GRADE 12 APPLIED

WORKBOOK

UNIT E – PERMUTATIONS AND COMBINATIONS

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Show work! So you can see how you calculated the solution
Label the answers so they can be found! Show units.
Try manually and with a calculator, different calculators even, or your favourite App. *But* you are not allowed your device Apps on Quizzes, Tests, Exam.

FUNDAMENTAL COUNTING PRINCIPLE (FCP)

1-1. A restaurant offers three sizes of pizza, three different types of crust and twelve different toppings. How many different:

a) One topping pizzas can be made?

b) Two topping pizzas can be made if you cannot use the same topping twice?

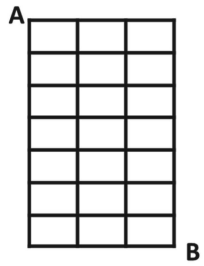
1-2. A baseball player has two different pairs of cleats, four different bats, three different fielding gloves and two pairs of batting gloves to choose from for each game. In how many ways can they choose one of each piece of equipment for a game?

1-3. Lyric goes to the movies! She can choose from the following snacks: nachos, popcorn or a hot dog. There are 9 different soft drink flavours and four different types of candy. How many different combinations of things can Lyric buy if she wants a snack, a soft drink and one type of candy?

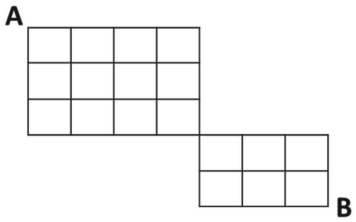
1-4. How many different license plates can be made if one is made using three letters followed by three numbers? Repetitions **are allowed.**

1-5. Postal codes in Northern Manitoba are made using R0B NLN, where N represents any number and L represents any letter. How many postal code combinations are possible for Northern Manitoba?

1-6. How many ways can you go from A to B in the diagram below if you can only move right or down?



1-7. How many ways can you go from A to B in the diagram below if you can only move right or down?

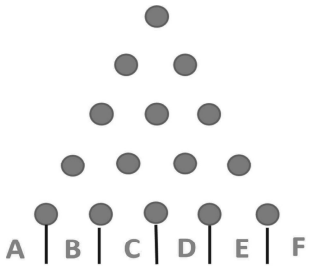


1-8. A pinball machine is constructed as follows. Exits are marked with letters at the bottom. Yellow circles represent pathways for the ball.

a) Determine the number of pathways to each lettered exit.

b) Which exit(s) has the highest probability of a ball exiting at?

c) What is the probability the ball lands in bin B?



FACTORIALS AND PERMUTATIONS

2-1. Express in factorial notation. Do not evaluate.

a. 4 x 3 x 2 x 1 =

b. 7 x 6 x 5 x 4 x 3 x 2 x 1 =

c. 5 x 4 x 3! =

d. 90 x 8! =

2-2. Evaluate:

a. 3! b. $\frac{5!}{4!}$ c. $\frac{12!}{10!2!}$ d. $\frac{10!}{5!5!}$

e. 0! f. 5.5!

2-3. Write in the permutation notation form ${}_nP_r$

a. 11 x 10 x 9

b. 5 x 4 x 3 x 2

2-4. A car dealership has a truck, a compact car, an SUV, a van, a motorcycle, a crossover and a full-sized car. In how many ways can they arrange the vehicles in a row in front of the dealership?

2-5. In how many ways can you arrange the letters in the word BRUNCH?

2-6. Gina can decorate her living room arranging different family portraits on a shelf. She has fifteen family portraits and wants to use three of them. In how many ways can Gina do this?

2-7. A ringette team of 12 girls chooses a captain and an alternate captain. In how many ways can they do this?

2-8. A band has 11 songs they sing at a concert. In how many ways can they make a set list if they want to sing the three most popular songs together?

2-9. In how many ways can 6 friends get their picture taken together in a line if the shortest and tallest person cannot be beside one another?

PERMUTATIONS WITH REPETITIONS AND RESTRICTIONS

3-1. Consider the following word arrangements:

a) How many arrangements can be made using the letters in the word SCHOOL?

b) How many arrangements can be made using the letters in the word SCHOOL if the "word" must start with C?

c) How many arrangements can be made using the letters in the word SCHOOL if the Os must be kept apart?

