GRADE 12 APPLIED

UNIT F

SINUSOIDAL FUNCTIONS

WORKBOOK

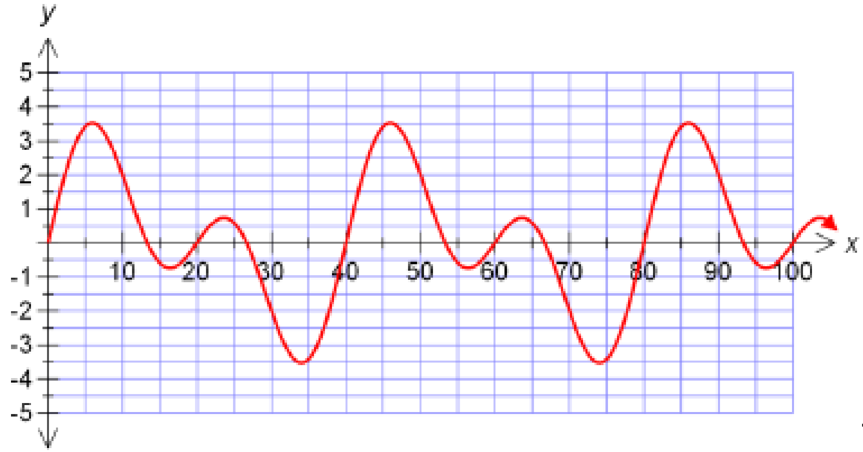
(answers will be provided elsewhere)

GRADE 12 APPLIED
UNIT F – SINUSOIDAL FUNCTIONS

Periodic Functions and Radian Measure

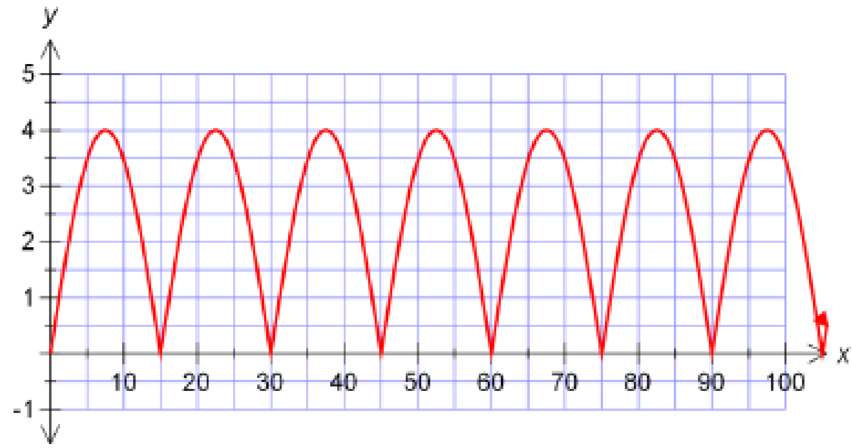
1. The graphs below are an amplitude with respect to time. State whether each of the following graphs is periodic or not. If they are periodic, state their period, **T**.

a)



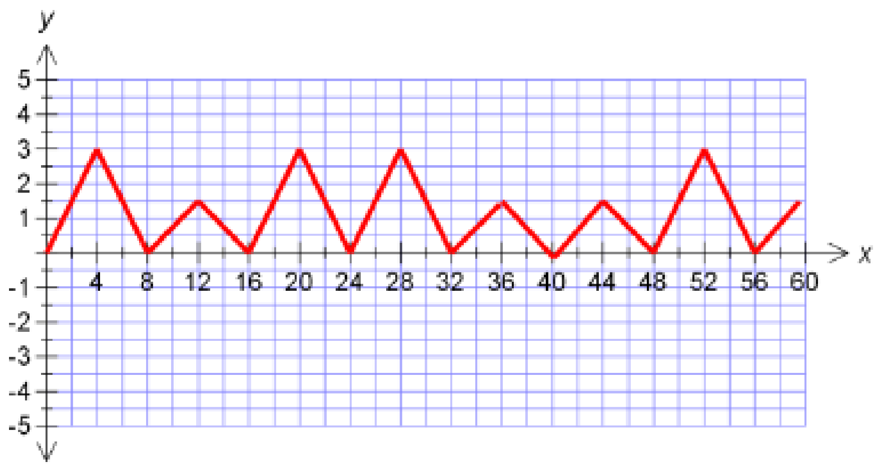
T = ______

b)



T = ______

c)



T = _______

2. Sketch and describe any periodic function that has a period, **T** (or wavelength, λ) of 10 units. Show the sketch here:
[λ is the Greek letter '*lamda*']

The '*standard*' sinusoidal function that we use and that the TI-83 uses by default is :

$$y = A*\sin(B*x + C) + D;$$

Be aware however that some references use the [better] form:

