GRADE 12 APPLIED
UNIT B – PERSONAL FINANCE

CLASS NOTES

1. Have you ever bought something on credit? (Then likely regretted it later?). Have you ever received mail offering you a cheque for $3,500 and wondered if you should use it? Are you planning on saving for your retirement or for your children's education or wedding(s)?

This unit is all about borrowing money and saving money.

Objectives

2. You will learn about simple interest and compound interest using manual formulas and using technology apps.

You will learn about effective use of different credit options, lines of credit, overdrafts, etc.

Pre-Requisites

3. To readily succeed in this unit you will want to be familiar with decimal arithmetic, percentages and decimals and fractions, exponents, simple algebra equations and to know how to use the internet. Knowing how to use a Spreadsheet (such as EXCEL or any of the ones available on-line) would be advantageous as well.

4. The first section (14 Pages) of these notes are an identical copy of Grade 11 Essential Lessons. You should just jump to page 15 of these notes. These Grade11 Notes are provided for review.

SIMPLE INTEREST

5. Interest is the fee paid (or earned) for the use of money. The fee is usually at a pre-determined percentage rate of the money borrowed or invested.

Simple Interest Example

6. You **borrow** $1000. You pay back $1100 at the end of some period of time. So it has cost you **$100** to get **$1000**.

The original amount you borrowed (or if you are the bank the amount you loaned) is called **Principal** amount. The *extra* amount you pay back for the use of that money is called the **Interest**. The **Amount Owing** at the end of the period for that initial **$1,000** is **$1,100**.

7. Complete the table:

Principal (P)	Interest (I)	Amount Owing (A)
$2,500	$200	
$5,800		$6,400
	$1,200	$10, 800
	$1,825.72	$22,352.35
$4,625		$5200.34

A fancy formula for this:

Amount Owing = Principal plus Interest

or

A = P + I

(so also using algebra I = A – P and P = A – I)

Types of Interest. There are two types of interest:

Simple Interest and

Compound Interest.

SIMPLE INTEREST

8. Simple interest is a percentage paid on only the principal and at fixed intervals of time.

See the table below for an example of a Principal Amount of $1,000 borrowed over a period of three years.

End Year	Principal [$]	Interest Rate [%]	Interest Owing [$]
1	$1,000	10%	=$1,000*10% = $100
2	$1,000	10%	=$1,000*10% = $100
3	$1,000	10%	=$1,000*10% = $100
		Total Interest	**$300.00**
		+ Principal	**$1,000.00**
		=Amount Owing	**$1,300.00**

9. So in the table above if you borrow $1,000 it costs $100 each and every year in simple interest. After three years you owe $300 in interest plus the original principal amount. So your original *principal* of $1,000 has become a final **amount owing** of $1,300 after three years.

Doing calculations like this line-by-line is called doing a ‘recursive’ or an ‘iterative’ method. **Spreadsheets** are especially well adapted for doing these types of recursive calculations.

You Try: Simple Interest Calculation using a recursive table.

10. You invest **$2,500** in a simple interest bearing product that has an interest rate of 6%. You keep it in that account for 10 years. What will the total amount owing (to you) be after 10 years?

End Year	Principal [$]	Interest Rate [%]	Interest Owing [$]
1	**$2,500**	**6%**	
2			
3			
4			
		Total Interest	
		+ Principal	
		=Amount Owing	

of course the total amount you have owing at the end of the loan is the Principal plus the Interest you have *accrued*. So

A = P + I

where **A** is the total amount owing at the end of the loan period.

So of course with some rudimentary algebra you could derive a new formula as follows:

A = P + I; so: **A = P + Prt**; and by factoring … **A = P(1 + rt)**

13. The formula works for monthly or any other interest rate calculation also, but it is law that the rate be always quoted as yearly (ie: annually).

Be aware: In some smart phone and on-line Apps you will find that the **A**mount is called **FV** for Future Value and **P**rincipal is called the **PV** for Present Value.

15. Leo borrows $1200 from his uncle to be paid back over 2 years. His uncle charges 10% interest *per annum (Latin for 'per year')*. How much does Leo pay back? How much interest was paid?

$I = P \cdot r \cdot t$; $I = 1200 * \frac{10}{100} * 2 = \$240$ Leo has to pay $240 in interest.
And he has to pay back the principal also of course. The total he pays back is $1440.

Notice! A percentage rate is actually a fraction, an amount per 100! If you do not write the formula as shown above you will likely mess it up.

16. Simple interest is *not very common*. Neither a bank nor an uncle is likely to actually loan you money and tell you to come back in two years with the amount you owe! But **Canada Savings Bonds** (CSB) use simple interest and so do some **Guaranteed Investment Certificates** (GICs) at your bank.

Simple interest is easy to calculate at the end of any reasonable lengthy period and was the only type used until the advent of computers when the more prevalent compound interest could be readily calculated.

17. You try some Simple Interest problems:

a. Carla borrows $900 at 32% Annual Percentage Rate (APR) for three years. (sort of like buying a TV on credit!). How much does she owe after the three years. (Ans: 1764).

b. Erick finds an old Canada Savings Bond in some old files of his grandfather's. It was a $1500 bond bearing a simple interest rate of 7.5% per annum. The bond is 50 years old. What is the Savings Bond worth now? (Ans: $7,125).

18. **Combining Formulae**. We had said that A = P + I; and we had said that I = P*r*t; so logically:

A = P + P*r*t

If you recall any Grade 9 math you might recall the distributive property of mathematical expressions that says that a * (b + c) = a * b +a * c

Consequently, we can make a single formula to calculate any simple interest owing as :

A = P *(1 +r*t)

Useful particularly if you want to graph Amounts due from a simple interest account.

COMPOUND INTEREST

20. Compound interest is interest calculated at regular intervals on an amount of money to which interest from previous intervals has been added. So interest is paid on the principal and the accrued interest! Investments that receive compound interest can grow rapidly (exponentially like bunnies breeding).

End Year	Amount [$]	Interest Rate [% / yr]	Interest [$]	Total Amount [$]
1	$1,000	10%	$100	$1,100.00
2	$1,100	10%	$110	$1,210.00
3	$1,210	10%	$121	$1,331.00
4	$1,331	10%	$133.10	$1,464.10
↓	↓	↓	↓	↓
↓	↓	↓	↓	↓
↓	***Keep going for 16 more iterations*** **!**			↓
20				$6,727.50

21. **Compound Interest Compared to Simple Interest**. If you compare this Compound Interest calculation to the Simple Interest calculation you will find the Total Amount at the end of the 20 years is only $3,000 for the Simple Interest method. So clearly Compound Interest is a better way to earn interest. You earn **interest on the interest** from previous periods!

Caution! Presently it is unlikely you would receive 10% interest. You would be lucky to get 2% interest. However, some of your older family may recall in the mid-eighties when interest was at 18%. 18% is good if you are earning interest, bad if you owe it!

22. Notice that compared to simple interest, your savings grow rapidly with compound interest, especially the longer you leave your money in savings. Loans and credit cards also use compound interest though; so your debt can grow more rapidly also! We have studied this exponential growth function before!