3-2. Using the letters of the word VELTRI, how many four letter words can be formed

a) with no restrictions?

b) if the word must start with a vowel?

c) with no vowels?

d) with vowels and consonants alternating?

3-3. Consider the digits 0, 2, 4, 5, 8, 9. If repetitions are allowed, find
a) how many 3-digit numbers can be formed?
b) how many 3-digit multiples of 10 can be formed?
c) how many 3-digit numbers can be formed that are larger than 700 and even?
d) how many 3-digit numbers that have no repeating numbers can be formed?

3-4. Danny, Nick, Forrest, Rey, Jade and Rebecca are sitting in a row at the cafeteria.
a) How many seating arrangements are possible if there are no restrictions?
b) How many seating arrangements are possible if the girls want to sit together?
c) How many seating arrangements are possible if the girls want to sit together in the middle?
d) State the assumptions you used in questions b) and c).

3-5. How many different ways can three brother/sister pairs sit in a row on a park bench if
a) there are no restrictions?
b) every brother and sister must sit together?
c) there must be a man at each end of the bench?
d) there must be a man at each end of the bench and a woman beside him?

3-6. Five brothers and five sisters line up at a wedding. How many ways can they be arranged
a) if there are no restrictions?
b) with brothers and sisters in alternating positions?

3-7. At a coffee shop, they offer 10 different drinks and four different desserts. How many ways can two people order the same drink but a different dessert?

3-8. How many 6-digit numbers can be found using the digits from 1 to 6 if

a) the even digits occupy the even places?
b) the number is less than 400 000 and the even digits occupy the even places in descending order?

3-9. Using the digits 1, 1, 2, 3, 3, 3, 5, 9 how many eight-digit numbers can be found if

a) there are no restrictions?
b) the number is even?
c) the number is even and larger than 50 000 000?

3-10. If all the letters in the word MATHEMATICS are rearranged,

a) find the number of arrangements beginning with the letter M
b) find the number of arrangements beginning with exactly one M

3-11. How many three-digit numbers larger than 638 are possible using the digits 3, 5, 6, 8, 9 at most once each?

3-12. Mrs. McKenzie signs books out from the library for reading period. She signs out 15 books for boys and 15 for girls. When a large number of students come into class late, there are 6 books left for the boys and 4 for the girls. How many ways can 3 late boys and 2 late girls be assigned books?

COMBINATIONS

4-1. Tommy has 7 marbles. In how many ways can he choose 5 of them?

4-2. A teacher is going to choose 4 students from his class of 18 students to organize a class trip. In how many ways can he do this if

a) there are no restrictions?

b) Meredith, a student in the class, must be one of the students chosen?

c) Alex, Trey and Yvonne, students in the class, must not be chosen?

4-3. How many 5-card hands will be dealt out from a deck of 52 cards if

a) there are exactly 3 face cards?

b) there is at least 3 face cards?

4-4. A mother wants balloons and party hats for her son's birthday. The store has 8 different types of balloons and 4 types of party hats. She wants to buy 3 types of balloons and 2 types of party hats. In how many ways can she do this?

4-5. An artist specializes in paintings and pottery. She can bring 6 items to display at an art show. She has 10 paintings and 6 pieces of pottery she would like to display. In how many ways can she display her artwork if she wants to display at most 2 pieces of pottery?

4-6. A community group is interested in planning activities for youth. The group is comprised of 7 men and 11 women. In how many ways can the group name three people to the executive committee, if there must be at least one woman chosen?

4-7. A gardener wants to plant 5 flowers in a line. What is the probability that one of the 5 flowers, a rose, will not be on either end?

4-8. A school has an athletics meeting. At the meeting, there are 8 hockey players and 13 basketball players. What is the probability of randomly forming a sub-committee of 5 students with no hockey players?

REVIEW

5-1. Construct a tree diagram to illustrate the number of 3 topping MCV (meat cheese veggie) pizzas if there are 3 different meats, 2 cheeses and 3 vegetables to choose from.