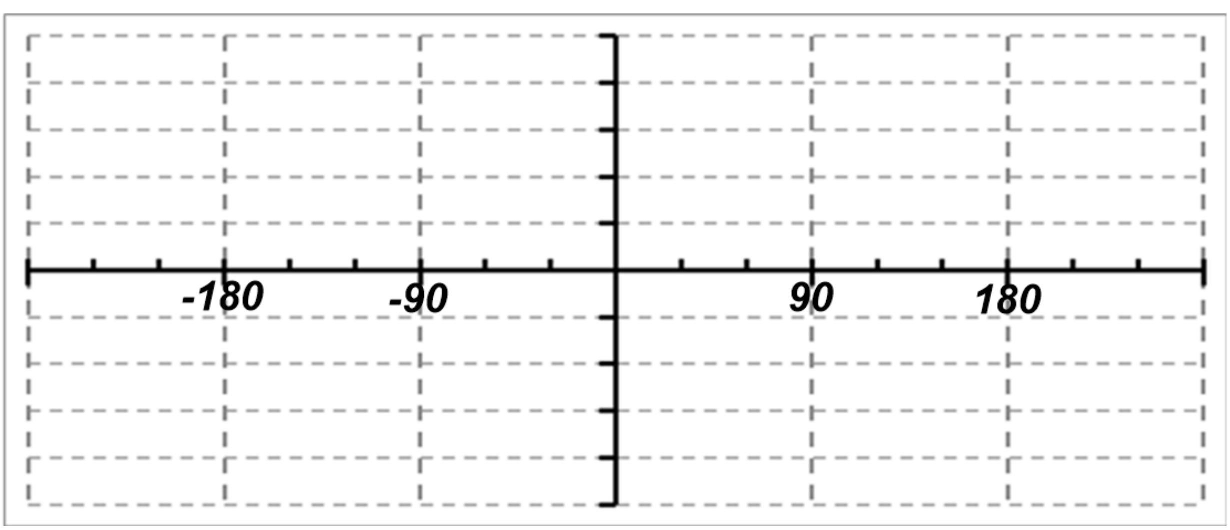
$$y = A * \sin[B*(x + C)] + D$$

This latter form is only different in the way that a 'horizontal phase shift' is represented.

Graph on a graphing tool [using x in degrees mode] the two equations:

$$y_1 = 3 \sin (2*x + 60) + 1; \text{ and}$$
$$y_2 = 3 \sin [2*(x + 30)] + 1$$

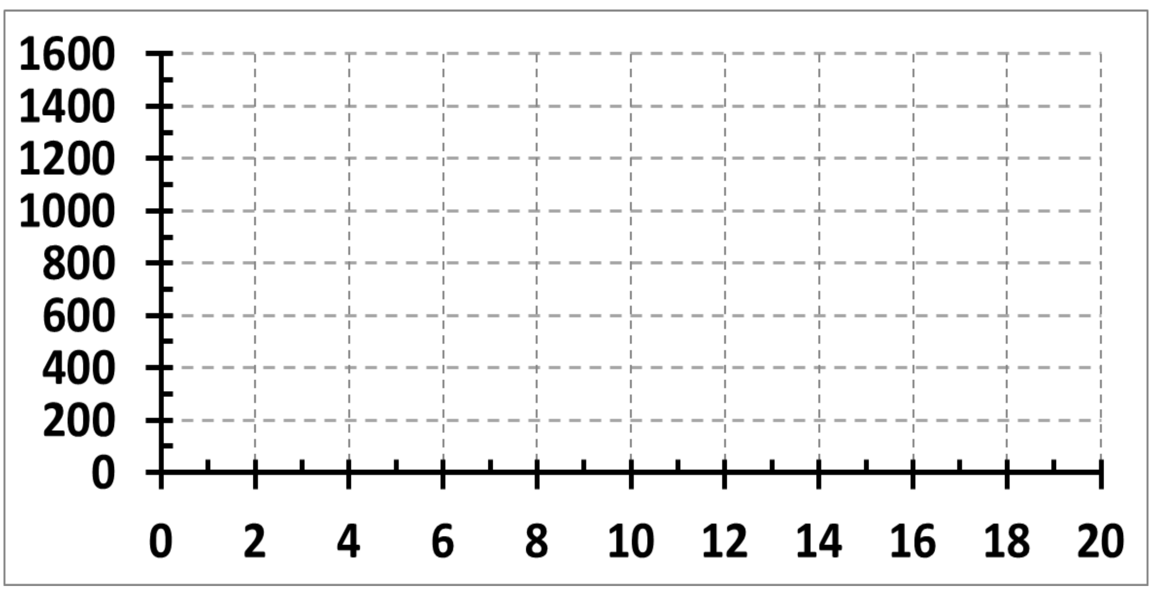
Now note from a TRACE or TABLE a few of their significant points and copy them to the graph below, connect the dots smoothly:



3. Monthly ice cream sales at a local vendor are given in the table below:

Monthly Ice Cream Sales			
Month	Sales	Month	Sales
May 2011	$1020	Feb 2012	$280
Jun 2011	$1340	Mar 2012	$560
Jul 2011	$1500	Apr 2012	$820
Aug 2011	$1360	May 2012	$1000
Sep 2011	$980	Jun 2012	$1320
Oct 2011	$800	Jul 2012	$1480
Nov 2011	$550	Aug 2012	$1300
Dec 2011	$300	Sep 2012	$970
Jan 2012	$100	Oct 2012	$820

a) Construct a properly labeled graph to plot the data from the table above [Start with May 2011 as month 0].
b) Label and describe the period of the graph.
c) State the domain of the situation: ________ ≤ month ≤ ________
d) State the range of the given situation. _______ ≤ sales ≤ ________
d) Assume the pattern continues. How many monthly ice cream sales (approximately) do you predict for May 2013?