Notice the above table can be easily done as a ‘recursive’ table in EXCEL for those students familiar with really useful financial spreadsheet programs.

25. The formula for compound interest is:

$$A = P\left(1+\frac{r}{s}\right)^{n*s}$$

Notice the 's' occurs in two places. Be aware some books may use different letters; the formula is still the same!

A is the final **A**mount of value of the investment ***or*** the total **Amount** paid on a loan at the end of '**n'** years
P is the **Principal** or initial value, i.e., the amount that has been invested or borrowed
r is the **rate** of interest expressed in decimal form
n is the **number** of years of investment
s is the number of times that the interest is calculated per year

26. Compounding Periods. The '**s**' in this exponential growth equation for compound interest is the most confusing part. It is the number of times the interest is calculated per year. It occurs **twice** in the formula. Here is how you know what value to use for the '**s**'.

Compounding Frequency	**Annual**	**Semi-Annual**	**Quarterly**	**Monthly**	**Daily**
s =	**1**	**2**	**4**	**12**	**365**
Meaning	on interest calculation per year	two interest calculations per year	four times per year (every three months)	12 times per year	365 times a year

Example formula calculation.

27. Jay borrows $1,000. He will pay it back after three years with interest. The Annual Percentage Rate (APR) is 10%. The interest on the loan is calculated and compounded once per year. How much does Jay pay back?

$A = P\left(1+\frac{r}{s}\right)^{n*s}$ so $A = 1{,}000\left(1+\frac{10/100}{1}\right)^{3\bullet 1} = 1{,}000(1+0.1)^3 = 1{,}000(1.1)^3$ so

$$A = 1{,}000*(1.1)^3 = 1{,}000*1.331 = \$1331$$

You try some Compound Interest Calculations

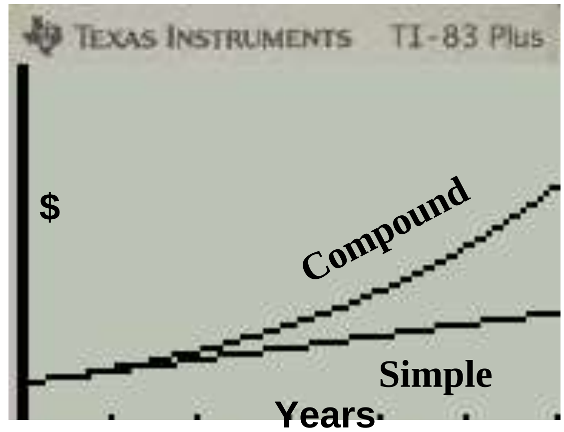
28. **CSB for a Newborn Calculation**. Brandon buys a $3,000 Canada Savings Bond (CSB) for his newborn son as a way to save for his child's education. The bond pays Compound Interest at a rate of 12% APR compounded annually. What will the CSB be worth when his son turns 18? (Ans: $23,069.90)

30. **New Credit Card Calculation**. Lyle gets a new credit card in the mail! The terms of the card say he does not need to make any payments for the first six months. However, of course, they still calculate how much he owes every month, he just gets a 'breather' before he has to pay! The interest rate is a somewhat normal 23% APR. Lyle takes a $2,500 advance on his new credit card to buy a nice new plasma TV for his girlfriend. How much will he owe on his credit card when he gets his first statement in six months? Assuming he only bought the TV on his new card! (Ans: $2,801.63)

31. **Unclaimed Balance**. Your great grandfather had $7,800 squirreled away in a secret bank account back in the 1930s. No one in the family knew this and the money has been sitting in Ottawa for 85 years earning compound interest at 3.5% interest compounded monthly. If no one in the family claims the money within 99 years the government keeps the money. You are a descendant and put in a claim as per the website. (*Bank of Canada Unclaimed Balance*). How much is the $7,800 worth after those 85 years if you bother to claim it! (Ans: $152,138.85)

32. Notice that Amount Owing or Due (aka: **Future Value)** of a Simple Interest is not that much different than a Compound Interest investment in the short term the first few years, but the *exponential* character of the Compound Interest formula kicks in eventually.

The value of a simple interest situation just grows along a straight line, the value of a compound interest situation takes off like a rocket after a while.



Try this on a graphing tool!
y = 1000 (1 + 0.1x) (simple)
y = $1000\left(1+\frac{0.1}{1}\right)^{(1*x)}$ (compound)

33. You try graphing this in a spreadsheet or especially on the TI-83 Graphing Calculator.

35. **Rule of 72**: A quick mental way to calculate the time to double a fixed investment receiving compound interest. Take the interest rate; divide it *into* 72, the result gives the approximate time to double an initial amount of money growing with compound interest. E.g.: 8% interest, your money doubles every 9 years. Only works well for rates between 3 to 10%.

36. Do the quick mental math with the **Rule of 72**! Time to double your money. Then test it with the compound interest formula to see how closely it works. Test it with a principal of $1, just to make the calculation easier.

APR (%)	Years to double	Test with the actual formula with $1
2	36	$A = P\left(1+\frac{r}{s}\right)^{n*s} = 1\left(1+\frac{0.02}{1}\right)^{36} \cong 2.04$ Pretty darn close to double!
6		
12		
	15	
	20	
	6	
	5	
9		

Credit Card Buying (Buying on Credit)

37. When you use a credit card or borrow on credit from stores like MasterCard or Leon's, you receive a statement each month listing your transactions for that month. It tells the new balance owing as well as the minimum amount due to be paid. If you do not pay the entire balance, but pay only the minimum, your lending institution interest will charge you interest on what remains unpaid.

38. **Exampl**e: Complete the following example table to determine the cost of credit for using a credit card. **Minimum Payments** are **5 %** of the **Balance Due** at the end of each month. You (foolishly) make the very **minimum payment** each month. The store credit card charges interest of 2.5% per month, like 30% per year (a bit more actually). Say you buy a big fancy entertainment centre for **$1,800** because the card was so easy to get. You only use the card once, to buy the entertainment centre.

Of course a spreadsheet would be the smartest way to do this!

Statement Date **26 Feb 2005**		Payment Due By: **12 Mar 2005**		Credit Limit **$2,000**		
Month	Previous Balance Owing	Payment Made	Purchases Charged	Interest Charges	New Balance Owing	Minimum Payment 5%
Feb 2005			$1,800	+$45.00	$1845.00	$92.25
Mar 2005	$1845.00	–$92.25	+ 0	+$43.81	$1796.56	
Apr 2005			+ 0			
May 2005			+ 0			
Jun 2005			+ 0			
Jul 2005			+ 0			
↓	↓	↓	↓	↓	↓	↓
Jul 2010					$327.40	
TOTALS		**-$2890.66**	$1,800	**$1,418.06**	**$327.40**	

39. So by July 2010, 65 months later, you have made **$2890.66** in payments and *you still owe* **$327.40**. So this $1,800 entertainment centre has *so far* cost you **$3218.06** and you still have not finished paying for it.

40. Interest rates on credit cards are usually anywhere from **18** to **32%** (APR). And if you miss a payment (on this or any other card or loan you have) it goes even higher for a year until your credit report improves! Have you ever checked your credit report?