5-2. How many different ice cream sundaes can be made at a restaurant if they have four different flavours of ice cream, four different toppings and five different sauces if a sundae must consist of one each of ice cream, toppings and sauces?

5-3. Kevin is doodling in math class. If he has five different colours and he chooses at least 2 colours to doodle with, how many different colour combinations can Kevin choose?

5-4. In North America, area codes are assigned using 3 digits. If the digit 0 cannot be used as the first digit, how many different area codes can be used? Is repetition allowed in this situation? Give an example to support your answer.

5-5. Evaluate $\frac{8!}{5!}$

5-6. Express in the permutation notation for form ${}_nP_r$:
15 * 14 * 13 * 12 * 11 * 10

5-7. Two brothers play in an adult hockey league. How many ways can the starting lineup (6 players) stand in a line for the national anthem if
a) There are no restrictions?
b) The two brothers must stand together?

5-8. The top five competitors in badminton are standing in a line in the gym. A photographer wants to get a picture of them in the correct order of how they placed. How many incorrect ways are there for the competitors to stand?

5-9. The nine members of student council must get their picture taken for the school yearbook. How many ways could they form a single line if
a) There are no restrictions?
b) Barbara wants to be in the middle?
c) Council members, Leanne & Darrel do **not** want to be next to one another?

5-10. How many permutations are there of the letters of the word SASKATOON?

5-11. How many six-digit numbers are possible using the following digits once each: 2, 4, 4, 5, 5, 5?

5-12. There are four scientific calculators, three Brand A graphing calculators and five Brand B graphing calculators. In how many ways can they be shared by 12 students if the calculators are not numbered?

5-13. How many arrangements can be made using the letters in the word TORONTO if

a) There are no restrictions?
b) The word must start with an N?
c) The T's must be kept apart?

5-14. Using the letters of the word LOGARITHM, how many five letters words can be formed if

a) There are no restrictions?
b) The word must start with a consonant?

5-15. The digits 1, 3, 5, 7, 8 and 9 are used to form a 3-digit number. If repetitions are allowed, find

a) The number of 3-digit numbers that can be formed.
b) The number of 3-digit odd numbers that can be formed.
c) The number of 3-digit even numbers that can be formed.
d) The number of 3-digit numbers that are larger than 500 and even.

5-16. How many different ways can five married couples sit in a row on a park bench if

a) There are no restrictions?
b) Every husband and wife must sit together?
c) There must be four women in the middle four positions?

5-17. A lodge in Gimli is serving four different salads and five different entrées on one particular day. How many ways can a pair of brothers have the same salad but a different entrée?

5-18. A teacher has calculators and rulers at the front of the classroom for students. The bin at the front of the class has 20 calculators and 15 rulers. When the late bell goes, the teacher has signed out 12 calculators and 10 rulers. There are five students that come in late. If all five require a calculator but only three of them require rulers, in how many ways can they choose the numbered rulers and calculators?

5-19. A teacher has a bank of 15 multiple choice questions and she wants to choose 10 of them for a test.

a) In how many ways can she do this if the order of the questions does not matter?

b) If she wishes to randomize the order of the questions chosen, how many question orders are possible?

5-20. In a standard 52-card deck, with 5 cards dealt in every hand, determine the number of hands with:

a) All five spades.

b) Exactly two hearts

c) At least three face cards

5-21. A school decides to form a peer mentor group of 5 students from the 16 students in the grade twelve math course. There are 9 males and 7 females. How many groups can be made if

a) There are no restrictions?

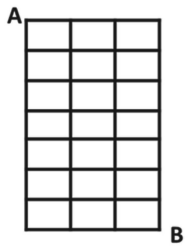
b) There must be three males?

c) There must be at least one female?

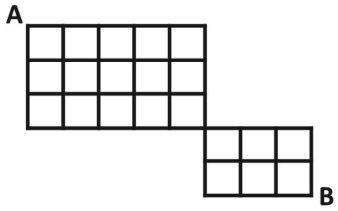
d) The group is made up of female students only?

5-22. At an ice cream parlor, there are six different things that can be put on a sundae – nuts, fudge, sprinkles, butterscotch, bananas and strawberries. How many different combinations of toppings can a customer have if they have at least one?