4. Convert the following radian measures into earthling degrees to the nearest whole degree.

a. $\frac{3\pi}{4}=$

b. 4 radians (ie: 4^r)

5. Convert the following degree measures into exact radians in terms of π. (Knowing how to handle fractions is really necessary here!)

a. 120°

b. 330°

c. 60°

d. 30°

e. 15°

f. $^-30°$

6. Convert the following into radians to 3 decimal places

a. 200°

b. 480°

c. 57.3°

d. 720°

e. 1440°

f. $^-30°$

Sine Curves (Sinusoidal Functions)

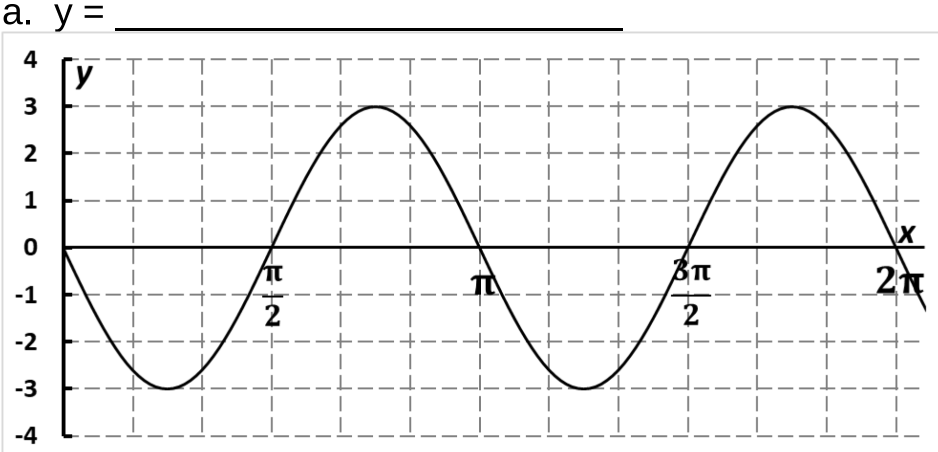
Sinusoidal functions can be represented by the function:

f(x) or y = Asin(Bx + C) + D;

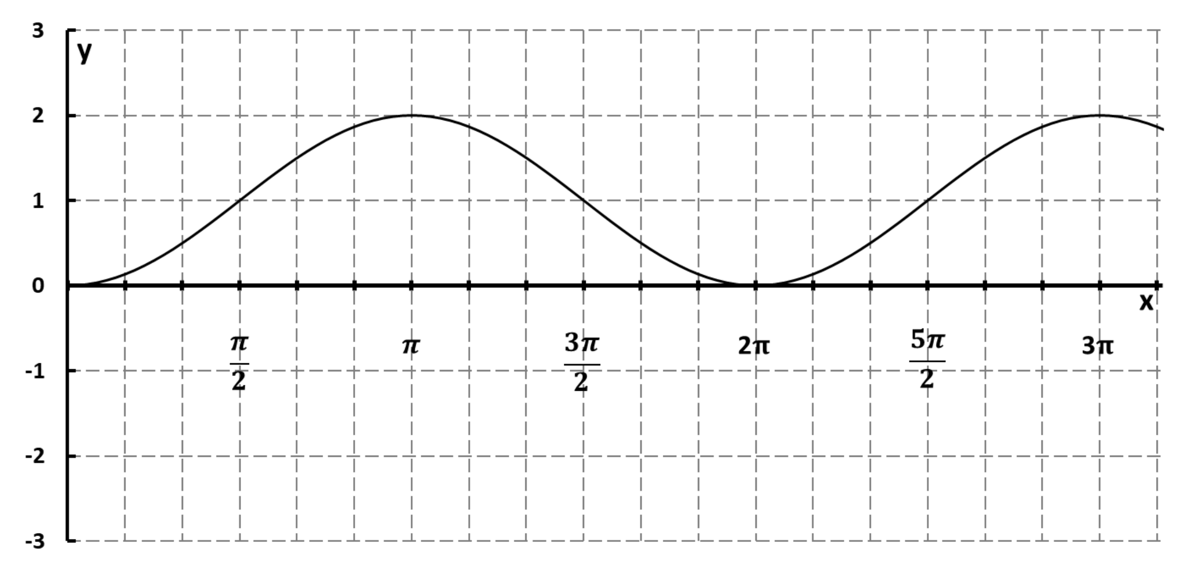
where **A** is the **A**mplitude either side of the median [mid value D] ; **B** relates to the period, **T**, and is the number of periods squeezed into 360° [or 2π radians] and **T** can be calculated from $\mathbf{T} = \frac{2\pi}{B}$; **C** relates to the horizontal phase shift (a little bit advanced calculation where phase shift = **⁻C/B**, and **D** is the vertical displacement of the Median, D [mid-value].