Personal Loans

42. We had looked at interest rates and loans using compound and simple interest formulas. The examples we had seen were that you borrow a Principal and in a certain time you paid everything back at once. In practice however, loans usually involve a monthly payment. The formulas for calculating the payments are complicated, normally we just use a Loan Payment table. (There are lots of Apps for you smart phone too.)

Amortization Period Monthly Payment Per $1000 Loan Proceeds					
Annual Rate	**1 Year Monthly**	**2 Years Monthly**	**3 Years Monthly**	**4 Years Monthly**	**5 Years Monthly**
6.00%	$86.07	$44.33	$30.43	$23.49	$19.34
6.25%	$86.18	$44.44	$30.54	$23.61	$19.46
6.50%	$86.30	$44.56	$30.66	$23.72	$19.57
6.75%	$86.41	$44.67	$30.77	$23.84	$19.69
7.00%	$86.53	$44.78	$30.88	$23.95	$19.81
7.25%	$86.64	$44.89	$31.00	$24.07	$19.93
7.50%	$86.76	$45.01	$31.11	$24.19	$20.05
7.75%	$86.87	$45.12	$31.23	$24.30	$20.16
8.00%	$86.99	$45.24	$31.34	$24.42	$20.28
8.25%	$87.10	$45.34	$31.45	$24.53	$20.40

A larger table is enclosed at Appendix B in these notes.

Example: Someone borrowing $10,000 over 5 years at 6% would make monthly payments of $19.34 ***per thousand*** borrowed; or $193.40 per month. For a total payment of:

______________ **$11,604.00**

43. The formula for this type of calculation is somewhat onerous:

$$P = \frac{r * M}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} \div n$$

where '**P**' is the monthly payment, '**r**' the annual interest rate; '**M'** the mortgage amount; '**t**' the number of years to 'amortize'; and '**n**' is the number of payments per year.

44. Notice most formulas you have ever used only have two variables (just an ‘x’ and a ‘y’), this one has a **P**, **r**, **M**, **s**, and **t;** it is has five variables!! Clearly our simple two-dimensional graphs and tables will not be overly useful if we change all the variables at the same time.

We will generally need a computer application to calculate these type of loan payments or else use pre-computed tables.

45. There are many Apps and websites that also do the calculations for you. Check out a few.

46. Using **Appendix B Tables** Try the calculation for a loan of **$75,000** at **8%** interest for **20** years. You should get **$627.00** for the monthly payment.

So how much in payments would you pay out over the 20 years.
________________ **$150,480.00**

How much **extra** (ie: interest) did you end up paying beyond the principal amount of the mortgage? __________________ **$75,480.00**

Now try using an App or a website. Your answer will be a bit more accurate since the loan table entries had been rounded.

47. Try the calculation(s) for a more realistic numbers like a $300,000 mortgage at 5% for 25 years. Using Tables, App, or website your answers may differ by a couple dollars.

Total Payments: _____________ **~$526,500.00**
Amount of Interest Paid: ____________ **226,500.00**

48. Fortunately there is an App for those types of calculations! You can find mortgage and loan calculators everywhere!

This completes the quick review of the Grade 11 Studies!

PERSONAL FINANCE USING TECHNOLOGY

51. We have learned how to do personal finance the ‘manual’ way using formulae and tables. Tables need interpolation. It is often tedious and repetitive.

In **Applied Math we use technology** to do all the tedious, complicated, and boring stuff! Perfect for an App or computer.

Our Primary technology device for this unit will be the **TVM Solver** App on the TI-83 Plus Graphing Calculator. You may find equivalent Apps to install on your phone (at your own risk). The TI-83 Graphing Calculator is readily available to download onto any Google Device.

An alternate and equally useful and similar App is the EZ Financial App available on-line for a laptop/desktop: https://www.fncalculator.com/

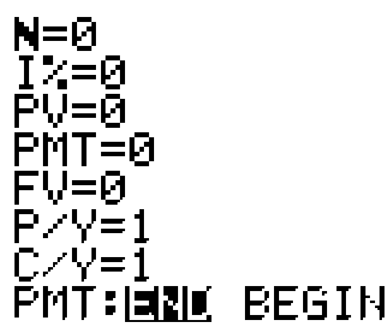
It is also available for **Android** in Google Play and also for **Apple** Devices in the Apple Store.

EZ Financial Calculators

Spreadsheets. Those familiar with Spreadsheets will know that Spreadsheets can readily do financial calculations also.

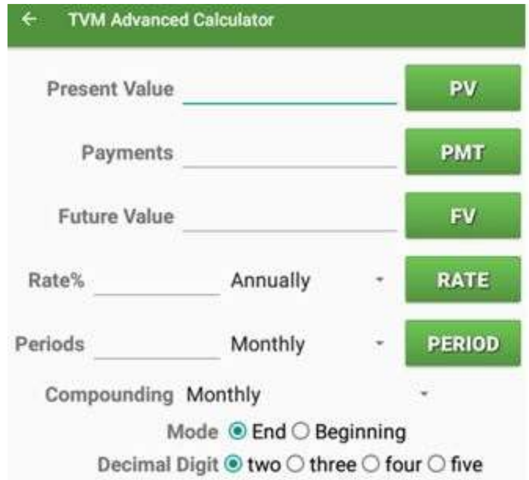
52. Here is a screenshot of the App on the TI-83 Plus Graphing Calculator. The TVM App that we will use frequently.

The TVM Solver (**T**ime and **V**alue of **M**oney)



Here is a screen shot of the EZ Financial App on an Android and an Apple Device

The EZ Financial TVM App that is available for download on Android and Apple Devices.



Calculating Simple Interest on a Single Investment

53. A = P(1 + rt) is trivial and is not done in the TVM App regardless.

54. It is a simple linear function so could just be entered in the Equation Editor and read the table of a Graphing App (Desmos or TI-83 Graphing Calculator).

Example: A simple $100 investment at 10% for ‘**x**’ years would be entered as:

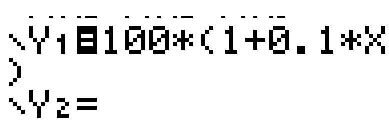
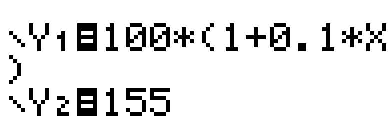


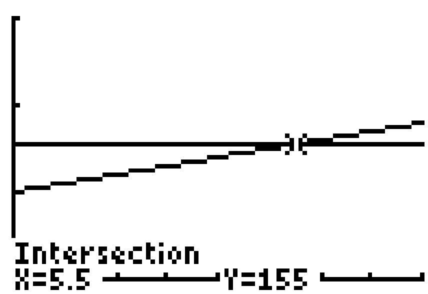
Table:

X	Y_1
0	100
1	110
2	120
3	130
4	140
5	150
6	160

55. And of course, working the graph backwards is an excellent way to solve any equation. Especially if your algebra is weak!

E.g.: Determine when does the investment grow to a total Amount of $155 at 10% APR





So at 5.5 years, your $100 is worth $155.

