

$Prob(K \text{ OR } S) = P(K) + P(S) - Prob(K \text{ AND } S) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} = 30.7\%$  We are subtracting out the common card to set K AND set S; that is the  $K\spadesuit$  so we don't double count it. **Mutually exclusive Events.** The two events or sets that share no common outcomes! ie: Kings AND Queens are mutually exclusive. **\*\*\*OR  $\equiv$  Add ; AND  $\equiv$  Multiply\*\*\***

**Pathways.** As per usual PASCAL triangle method, or use the secret combination!  ${}_nC_r$  where n is the total number of steps, r is either the downs or the rights. Doesn't matter what you count (downs or rights)! Just like selecting non-distinguishable objects.



**UNIT B – PERSONAL FINANCE**

**Simple Interest:**  $I = P * r * t$  and  $A = P + I$ ;  $A = P(1 + r*t)$

**Compound Interest Investment:**  $A = P \left(1 + \frac{r}{n}\right)^{(n*t)}$ ; where A =

Total Amount or FV [\$], P = Principal [\$] or PV, r = annual percentage rate[%/yr], little n is compounding frequency (C/Y times per year), t is time [years].

**Rule of 72:** If  $I\% * \text{years} = 72$  then double

$\text{Net Worth} = \text{Total Assets} - \text{Total Liabilities}$   
**Debt – to – equity ratio [max 50%]** =  $\frac{\text{Total Liabilities} - \text{Mortgage}}{\text{Net worth}}$

**Gross Debt Service ratio [max 32%]** =  $\frac{\text{Monthly Mortgage Payment} + \text{Monthly Property Taxes} + \text{Monthly Heating Costs}}{\text{Gross Monthly Income}}$

Mode  End  Beginning

**Present Value** Account now. Negative if a deposit. Positive if a loan

**Payments** Periodic top-up annuities (-) or withdrawals (+)

**Future Value** Value in future. Zero if a loan

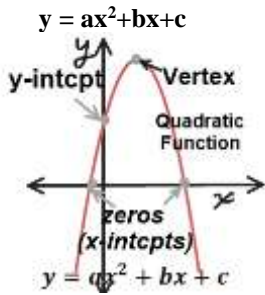
**Annual Rate (%)** Enter % value. eg: 7.5

**Periods** Nbr of Periods  
48 monthly frequency = 4 years

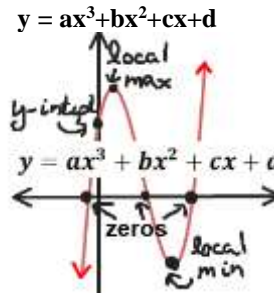
**Compounding** Frequency type  
monthly 12 pd / yr  
quarterly 4 pd / yr

**UNIT C: FUNCTIONS.** Intercepts, zeros, intersections of two curves, max, min, asymptotes

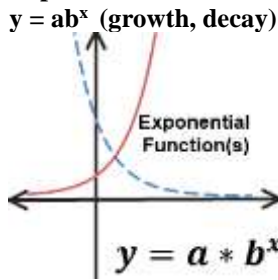
**Quadratic**



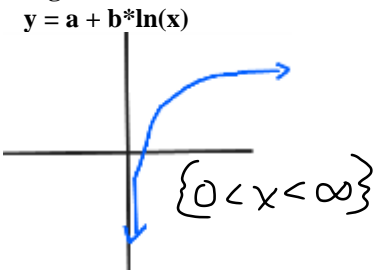
**Cubic**



**Exponential**



**Logarithmic**



**Domain:** allowed 'x's  $\{-\infty < x < \infty\}$  for most functions. **Range:** resulting y's;  $\{? \leq f(x) < ?\}$ . Often limited by a max or min value (vertex or asymptote).

**Logs.** If  $2^3 = 8$  then  $\log_2 8 = 3$ . **Change of Base:**  $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ ; eg:  $\log_2 8 = \frac{\log_{10} 8}{\log_{10} 2} = 3$

**UNIT F – SINUSOIDAL FUNCTIONS**

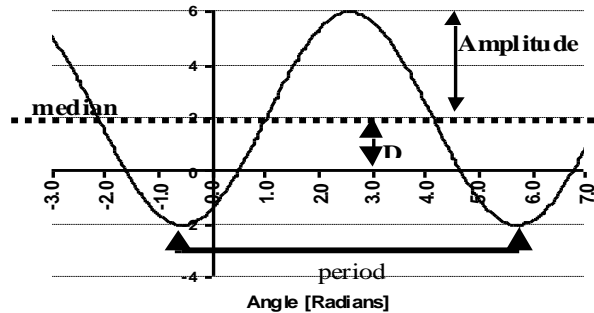
$y = A \sin(Bx + C) + D$  A is the Amplitude, B is the number times a cycle fits into  $2\pi$ . (or  $360^\circ$ ). Large B squishes the cycles in the x-domain.  $T = \text{Period} = \frac{2\pi}{B}$  or  $\frac{360^\circ}{B}$ .