1. Find a possible equation for each of the following graphs: (You will need the SinReg regression tool, but they can be done contextually without a tool if you think about it!). Make sure the calculator is in RADIAN mode!

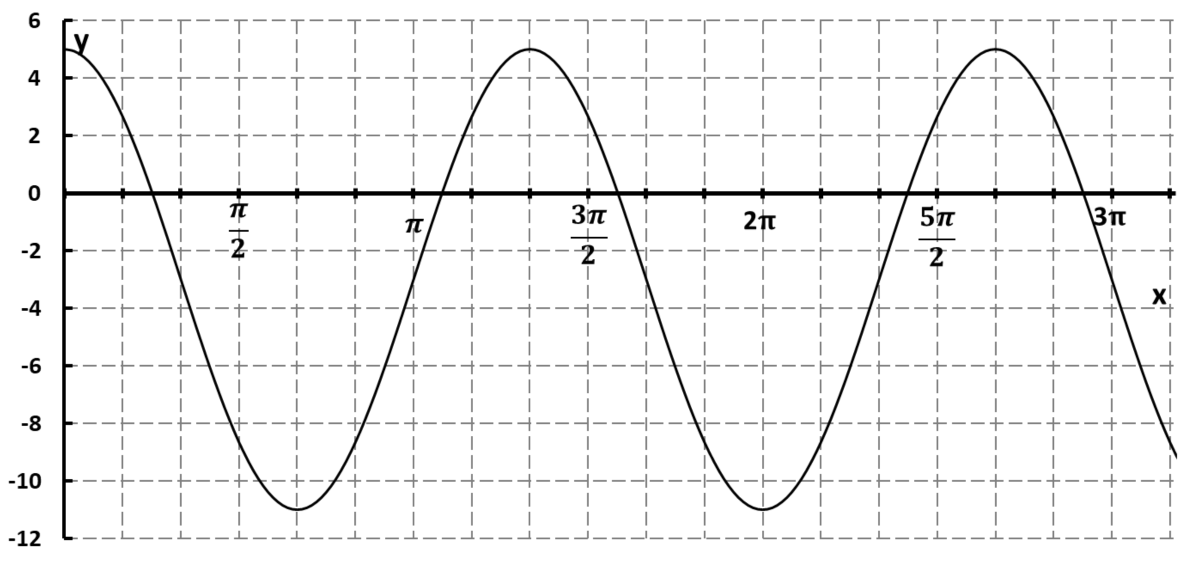
a. y = ______________________________