You had learned how to solve basic linear equations like this using Algebra in Grade 11:

$$I = P \cdot r \cdot t$$
$$55 = 100 \cdot \frac{10}{100} \cdot t$$
$$55 = 10 \cdot t$$
$$t = \frac{55}{10} = 5.5 \text{ yrs}$$

You Try. Use both algebra and a graphing tool to solve the simple interest problem:

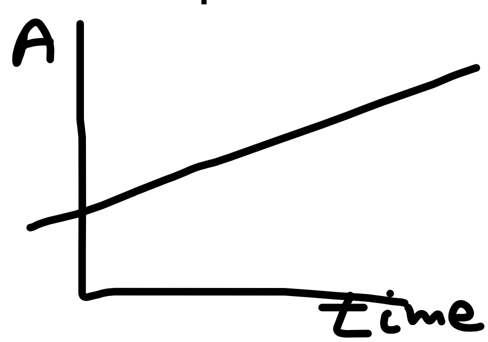
Joanne borrows $750 from her mom for 9 months. Mom charges simple interest of 12% per annum (12% per year). Determine how much does Joanne pays back to her mom after those 9 months.

Solve using algebra and by sketching the graphical solution you got with a graphing tool.

Algebra:

Ans: $817.50

Sketch of Graphical Solution:



Compound Interest on a Single Investment

56. The TVM app quite readily solves compound interest for a single investment.

Recall the formula for a single investment (or single loan repayment) was:

$$\boldsymbol{A = P * \left(1 + \frac{r}{s}\right)^{ns}}$$

57. At right; the '**N**' is the number of **payment periods**. So, this is really the (**n*s**) exponent in the manual formula.

```
N=0
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

The **I%** is the Annual Percentage Rate (**APR**) (enter in %, no need to convert to decimal)

The **PV** is the **Present Value**, what the initial value was.

58. The **PMT** is any regular periodic payment amount, so for example say you contribute $100 at the end of every period. If instead, it is a payment you receive it is called an Annuity.

59. The **FV** is the Future Value, the total amount that the investment pays or owing after the N periods.

The **P/Y** is the number of payment periods per year; so, 12 for monthly, 52 for weekly, how often the regular payments are made.

60. The **C/Y** *defaults* to the same as the **P/Y**, is the number of times per year that interest is computed, the compounding periods per year, which is *usually* the same as the payment period.

The **END BEGIN** is for when the regular PMT annuity is paid at the start of a period or at the End. It defaults to END which is the much more common selection.

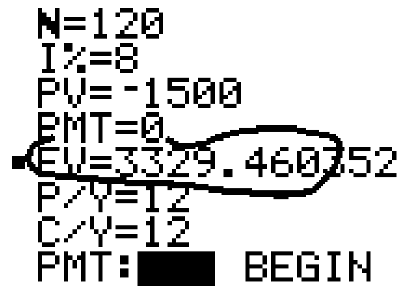
61. **Example**: Solve Compound Interest problem.

You invest $1,500 in a GIC that pay 8% interest APR compounded monthly. Determine the value, **FV**, of the investment after 10 years. →

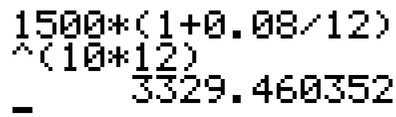
Enter:

```
N=12*10
I%=8
PV=-1500
PMT=0
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

We want to solve for the FV, so *cursor up* to the FV to highlight it, and then select buttons ALPHA ENTER . (ie: **SOLVE** on top of the ENTER button)

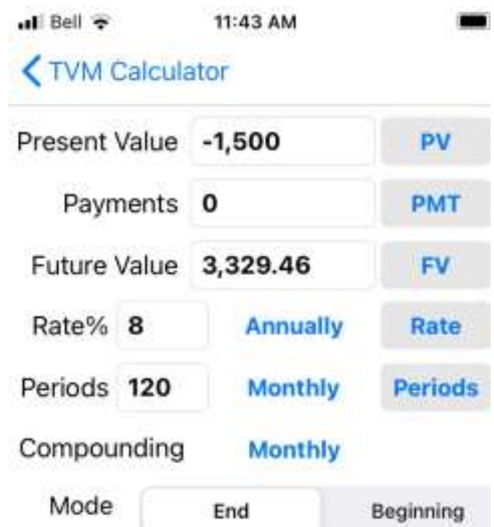


Of course, you could readily check this simple calculation using the manual formula or graphing methods as in Grade 11.



The EZ Financial App on your Android or Apple device would give the same answer; just slightly different layout!

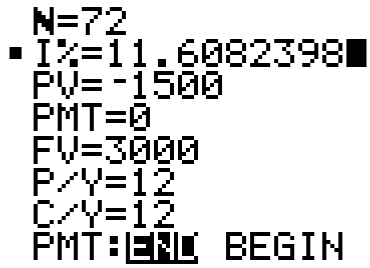
Have you noticed that when you take money out of pocket and put it in the bank it is a negative value!



62. Where the TVM calculator is most useful is in solving for the more complicated calculations and manipulations of the Compound Investment formula!

Asking the question differently! What interest rate do I need if I want to turn my $1,500 (PV) investment compounded monthly into $3,000 (FV) after 6 years?

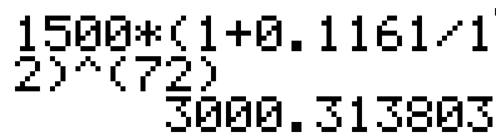
Of course, you know the answer is close to 12% from the rule of 72, but let's get it super accurate using the TVM.



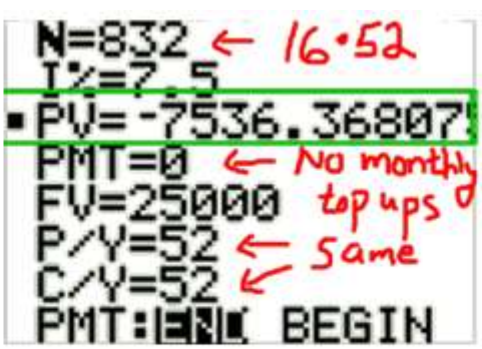
Notice when we put money into something (ie: take it out of our pocket) we enter that as a negative amount.

The solution is that we need an 11.61% APR

You should of course **check** the answer by plugging it back into the manual formula to see that the solution is *likely* correct.



63. Let us solve for a different variable in a single compound investment. I want to save $25,000 for my daughters University. I know I can get 7.5% APR compounded weekly at the bank in a savings device. What lump sum single investment of money do I have to put into the bank **today** so that it will be worth $25,000 after 16 years.



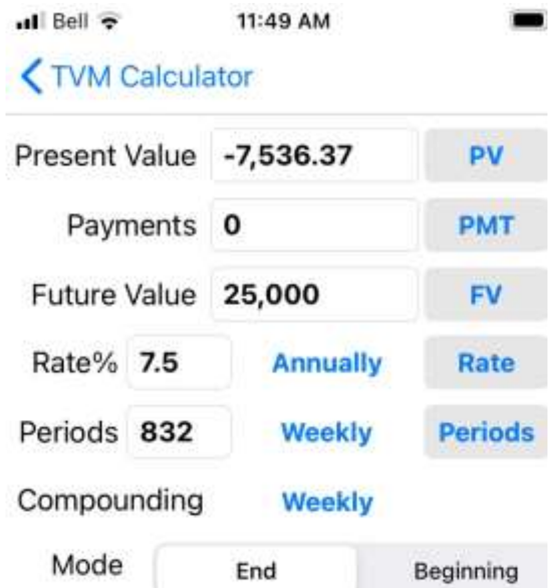
You had to enter the **N** (16*52)

You entered the **P/Y** and **C/Y** as 52 (since weekly).