5-23. How many ways can you go from A to B in the diagram below if you can only move right or down?



5-24. Determine the number of routes from A to B by only moving east or south:



5-25. A gardener wants to plant 5 distinct flowers in a line. What is the probability that the rose, one of the flowers, will not be on either end?

5-26. A school has an athletics meeting. At the meeting, there are 8 hockey players and 13 basketball players. What is the probability of forming a sub-committee of 5 students with no hockey players?

ANSWERS

Full solutions , with steps, are available at the Mb Distance Learning website.

1-1. a. 108 pizzas b. 1188 pizzas
1-2. 48 equipment combinations
1-3. 108 combinations

1-4. 17, 576, 000 license plates
1-5. 2600 Postal Codes
1-6. 120 ways

1-7. 350 ways
1-8. a. 1 5 10 10 5 1 b. c and d c. 5/32

2-1. a. 4! b. 7! c. 5! d. 10!
2-2. a. 6 b. 5 c. 66 d. 252
2-3. a. $_{11}P_8$ b. $_5P_4$

2-4. a. 7! = 5040
2-5. 6! = 720 ways
2-6. 2730 ways

2-7. 132 ways
2-8. 2, 177, 280 set lists
2-9. 480 ways (hint: calculate total ways with no restrictions, subtract ways of what you don't want)

3-1. a. 360 ways b. 60 ways c. 240 ways
3-2. a. 360 ways b. 120 ways c. 24 ways d. 48 ways
3-3. a. 180 numbers b. 30 numbers c. 48 numbers d. 100 numbers

3-4. a. 720 ways b. 240 c. 48 d. Jade and Rebecca are girls
3-5. a. 720 ways b. 48 ways c. 144 ways d. 72 ways
3-6. a. 3,628,00 b. 28,800 c.
3-7. 120 ways

3-8. a. 36 b. 4
3-9. a. 3360 b. 480 c. 120
3-10. a. 907,200 b. 816,480

3-11. 34 (hint: one nbr starts with 63, 9 are $\geq$ 640, 24 nbrs start with 8 or 9)
3-12. 1440

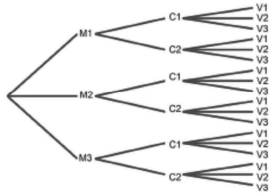
4-1. 21 ways to chose marbles
4-2. a. 3060 b. 680 c. 1365
4-3. a. 171,600 b. 192,192

4-4. 336 to select party stuff

4-5. 4872 (210 with no pottery, 1512 with one pottery, 3150 with 2 pottery)
4-6. 781 ways with at least one woman. (816 ways to make a committee with no genre restrictions, 35 ways to make a committee with no women)

4-7. a. P(rose no on ends) = 0.6
4-8. 0.063 (20,349 ways without restriction; 1,287 ways with no hockey players)

5-1. 18 different pizzas



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5-2. 80 sundaes
5-3. 26

5-4. 900
5-5. 336
5-6. $_{13}P_6$

5-7. a. 720 b. 240
5-8. a. 118 (hint: 120 unrestricted ways to stand, 2 ways to stand correctly depending on which end they are at)
5-9. a. 362,800 b. 40,320 c. 282,240 (hint: 9!-8!2!)
5-10. a. 4,360 distinguishable ways

5-11. 60
5-12. 27,720
5-13. a. 420 b. 60 c. 300

5-14. a. $_9P_5$ =15, 120 b. 6*8! = 241,900
5-15. a. 216 b. 180 c. 36 d. 24
5-16. a. 3,628,000 b. 3,840

5-17. 80 ways
5-18. 560
5-19. a. 3003 b. 3,628,800

5-20. a. 1,287 all 5 spades b. 712,842 two hearts exactly
c. 171,600 + 19,800 + 792 = 192,192 at least 3 face cards
5-21. a. 4,368 b. 1,764 c. 4,242 d. 21

5-22. 63 combos
5-23. 120 different routes

5-24. 840 different routes

5-25. 0.6 probability
5-26. Prob (committee with no hockey players) = 0.063