GRADE 11 ESSENTIAL MATHEMATICS
UNIT C – 3-D GEOMETRY
CLASS NOTES

INTRODUCTION

1. These notes are designed to guide the student through the Three Dimensional (3-D) Geometry Unit of Grade 11 Essential Mathematics. They are written in a note frame form, so students are expected to fill in some of their own notes as the course progresses.

2. **Specific Outcomes**. It is expected that students will:

solve problems involving Metric and Imperial units in surface area measurements;

solve problems involving Metric and Imperial measurements in volume and capacity measurements; and

solve problems that involving the manipulation and application of formulas (algebra)

PRE-REQUISITES

3. You should already be familiar with:

a. how to read a metric and imperial ruler (Grade 10 Essential)

b. how to convert units of length and capacity and weight to different units (Grade 10 Essential)

c. the area and perimeter of 2-D shapes (Grade 10 Essential)

d. basic algebra (Grades 8 – 10)

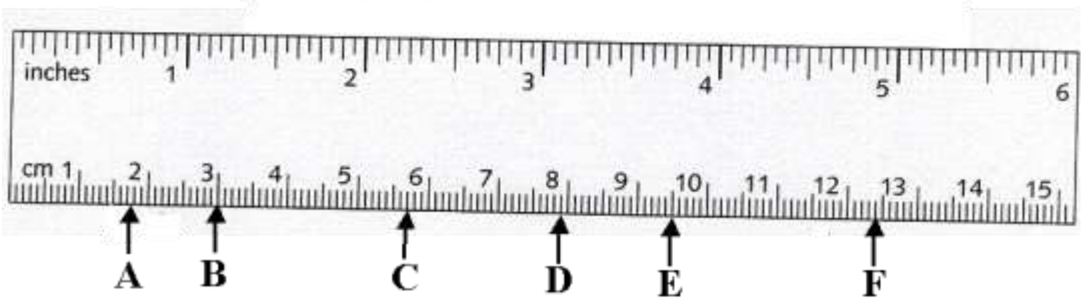
4. If you are unfamiliar with these you will have extra effort to apply and you will likely want to consult the notes and resources from Grade 10 Essential Unit C (Measurement) and Unit D (2-D Geometry).

HASTY REVIEW OF GRADE 10 ESSENTIAL

READING A RULER

Reading a metric ruler

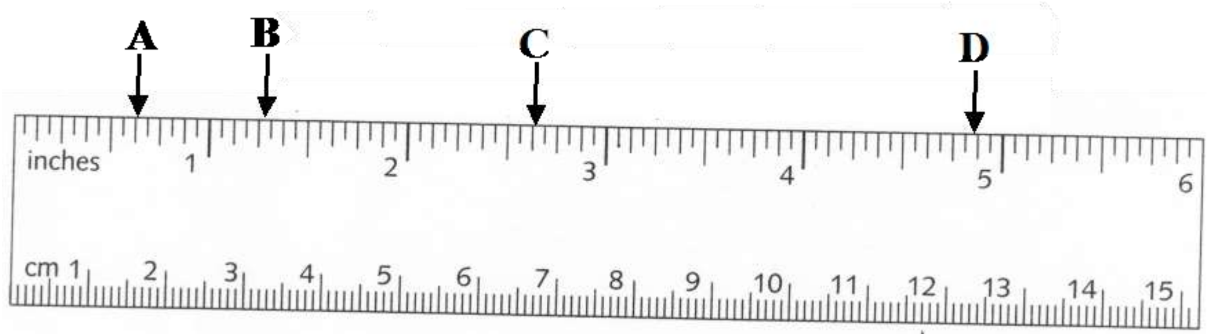
5. Complete the table for the indicated measurements: (to nearest *tenth* of a cm):



Position	A	B	C	D	E	F
cm						

Reading an Imperial Ruler

6. Complete the table for the indicated measurements: (nearest *eighth* of an inch):



Position	A	B	C	D
inch				

CONVERTING UNITS OF MEASURE

7. There are two common methods to convert units: The Proportion Method and the Unit Factor Method. The latter is the preferred method. Conversion tables are at the end of these notes although you should know the common ones. The metric conversions should be especially familiar since they are Canadian for over 40 years. Only Americans still use feet and inches and miles and pounds and gallons, etc.

8. **Example**: Convert 30 inches (in) to centimetres (cm).