The solution then is you need to take **$7,536.37** (**PV**) of your money today and put it in the bank if you want it to grow to **$25,000** (**FV**) after 16 years.

In the EZ Financial App it looks similar; like this:

→



Notice in the EZ Financial App you need t manually calculate that the number of periods is 832.

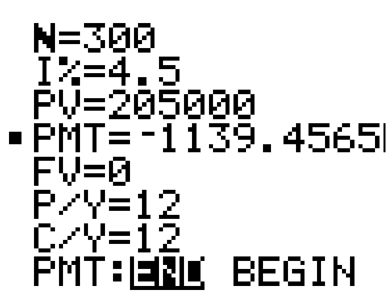
On the TI-83 you can type in 16*52 and it will calculate the periods for you.

64. Check our **Unit B Personal Finance** workbook for lots more practice questions of this type.

REGULAR LOAN REPAYMENTS

65. Most banks will not loan you money and say, "*come back in 20 years and pay us back*". They want regular monthly payments. That works out much better for you for paying off the loan too since it is nibbled away slowly instead of one massive payment with lots of accrued interest at the end of a lengthy period.

66. **Example**: You are buying a house. You need to borrow $205,000. The loan (mortgage) is at 4.5% for 25 years with monthly payments and compounding of interest owed on the balance. Determine your monthly payments.



There are 25 * 12 is **300** Periods

The **FV** is equated to zero since that is what you want it 'whittled' down to after the 25 years in the Future.

Solving for **PMT**, the monthly amount that has to be paid is **$1,139.46, t**he end of every month out of your pocket (so a **negative** indicated) for 300 regular monthly payments.

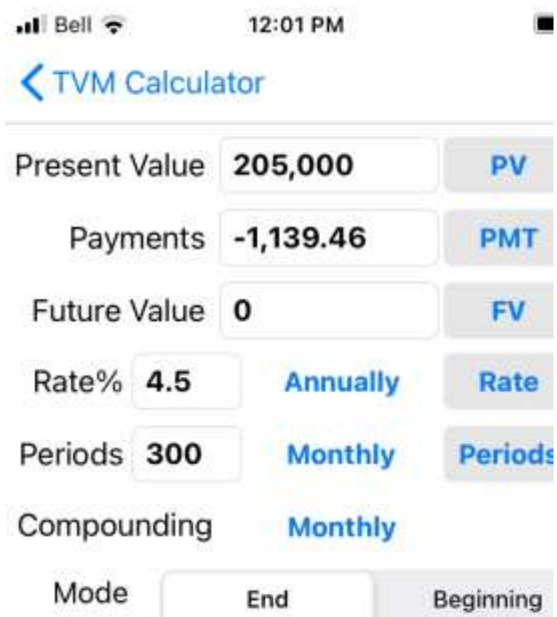
You may want to check that with the manual table look up method.

I get from interpolating the rounded table entries:
\$5.565* 205 = \$1140.83. So, we are in agreement with the TVM App.

This cannot be readily solved using a graphing tool or a spreadsheet since the formula (the function that relates PMT to the PV for example) is completely unwieldy!

The EZ Financial App gives the same identical result.

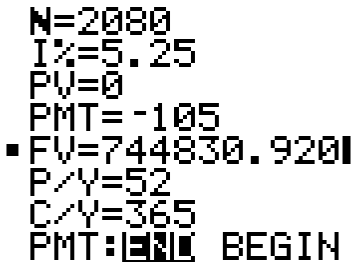
I tend to use the advanced mode of the EZ Financial App, did you notice the difference between the basic and the advanced mode?



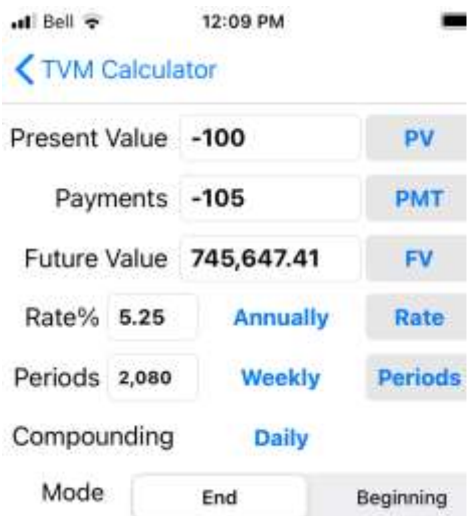
STOP SMOKING

67. How much money could you have if you quit smoking and invested that for 40 years? (and did not spend it on anything else!)

Assume \$15/ pack, one pack a day, thus \$105/ week. You decide to take this \$105 instead and start to put into a daily saving account at 5.25% interest at the end of the week and every week thereafter. How much would you have after 40 years of not smoking?



There are 40 years * 52 payments/yr = **2080** periods of contributions you make. The bank calculates interest **daily** (365 times a year).
You would have **$744,830.92** if you quit smoking for those 40 years and invest the money instead with regular deposits.



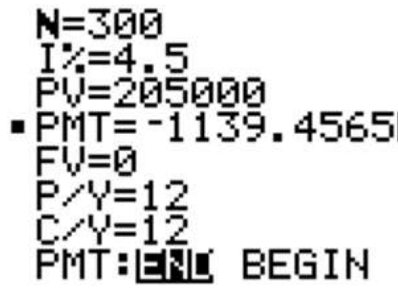
What would the result be if instead of waiting a week to put your smoking money in the bank you made an initial $100 payment immediately and then topped that up every week with $105 for 40 years. The only difference is that that initial immediate $100 deposit that grows for 40 years!

The extra 816.49 for that initial deposit that grew for 40 years.

BALANCE OWING AT ANY PARTICULAR PERIOD

68. An extra useful feature of the TVM program that contains the TVM Solver is the ability to calculate a balance of what is owing at any particular time. Examine our home buying example above.

69. Finding a balance on a **mortgage amortization schedule.**

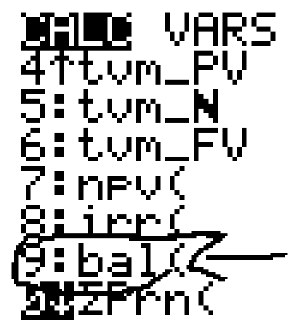


In a previous example we were making mortgage payments. Paying off our loan for 25 years at $1139.46 per month.

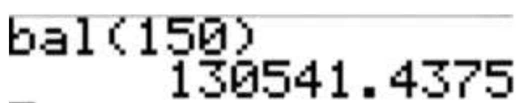
But what amount do we owe still after 12.5 years? You would think it would be *exactly (?)* half? So, about $102,500 still owing! (?)