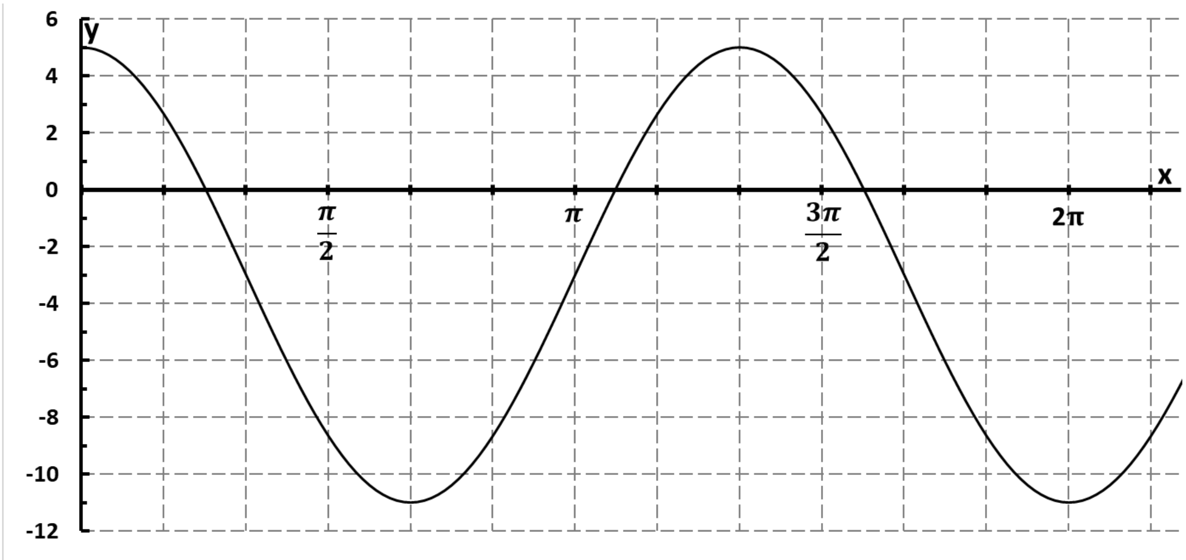
b. y = ___________________________



c. y = ___________________________

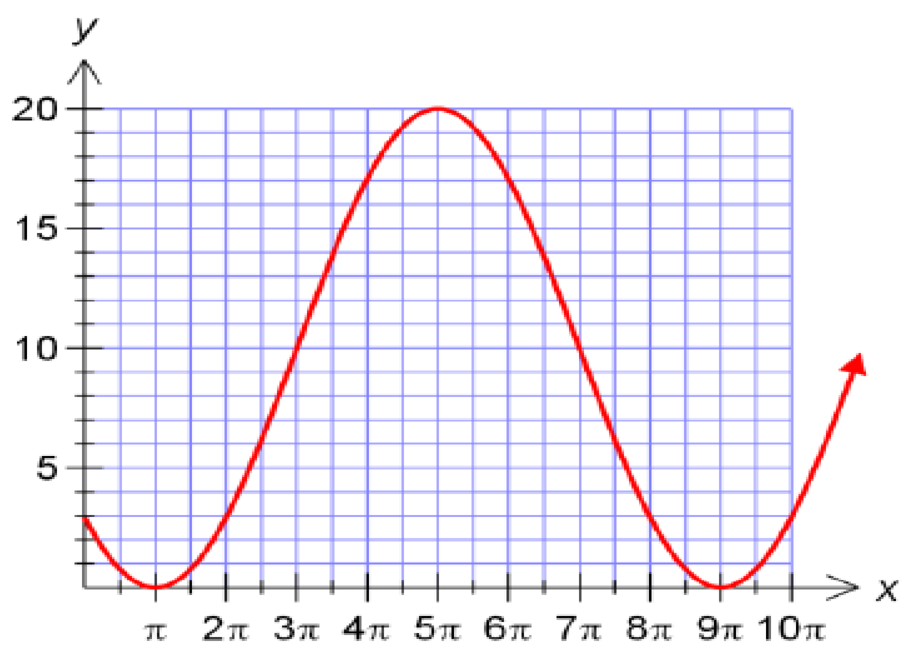


c.



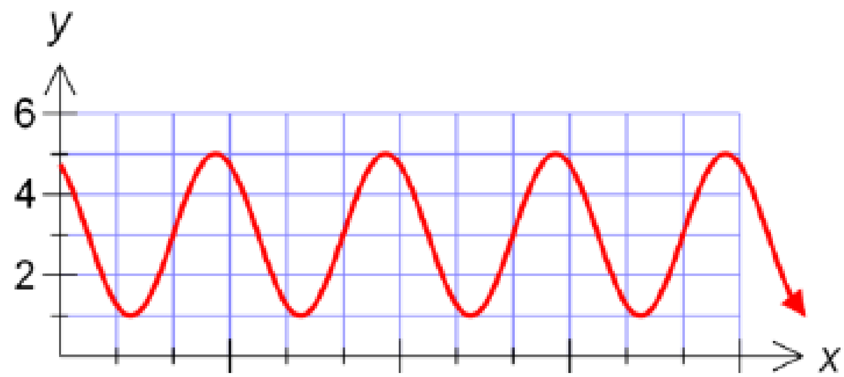
6
4
2
0
-2
-4
-6
-8
-10
-12
y
x
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

d)