Proportion Method

$$\frac{2.54\,cm}{1\,in} = \frac{x\,cm}{30\,in}$$ cross multiply

$$\therefore x = \frac{2.54\,cm * 30in}{1\,in} = 76.2\,cm$$

Unit Factor Method

$$30\,in * \frac{2.54\,cm}{1\,in} = 76.2\,cm$$

multiply by (new units / old units)

You Try. Convert. Round decimal answers to two decimal places!

Workspace:

a. 5 km = __________mi

b. 30 m = __________ ft

c. 60 cm = __________ in (to nearest 1/8th in)

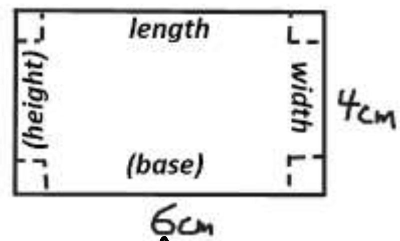
d. 4 m^2 = __________ft^2

Ans: a. 3.11 mi b. 98.42 ft c. ~~23.62 in~~ $23\frac{5}{8}$ in d. 43.05 ft^2

FORMULAE FOR CALCULATING VARIOUS PERIMETERS AND AREAS OF SELECTED TWO DIMENSIONAL (2-D) SHAPES
REVIEW

10. **Lengths**:

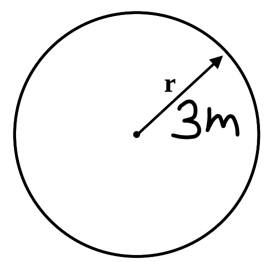
a. Distance around a rectangle (Perimeter)
Add the length of sides
Perimeter = 2w + 2l where w = width and l = length (all measured in the same units)



Often length & width are called base & height

Of course, if it is a square (a special rectangle) then it is 4 * the length of one side

b. Distance around a circle (Circumference, '**C**')
Circumference is really a perimeter, but a different word is used for circles.



C = 2πr; where r is the radius measurement or since 2*r is a diameter ; or **C = πd** where d is the diameter measurement.

The circumference of this circle is ~18.85 m. Check it yourself on a calculator.

Note: Value of pi, π. Pi is an 'irrational number', the decimal portion has no end. Generally, we use the π button on our calculator which for *most* calculators is 3.141592654. Sometimes you will be asked to use the more inaccurate value of '**3.14**' (for those who have no calculator and do the calculations the old manual way!) or like in the old days, before calculators and the rampant use of silly decimal numbers, many students used the fraction $^{22}/_{7}$ as a good approximation of π**.**

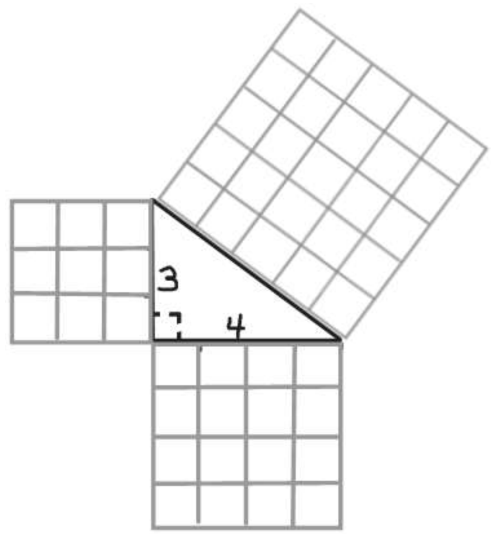
Finding a Length of a triangle side (Pythagorean Theorem)

11. Recall the Pythagorean Theorem. [A critical idea!]

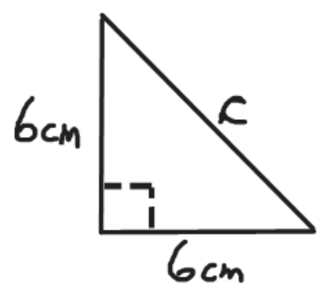
"For a right triangle, the sum of the squares of the short two sides equals the square on the hypotenuse"

$\mathbf{c^2 = a^2 + b^2}$, where c is the hypotenuse length

12. Example: Determine the length of the hypotenuse:
(the hypotenuse is the longest side of a right triangle, across from the 90° corner)

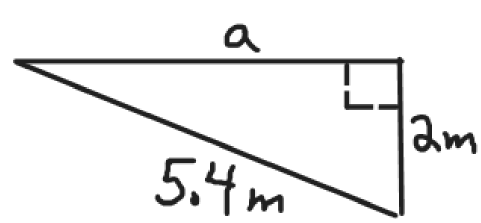


13. **You try**. Determine the unknown length, c, of the right triangle.