Let us see…..

70. Find the **'bal('** *balance* function in the Finance App, it is not in the TVM Solver itself. Go to the main calculator screen and paste this **'bal('** function into the main home calculation screen.



Insert **150** periods (ie: 12 periods per year for 12.5 years)



So even after having paid for 12.5 years you still owe the bank $130,541.

71. The balance **still owing** on your mortgage is **$130,541.44** halfway through your mortgage. Consequently, after 12.5 years you have only really paid off **~$74,500** of the principal (205K - 130K) that you borrowed. WTH!!!

Your '**equity**' is ~$74,500. The amount of the house that you have paid off.

Much of your payments so far have mainly just paid interest! Halfway through the loan you have only 'paid off' a ~ third of the mortgage.

The EZ Financial App will do an Amortization Schedule too that shows how you are steadily (but not linearly) paying off your mortgage. You must select the Loan Calculator App from the main menu and generate a spreadsheet.

You will figure it out!

Fully explore an amortization schedule! See how the first few years you are really only paying off interest.

Bell 12:33 PM

Back **Amortization Schedule**

No	Amount	Interest	Principal	Balance
145	1,139.46	503.96	635.50	133,754.17
146	1,139.46	501.58	637.88	133,116.29
147	1,139.46	499.19	640.27	132,476.01
148	1,139.46	496.79	642.67	131,833.34
149	1,139.46	494.38	645.08	131,188.25
150	1,139.46	491.96	647.50	130,540.75
151	1,139.46	489.53	649.93	129,890.82

RETIREMENT ANNUITIES

72. Many people save money for retirement and at a certain age start to draw out of that fund, meanwhile, **it still collects interest**. It is a pool (of money) that they are slowly draining (taking out regular payments) while it is still filling with interest (money).

73. **Example Retirement Annuity**. You have $400,000 saved (since you quit smoking 40 years ago). You are 65 and want to start taking out $3,000 / month from your savings. Of course, the savings is still earning interest, at the same time as you are taking out money, at 7% APR compounded monthly. When will your retirement savings run out?

```
▪N=220.271307
 I%=6
 PV=-400000
 PMT=3000
 FV=0
 P/Y=12
 C/Y=12
 PMT:END BEGIN
```

Solving for **N** the TVM gives 220 periods. So, since it was 12 periods a year, that is 220 months, or 18.33 years. So you can party until you are 83.33 years old.

Notice the **PV** is –400,000 since you took it out of your pocket to invest.

A Nice Retirement. $3,000 / month from the that RRSP retirement savings account that you grew with your non-smoking money plus Canada Pension Plan (CPP) Benefit of $1,000 per month plus Old Age Security (OAS) of $700/month plus maybe a $1,500 company pension from the city or a union plan. A rather comfortable retirement.

Glad you quit smoking?

Healthy enough to enjoy it too till you are at least 85!

PRACTICE PROBLEMS [workbook(s)]

Check out the dozens of pages of practice problems with answers that I have provided you**!**

APPENDIX A
GLOSSARY AND FORMULAE

TERMS

Accrued. A fancy financial term for ‘earned’, ‘built up’. As in ‘accrued interest’.

Amortization Period. The period of time over which a loan is made.

Annuity. A regular periodic; usually monthly, payment.

APR: Annual Percentage Rate. By law, all loans must be stated in terms of the equivalent annual rate! APR often includes extra fees also, but it is not a perfect comparison device. (for example, Payday Loans will charge extra Fees that are not included in the APR)

Balance. The amount still owing on a loan at any particular period of the repayment schedule.

Bi-Weekly. Every two weeks. There are 26 bi-weekly periods in a year.

Budget: A financial plan involving income, expenditures, and savings.

Canada Savings Bonds (CSB). A type of investment device issued by the government of Canada. In effect you are loaning the government money so they can build a stronger Canada and give you back some profit too after several years. These were just recently (2017) cancelled by the government.

Compound Interest: Interest calculated at regular intervals on an amount of money to which interest from previous calculations has been added.
$A = P(1+i)^n$

Equity. The difference between the market value of real estate and the amount still owing; the amount of the property that you have paid off.

Exponential. A type of growth that grows on itself. The rate at which it grows gets higher and higher since it grows on itself like mould or breeding bunnies. Exponential functions grow quicker and quicker the longer they grow.

Guaranteed Investment Certificate (GIC). An investment product at your bank. The bank guarantees a certain Interest Rate for a certain time. Usually the more you invest the better rate of interest you get.

Interest. The fee paid for the use of money, usually at a predetermined percentage rate of the money borrowed or invested.

Lease. To rent an item from the owner; the lease payments cover the depreciation of the item over the course of the lease plus interest on the outstanding balance of the full purchase price. Plus a little profit.

Line of Credit. An amount the bank allows you go into the 'red'. Many banks will allow you to have a debt (a negative amount) in your account. Of course, you will pay interest on it. So say you have a line of credit for $20,000. If you use the entire line of credit you will likely pay 6-7% per month on the balance owing, so $120 – 140 per month in interest. It is pretty much a continuous loan that you may never pay off.

Mortgage. A long-term loan on real estate that gives the person or firm providing the money a claim on the property if the loan is not repaid.

Overdraft. An arrangement you make with your bank whereby if you inadvertently spend more than you have in an account you will still be covered so that you do not get an NSF (Non Sufficient Funds) or Purchase Declined event. Of course, there will be a charge and a percentage applied.

Principal: The amount of a loan or investment.

Quarterly. Something that happens ever quarter of a year. That is: every three months. If you have a quarterly payment of $300 it is like $100 per month.

Residual Value. The value of a leased item at the end of the lease period. You lease a car, you drive it for three years, it gets regular wear and tear (depreciation); what the vehicle is worth after that lease period. There are websites, Blue Books, Black Books that will document the residual value.

Rule of 72. A quick way to estimate the doubling of a single compounding and one-time investment.

Semi-Annually. Something that happens twice per year. Every six months.

Per Annum: Per year

Formulas:

I=P*r* t
A = P + I or combining the formulae: **A = P*(1 + r*t)**

I is interest in $. P is principal in $. r is interest rate [%]. t is time (in years) and A is the final Amount. Principal is often called Present Value (PV) and final Amount is often called Future Value (FV)

$$\boxed{A = P\left(1+\frac{r}{s}\right)^{n*s}}$$

where

A is the final value of the investment
P is the Principal or initial value, i.e., the amount that has been invested
r is the rate of interest expressed in decimal form
n is the number of years of investment
s is the number of times that the interest is calculated in one year

Do not be confused by the fact that in one formula the years is a '**t**' and in the other it is an '**n**'. You will just have to get used to the fact that often formulas use different symbols for the same idea!