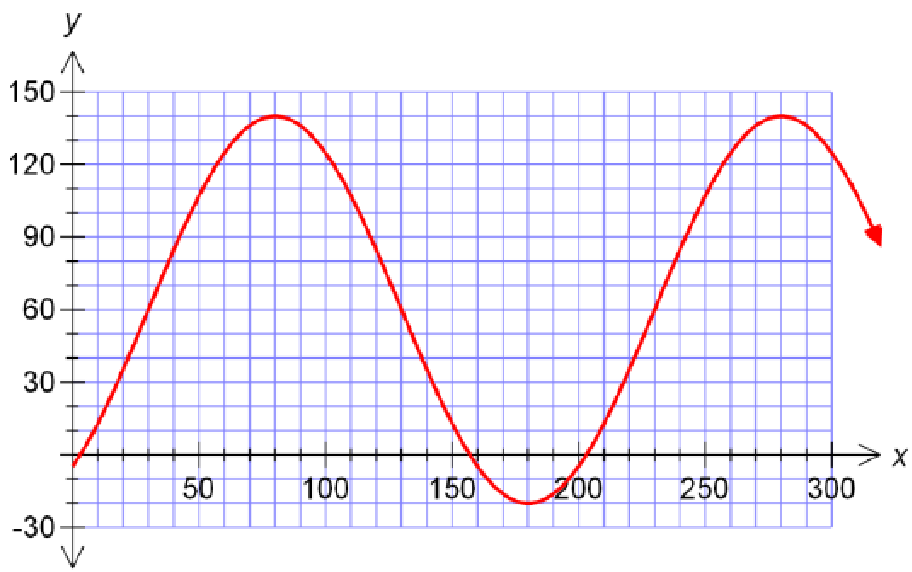


y
20
15
10
5
x
π 2π 3π 4π 5π 6π 7π 8π 9π 10π

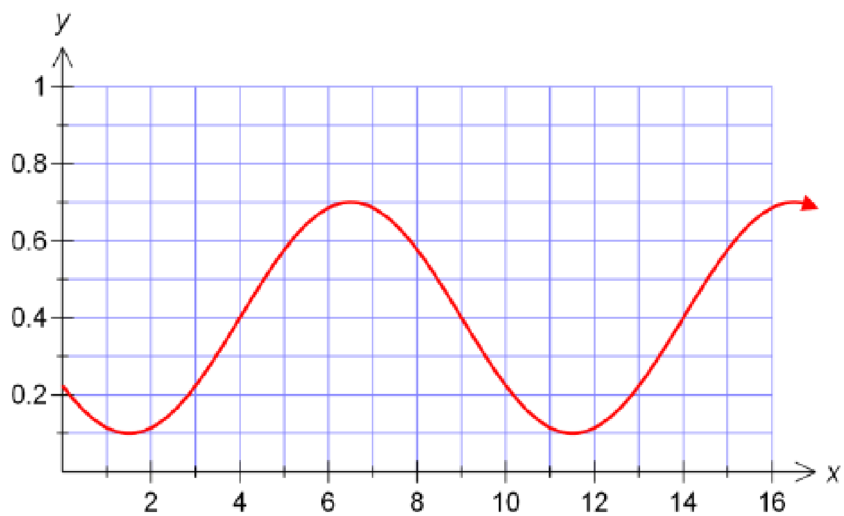
f)


y
6
4
2
x

g)

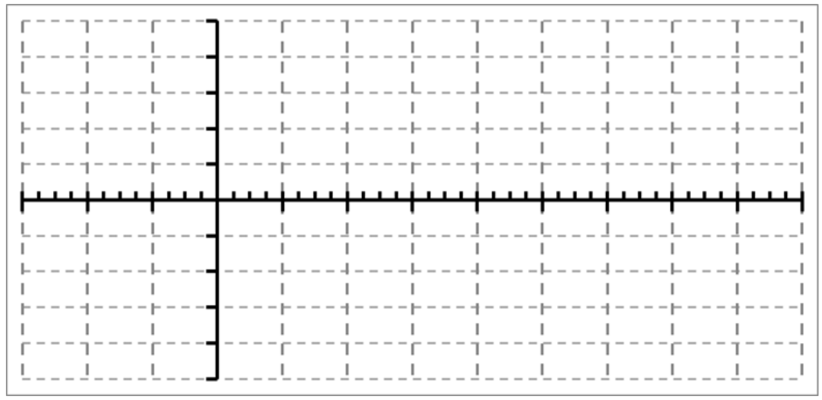

y
150
120
90
60
30
-30
50
100
150
200
250
300
x

h)


y
1
0.8
0.6
0.4
0.2
2
4
6
8
10
12
14
16
x

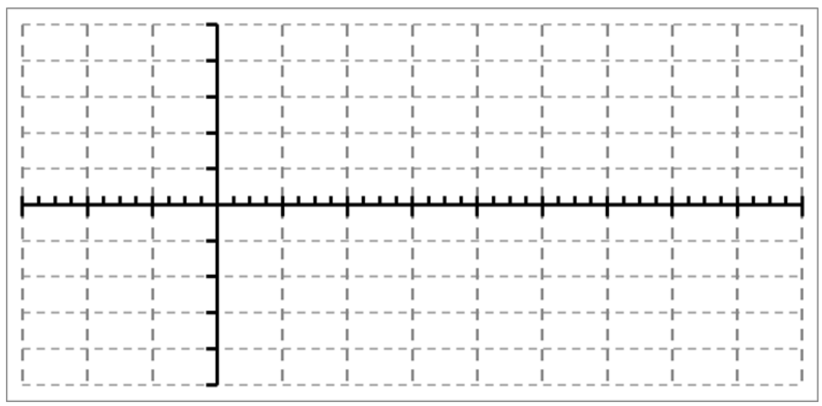
2. Graph on a graphing tool at least one period of each of the following sinusoidal graphs: **graph** them (as opposed to a simple freehand 'sketch') to the right as an accurate 'screen shot'.
Working in radians of course! Only an earthling would use degrees!

a. **y = 0.5sinx – 3**



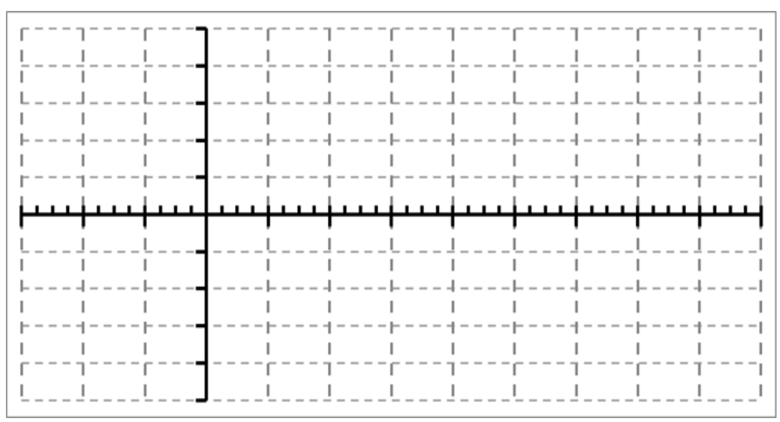
for the function state:
Domain:
_____ < x < _____
Range:
_____ < y < _____
Median: y_{med}=_______
Amplitude: ________

b. y = 4sin(3x – π)



for the function state:
Domain:
_____ < x < _____
Range:
_____ < y < _____
Median: y_{med}=_______
Amplitude: ________