Ans: 8.49 cm

14. **You try**. Determine the unknown length, a, of the right triangle.



Recall: We pretty much always round decimal answers to two decimal places in all our math

Ans:5.02 m

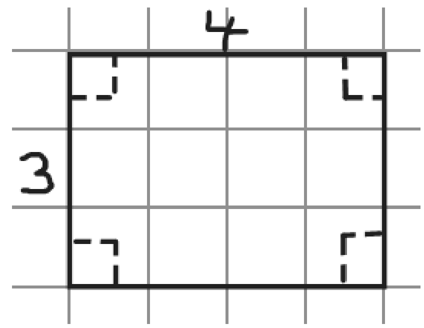
15. **Review of Areas**: How many 'squares' would fit onto a surface.

a. **Rectangles, squares and parallelograms**

Area = base * height
Area = b * h

This area is 4 units* 3 units = 12 square units or **12 units^2**

Notice that the word 'height' is a measurement that is perpendicular to a base length.



b. **Circles**

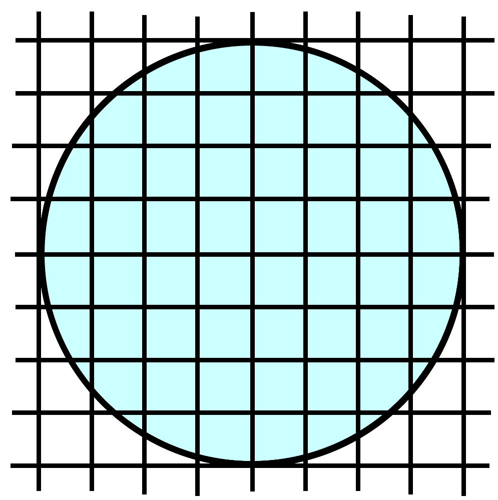
Area = π**r^2**

Where **r** is the radius (or half the diameter)

How many square cm are in a circle of *diameter* **8 cm**?

____

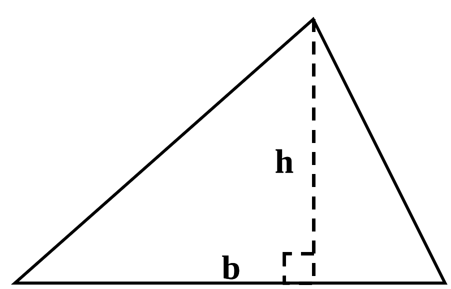
Ans: 50.27 cm^2



c. **Triangles**

Area = ½ * base * height

Note that '*height*', **h**, is always measured perpendicular to the base! When you measure your kid's height they stand up straight I hope!



*Any triangle is **half** a parallelogram.*

12. What is the area of a triangle having a **base** of 4 meters and a **height** of 2 meters?

Your Solution:

You are encouraged to sketch geometric shapes!

ANS: 4 square metres

13. A formula sheet for all the geometric formulas you used in Grade 10 and the ones you need for Grade 11 are attached at the end of these notes in an Appendix.

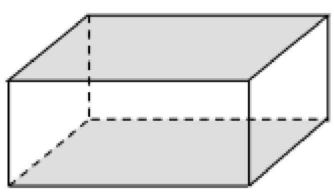
NAMING THREE-DIMENSIONAL (3-D) SHAPES

20. You had learned the two-dimensional shapes: rectangles, squares, circles, triangles, trapezoids, parallelograms, rhomboids, etc. Check especially the **glossary** at the end of these notes.

Two of the most common types of 3-D or solid objects are: Prisms and Pyramids.

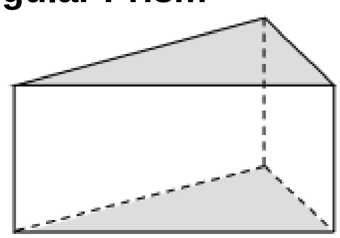
21. **Prism Definition**. A prism is two congruent (same) 2D shapes that are moved apart and their edges connected with rectangles. The two shapes that were moved apart are called the **base** shapes, (or simply the base) the rectangles that join the edges are called the **lateral** sides.

a. **Rectangular Prism**



of course, if all the edges and faces are the same (congruent) it is called a cube.

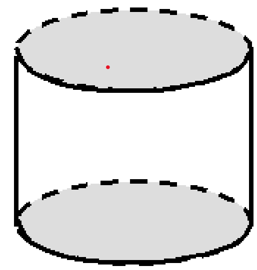
b. **Triangular Prism**



if the rectangles go straight up from the base it is called a '*right*' prism.