APPENDIX B
TO GRADE 12 APPLIED
UNIT B PERSONAL FINANCE NOTES

MONTHLY LOAN PAYMENT TABLE FOR A LOAN OF $1,000

Annual Rate	1 Year Monthly	2 Years Monthly	3 Years Monthly	4 Years Monthly	5 Years Monthly	10 Years Monthly	15 Years Monthly	20 Years Monthly	25 Years Monthly
2%	$84.24	$42.54	$28.64	$21.70	$17.53	$9.20	$6.44	$5.06	$4.24
3%	$84.69	$42.98	$29.08	$22.13	$17.97	$9.66	$6.91	$5.55	$4.74
4%	$85.15	$43.42	$29.52	$22.58	$18.42	$10.12	$7.40	$6.06	$5.28
5%	$85.61	$43.87	$29.97	$23.03	$18.87	$10.61	$7.91	$6.60	$5.85
6%	$86.07	$44.32	$30.42	$23.49	$19.33	$11.10	$8.44	$7.16	$6.44
7%	$86.53	$44.77	$30.88	$23.95	$19.80	$11.61	$8.99	$7.75	$7.07
8%	$86.99	$45.23	$31.34	$24.41	$20.28	$12.13	$9.56	$8.36	$7.72
9%	$87.45	$45.68	$31.80	$24.89	$20.76	$12.67	$10.14	$9.00	$8.39
10%	$87.92	$46.14	$32.27	$25.36	$21.25	$13.22	$10.75	$9.65	$9.09
12%	$88.85	$47.07	$33.21	$26.33	$22.24	$14.35	$12.00	$11.01	$10.53
14%	$89.79	$48.01	$34.18	$27.33	$23.27	$15.53	$13.32	$12.44	$12.04
16%	$90.73	$48.96	$35.16	$28.34	$24.32	$16.75	$14.69	$13.91	$13.59
18%	$91.68	$49.92	$36.15	$29.37	$25.39	$18.02	$16.10	$15.43	$15.17
20%	$92.63	$50.90	$37.16	$30.43	$26.49	$19.33	$17.56	$16.99	$16.78
25%	$95.04	$53.37	$39.76	$33.16	$29.35	$22.75	$21.36	$20.98	$20.88
30%	$97.49	$55.91	$42.45	$36.01	$32.35	$26.36	$25.30	$25.07	$25.02
35%	$99.96	$58.52	$45.24	$38.97	$35.49	$30.12	$29.33	$29.20	$29.17

EXAMPLES of loan payments ↓

Example A. You borrow $120,000 for 10 years at 14% Annual Rate. Your monthly payments are $13.22 for each thousand you borrow. So your monthly payment on $120,000 is 120 times as much or $1,586.40 per month. So your loan is paid off after 120 payments of $1,586.40 so a total of $190,368 in payments. So your $120K loan cost you $190K.

Example B. You borrow $200,000 for 25 years at 6% Annual Rate to buy a house. Your monthly payments are $6.44 *for each* thousand you borrow. So your monthly payment on $200,000 is 200 times as much or $1,288 per month. So your loan is paid off after 300 payments of $1,288 so a total of $386,400 in payments. So your $200K house cost you $386K over 25 years. Of course, hopefully you will be able to sell it for at least $350K, so it really only cost you $36K to live in a house for 25 years. Mind you now you need another place to live… but the kids are gone so you can get a smaller place!

APPENDIX C
TO GRADE 12 APPLIED
UNIT B – PERSONAL FINANCE

SOLVING FINANCIAL PROBLEMS WITH THE TIME-VALUE OF MONEY SOLVER ON THE TI-83 & 84 GRAPHING CALCULATOR

TI 83 Variable Label	N	I%	PV	PMT	FV	P/Y	C/Y
Meaning	Number of Payments	Interest Rate [%]	Present Value [$]	Payment Each Period [$]	Future Value [$]	Payment Periods per Year	Compounding periods per year

All of the above variables are closely inter-related. Any change in one will influence the others.

I% Interest Rate. The rate that the bank applies. It is different for loans than for savings. You have little control over this. It is what *they* charge *you on loans* (and mortgages) and what *they pay you* on your (meagre) *savings*.

PV Present Value. Present value is the amount of the loan or the savings. A negative PV means you have invested money (paid out, deposited).

FV Future Value. Future Value is:

a. what an investment of yours will be worth after a certain time , or

b. what amount remains to be paid on a loan at the end of a certain time. You usually want to know when this will be zero so your loan is done. A loan is done at the end of the amortization period when FV=0. You still owe money if FV is negative.

A (-) negative amount means you owe money.

P/Y Payment Periods Per Year.

a. The number of payments per year *you make* on a loan, or;

b. The number of payments per year the bank makes *to you* on your savings.

C/Y Compounding periods per year. The number of times per year that interest is compounded. *Compounding* just means that the bank *pays interest on your interest and your principal* in savings accounts! It also means, in the case of a loan, that you are paying interest on what you owe plus the *interest on the accrued interest* on what you owe. (which as you will see is why you typically pay $240,000 for your $100,000 house!)

PMT – Payment Each Period. This is the amount of dollars transacted each payment period.

a. In the case of savings, it is the interest that the bank pays to you.

b. In the case of loans it is the amount you pay to the bank. **On the TI 83** TVM Solver your **cash outflow is always a negative number**. So if you are repaying a loan off at $250.00 per month, it would be (–)250.00 in the TI-83 TVM solver. If you want to make this a lesser payment then you will have to pay longer (much longer) or hope for a reduced interest rate (I%).

N - Number of Payments. This is the total number of payments. If you want to make fewer payments on a loan then you need to change one of the variables above. (That is: either hope for a lower interest rate, make larger payments each payment period, or don't take out such a big loan!). **Often, you can't calculate N without first knowing P/Y.**

PMT: END BEGIN. This selection on the TI-83 determines when your deposit is made. At the **end** of each payment period or at the **beginning** of each payment period. It is almost always selected to **END**.

USING THE TVM SOLVER ON TI 83

Simply select APPS and TVM Solver.

Input values to each of the variables. Input zero if it something you will want to calculate.

When you have input the values, cursor to the value you want to calculate and press: ALPHA ENTER to solve.

CALCULATION EXAMPLES

Now that we understand that there are **seven different inter-related variables** in financial calculation of loans and savings, let us try several examples in which we want to calculate one of the unknowns.

In the table below, cells in grey are the resultant calculations given the known values that are input in the white cells.

	N	I%	PV	PMT	FV	P/Y	C/Y
Loans and Mortgages							
Example	Number of Payments	Interest Rate [%]	Present Value [$]	Payment Each Period [$]	Future Value [$]	Payment Periods per Year	Compounding periods per year
A	60	8%	+5,000	**-101.38**	0	12	12
B	**74.7**	8%	+10,000	-150	0	26	12
C	24	10%	+4,000	**-184.58**	0	12	12
D	**76.7**	23%	+4,000	-100	0	12	12
	76	23%	+4,000	-100	**-64.31**	12	12
E	25*26= 650	**4.26%**	+60,000	-150	0	26	12
SAVINGS AND INVESTMENT							
F(1)	1222	6%	$0	**-155.10**	$1,000,000	26	1
F(2)	1222	6%	-10,000	**-131.11**	$1,000,000	26	1
G (1)	3650	2%	-10,000	0	**$12,213.96**	365	365
G (2)	40	5%	-10,000	0	**$16,436.19**	4	4

EXAMPLE A. Example A is a loan *you take from* the bank. You take **$5,000** as a loan (**PV=5,000**) to buy a 1996 Ford and decide to pay it off over 60 payments (**N=60**) and at that time, at the end of the loan you owe nothing (**FV = 0**). You make 12 payments per year (**P/Y**); that is one per month. The bank is charging you **8%** interest (**I% = 8**) (you don't get a great interest rate because you have *no credit rating* yet). The bank compounds your indebtedness every month, that is (**C/Y**) is **12**. Payments are made at the end of each month so **P/Y** is **12** and '*PMT [Begin] [End]'* is [**End**].. Using your TI-83 TVM solver find:

> <u>The monthly payment - PMT</u>. Answer: **–101.38**. The TI 83 shows a negative sign because it is an '**out flow**' of money for you; that is; you are paying. So your payments are $101.38 a month for 5 years.
>
> <u>Total Paid</u>. You paid **$101.38** per month for 60 months. Multiply. So you paid a total of **$6082.80** for your car.