C and B work together for horizontal phase shift to affect where cycle starts.

Horizontal Phase Shift of sine curve =  $-C/B$ .

D is the Median value; half the data is above; half below so D is the vertical displacement upwards from x-axis of all the data.

$D = (\text{Max} + \text{Min})/2$ .  $\text{Max} = D + A$ ,  $\text{Min} = D - A$ .



**Radians:** Exactly  $\pi$  radians in  $180^\circ$ . Conversion factor is:  $\pi^\circ/180^\circ$ . Eg:  $60^\circ = 60^\circ * \pi^\circ/180^\circ = \frac{\pi}{3} = 1.22$  radians.

$$\frac{4\pi^r}{9} = \frac{4\pi^r}{9} * \frac{180^\circ}{\pi^r} = 80^\circ$$

Make sure calculator is in proper Degree or Radian mode!

**STATISTICS**

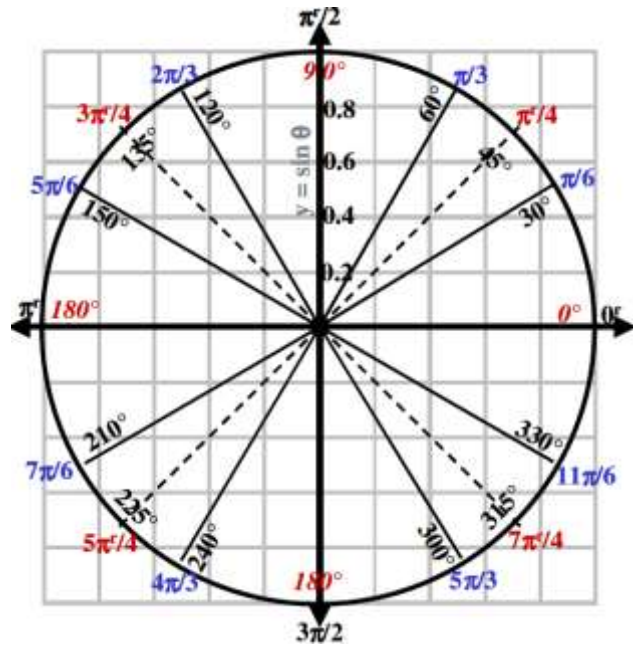
**Mean.**  $\bar{x} = \frac{\sum x_i}{n}$ ; sum up all the data and divide by the

data set size, n. **Weighted Mean:**  $\frac{\sum x_i f_i}{\sum w f_i}$

**Median,  $\tilde{x}$ .** Line data up in ascending order, find the data value at the middle place.

Middle place =  $\frac{(n+1)}{2}$ . Eg: n= 17 data  $\rightarrow$  middle place is the 9<sup>th</sup> place. With 20 data  $\rightarrow$  middle place is the mean between the 10<sup>th</sup> and 11<sup>th</sup> place, value in 10 and a 'halfth' place.

**Percentile Rank.**  $PR = \frac{B+1/2E}{N} * 100$ ; round up!; where B is the number of scores below, E is the number equal; and N is the total number. **Percentiles and Quartile Ranks.**  $P_{25} \equiv Q_1$ ;  $P_{50} \equiv Q_2 \equiv \text{Median}$ ;  $P_{75} \equiv Q_3$ .



**UNIT G – DESIGN AND MEASUREMENT** (you will be provided geometric formulae)

**Common Conversions:** 1 in = 2.54 cm. 1m = 3.28 ft. 1 m = 39.37 in. 1 Kg = 2.21 lbs. 1 ImpGal = 4.55 l.

1 km = 0.6214 mi. Example:  $3\cancel{mi} * \frac{1 km}{0.6214\cancel{mi}} = 4.83 km$  Kilo: k = 1,000. Centi: c = 1/100. Milli: m = 1/1000. 1ft =

12 inches. 1 yard = 3 ft **Common Geometric Formulas.** **Circle:**  $A = \pi r^2$ ;  $C = \pi d$  or  $2\pi r$ . **Triangle:**  $A = \frac{1}{2}bh$

$\text{Vol}_{\text{prism}} = \text{Base}_{\text{area}} * h$ .  $\text{Vol}_{\text{RecPrism}} = (l*w)*h$ .  $\text{Vol}_{\text{Cyl}} = \pi r^2 * h$ .  $\text{Vol}_{\text{Pyramid}} = 1/3 * \text{Vol}_{\text{Prism}}$

$\text{Vol}_{\text{Sphere}} = (\frac{4}{3})\pi r^3$ . SA is sum of area of all faces and sides.  $\text{SA}_{\text{Cyl}} = 2\pi r^2 + 2\pi rh$ .