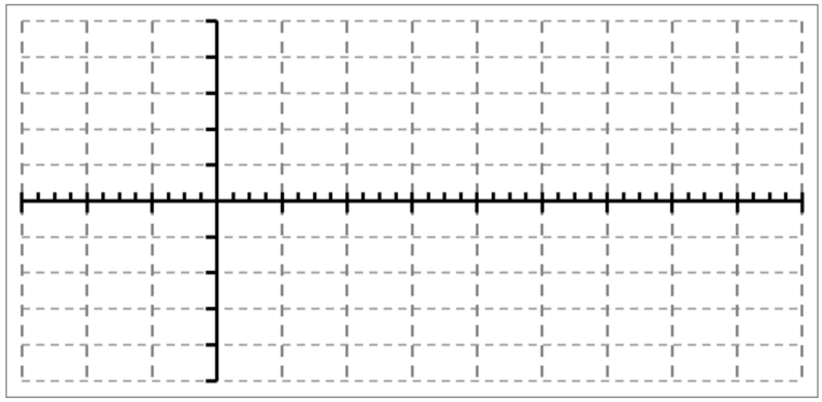
c.
$$y = 2\sin\left(x + \frac{2\pi}{3}\right) - 1$$



for the function state:
Domain:
_____ < x < _____
Range:
_____ < y < _____
Median: y_{med}=_______
Amplitude: ________

d.

$$f(x) = \boldsymbol{sin}\left(\boldsymbol{2x} - \frac{\boldsymbol{\pi}}{\boldsymbol{2}}\right) + \boldsymbol{4}$$



for the function state:

Domain:

______ < x < ______

Range:

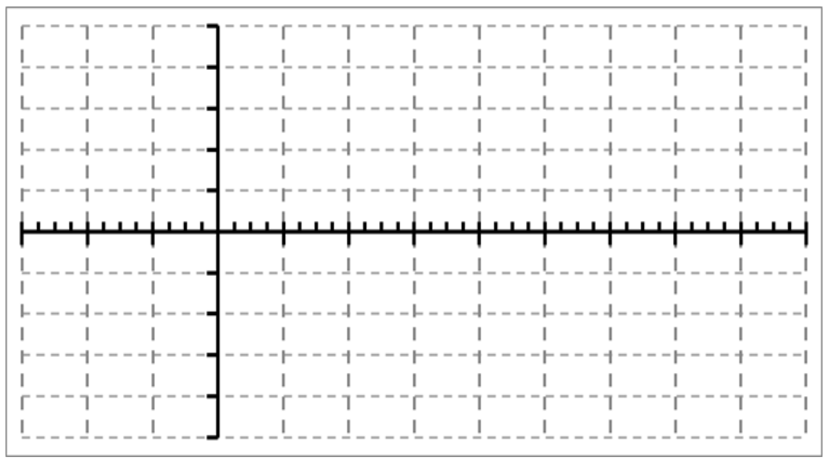
______ < f(x) < ______

Median: y_{med}=_______

Amplitude: _________

e.

$$f(x) = 4sin\left(\frac{1}{3}x - 3\pi\right)$$



for the function state:

Domain:

______ < x < ______

Range:

______ < f(x) < ______

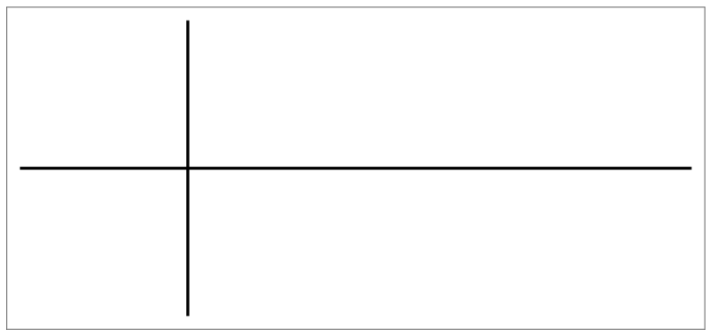
Median: y_{med}=_______

Amplitude: _________

3. Draw (ie: sketch) and label at least one period of a sinusoidal graph for each of the following using the properties given:

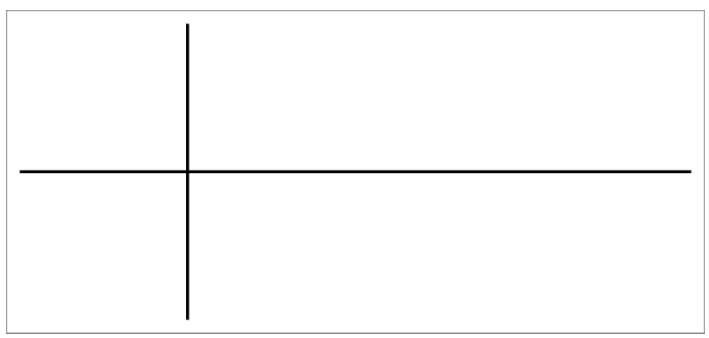
a.
(1) The Period, T, is π;
(2) the Amplitude, A is 4;
(3) one max value is at (0,5).

b.
(1) The distance between two consecutive maximums is 3π
(2) The amplitude is 12
(3) The central horizontal axis (ie: median) is at $y = {}^{-}3$
(4) One point on the curve is at $(\pi, {}^{-}3)$



c. Three full periods are spread over 108 radians.

One maximum value is at (8,20) and a same cycle minimum is at (26, 8)



Applications of Sinusoidal Functions

1. The number of tourists, N, at a resort varies according to the equation:

$$N = 450sin\left(\frac{\pi}{3}(t + 1.5)\right) + 600$$

where **N** is the number of tourists and **t** represents the time measured in months. January 2012 is month 0, February 2012, is month 1, etc.

a) How many tourists will there be in March?
b) Determine in what months there will be more than 825 tourists.
c) Determine the period for this sinusoidal model
d) Determine which month(s) in 2012 to expect to have at least 600 tourists.

2. The hourly traffic flow on a main street in a certain town on a Friday is said to model a sinusoidal function. The equation that models the traffic is:

$$\boldsymbol{V = 52\sin(0.785(h+2)) + 60}$$

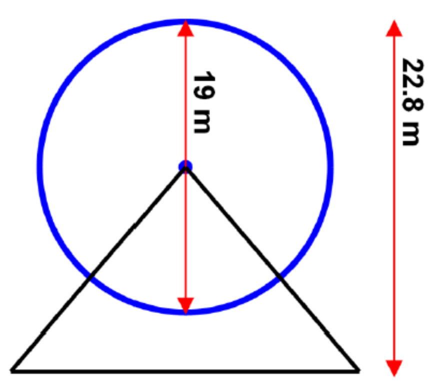
watch the brackets

where **V** represents the volume of flow of vehicles that pass through at an hourly rate for any time of day, h. When h = 0, the time is midnight.

a) Determine the volume of traffic flow at 8 pm.
b) What is the period of the sinusoidal model?
c) At what times of day will traffic be the heaviest?
d) approximately how many vehicles will pass through the intersection between 7am and 10am? [this is called doing an Integral in calculus! There is a TI-83 button will do it.]

3. A Ferris wheel is pictured below. A complete ride makes 6 revolutions and takes 900 seconds.

a) Find an equation that models the height, h, of a passenger on the wheel as a function of time if they start at the lowest point on the ferris wheel.
b) What will be the height of a passenger after 80 seconds?



4. In Prince Albert, Saskatchewan the earliest sunrise time is at 4:36 am on June 21. The latest sunrise time is on December 21 at 9:15 am. (for simplicity assume the difference between these dates is 180 days)

a) Write a sinusoidal equation that models the sunrise times in Prince Albert.
b) What would the sunrise time be on February 21?
c) What is the average time of sunrise?

5. The number of hours of daylight in Brandon on the first of each month is given in the table below:

Hours of Sunlight in Brandon July 2010 to December 2011			
Month	**Hours of Sunlight**	**Month**	**Hours of Sunlight**
July '10	16.3	Apr '11	12.9
Aug '10	15.2	May '11	14.7
Sept '10	13.5	June '11	16.1
Oct '10	11.6	July '11	16.3
Nov '10	9.8	Aug '11	15.2
Dec '10	8.4	Sep '11	13.4
Jan '11	8.2	Oct '11	11.7
Feb '11	9.3	Nov '11	9.8
Mar '11	11.0	Dec '11	8.4

a) Plot the data on a graphing tool and label and show a *sketch* below.
b) Using sinusoidal regression, create an equation that models the data.
c) Using the equation found in b), comment on the trends in the hours of sunlight.
d) Using the equation, estimate the amount of sun in Brandon on April 1, 2012.
e) could you actually derive *some* of the formula just in your head?

6. A mass is attached to a spring hanging from the ceiling. It is pulled down and released. Time is measured from the point of release. The mass moves up-and-down in a periodic manner. The height of the mass at various times is given in the table below.

Height of a Spring			
Time (s)	**Height (m)**	**Time (s)**	**Height (m)**
0	0.15	0.70	0.33
0.25	0.98	0.95	0.52
0.50	1.17	1.20	1.35

a) Plot the data on a graphing tool. Sketch and label it below.
b) Using sinusoidal regression, create an equation that models the data.
c) Using the equation, what is the height of the spring after 1.1 seconds?
d) How many periods (ie: cycles) are there in 10 seconds?

LOTS OF GRAPH PAPER