EXAMPLE B. You have found the perfect car. It is *only* **$10,000** and you need a loan for the full amount (that is you have no down-payment so you need the full amount so **PV=10,000**). You know you can afford to pay **$150** (PMT= **–150**) every two weeks for it to the bank (**P/Y=26**). You know the best interest rate (I%) you will get is **8%** (I% =8) and the bank compounds it monthly (**C/Y=12**).

> **Don't forget with TVM solver, any payment you make, or money outflow, must be entered as a negative number. You must use the (-) key on the TI 83, not the subtract button, otherwise you will get an error.**

a. **how many payments will you have to make**? Answer: you will need to make **N = 74.7** payments. So really 75 payments and the last one is a bit less than the others.

b. **What will be your total cost for the car**? You paid out **$150** for **74** two-week period pay checks so 148 weeks, or just 8 weeks short of 3 years. (the 75th payment was only $106.17), so you paid approximately **$150*75=$11,250** for your $10,000 car (which is now 3 years older by the time you paid for it, time for another one).

EXAMPLE C. You desperately need a vacation. The travel agency offers you one to Cancun, Mexico for **$4,000** all expenses included (**PV = +4,000**). You are willing to pay for the vacation for the next two years (**24** months). You take a loan from the bank at 10% compounded monthly (**C/Y = 12**). Your payments are monthly (so **P/Y = 12**) at the end of each month.

a. **How much are your monthly payments (PMT)?** Answer: **–184.58**. The (-) means it was an **outflow** from you, that is **you paid** it.

b. **What total amount do you pay?** You pay **$184.58 * 24** payments = **$4429.92** total for your **$4,000** vacation.

EXAMPLE D. You desperately need *another* vacation. The travel agency offers you one to St. Lucia in the Caribbean for **$4,000**; all expenses paid. The bank is unwilling to lend you money since you had a few financial problems before, so you (foolishly) charge it to your charge card. Your charge card interest is **23%** (this is no exaggeration!) You can only afford to pay it off at **$100.00** (PMT= **–100**) per month (**P/Y=12**).

a. **How long will it take you to pay off your credit card**? It will take **76.7** months to pay off your card. (ie: almost **6.5 years**) to get FV down to zero (FV=0). At the end of 76 months of payments you only owe for your last payment $64.31. (You can get the last payment by entering N=76 to see what remains as an FV on the loan. The FV shows (–)64.31 so it something you still owe.). (Hopefully you hadn't charged anything else on your charge card over those 6.5 years!) (hopefully you know also that any cash you withdraw on a credit card is compounded daily!)
   Another way to find the balance you still owe at the end of 76 months is the TVM **'bal('** function. Paste **'bal(76)'** onto the main calculation screen, it will reveal the balance at the end of the 76 months is 64.32

b. **How much will the vacation have actually cost**? Your vacation will have cost 76 months at $100 and the last payment of $64.31. A total of $7664.31 for a $4,000 vacation! You could have almost had two full vacations if you had that amount to start with!

EXAMPLE E. You are tired of paying rent and fighting with the landlord. You think you can afford your own house but are not sure. The house you want costs **$70,000** but you can put **down** cash **down payment** of $10,000 that your grandmother left you. You want to make payments every paycheck, so every two weeks (**P/Y=26**). You are confident you can afford **$250** per pay check (after allowing for the budget considerations and fact you have to keep some monthly money aside for house repairs, heat, electricity, water, insurance, property tax, school tax, ….). The bank offers you a house loan of $60,000 (called a mortgage) compounded monthly (**C/Y = 12**) over **25** years. (25 years is what most people take for a mortgage).

what minimum interest rate (I%) do you need so as to be able to afford the house? Answer: **4.26%**. If the bank interest rate is anything less than **4.26%** then you *can afford* the house. Presently however, the mortgage rates in Winnipeg for good solid bank customers are about **5.75%**. So you can't afford the house. (unless you get someone else to help you with an additional **$10,000** toward the up front cash down-payment to reduce the amount of the mortgage).

SAVINGS AND INVESTMENT

EXAMPLE F. You are **18**. You want to be a millionaire by the time you are **65**! You have studied investments closely and have determined you could make **6%** interest (compounded) on your investment until you are 65. You have a pretty good job and want to put away a certain amount each pay check. You get paid every two weeks (so P/Y = 26). You will make bi-weekly payments for **47** years, so **N =1222**. All your investments will compound yearly. (like stock dividends do if you invest in WALMART). So C/Y = 1.

How much do you have to put aside every paycheck until you are 65? Answer: **$155.10** every paycheck will make you a millionaire (**FV=+1,000,000**) at age 65 if you start right now as a high school student.

How much do you need to put aside each pay check if you put your grandma's $10,000 straight into stock right now and then add to it from your pay check? Answer. If you took Grandma's gift money of $10,000 (PV=(-)10,000) and invested it right now and then added your own amount every two weeks, you would have to add $131.11 every paycheck to become a millionaire at age 65.

EXAMPLE G (1). Your grandmother leaves you $10,000. You promised to invest it all for 10 years! You put it in your daily interest saving account. The bank pays you a whopping 2% interest (I%) on it but it pays it daily (P/Y=365) and the interest is computed daily (C/Y=365).

How much is your $10,000 worth (it's Future Value FV) after 10 years? Answer. After 3650 (365 days per year times 10 years) payments from the bank you have $12,213.96

EXAMPLE G(2). Just like example G(1) but you invest in a different portfolio that pays 5% but only pays out 4 times per year (*quarterly*) and only compounds quarterly also.

How much is your $10,000 investment worth? Answer: $16,436.19.

YOUR OWN EXAMPLES

If you want to invent some of your own examples and test different things try printing and using the blank table below.

Try playing with the difference between monthly and daily interest on savings accounts.
See what house payments might be if mortgage rates went to 18% like they did back in the 80s!
Try comparing the TI-83 results with other calculators and on-line websites and Apps on your device.

Have fun with TVM

	N	I%	PV	PMT	FV	P/Y	C/Y
Loans and Mortgages							
Example	Number of Payments	Interest Rate [%]	Present Value [$]	Payment Each Period [$]	Future Value [$]	Payment Periods per Year	Compounding periods per year