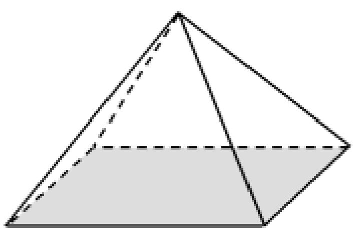
Draw a pentagonal prism!

c. **Cylinder**. A cylinder is type of prism except its base shape is a circle. It has one continuous rectangle wrapped around the two circles. The wrapped rectangle is the *lateral* side of the cylinder.



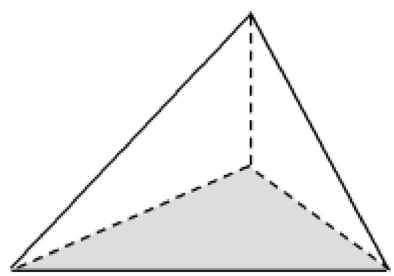
22. Pyramid Definition. A pyramid is one base shape that has its edges connected to a single point (vertex) by triangles (as opposed to a prism that has rectangles). The triangles that join the edges and meet at the point (vertex) are called the **lateral** sides.

a. **Rectangular Pyramid**



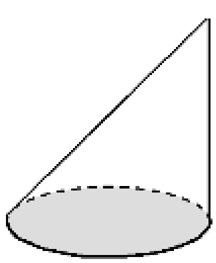
If the base is a square (a special rectangle) then it is called a **square pyramid**.

b. **Triangular Pyramid**



Of course, what is the base in this case is up for debate, and doesn't really matter any way.

c. **Cone**. Not really a pyramid is a 'circular pyramid', more commonly called the **cone**.

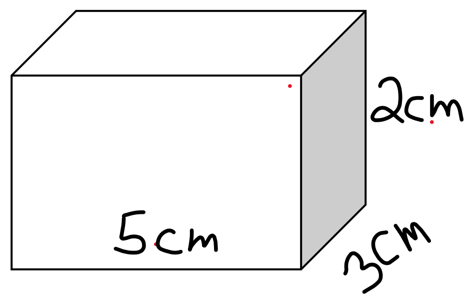


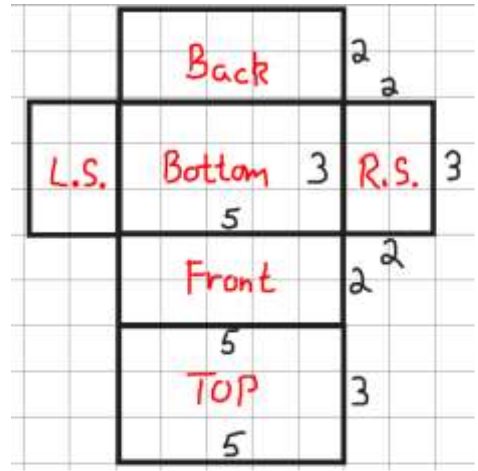
Notice this particular cone shape is not symmetrical. Symmetrical shapes that connect 'straight up' to a point or to another base shape are called right objects. If they do not go straight up, they are '*non-right*'

Sketch a hexagonal pyramid →

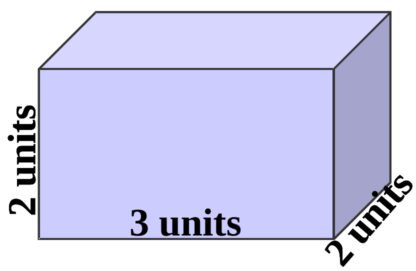
23. Surface Area (**SA**). The area of all the surfaces of a three-dimensional object if you covered the outside in squares of some size (m^2, ft^2, etc.). Sometimes when finding the surface areas of solid (3-D) figures it is easier to draw the '**net**' of the figure:

NET

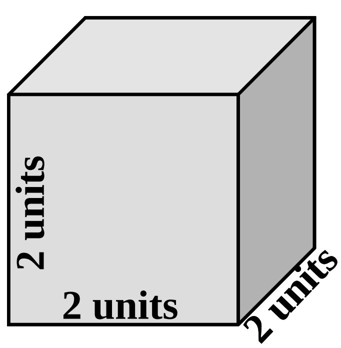




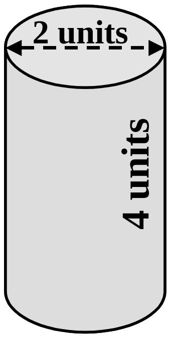
24. Accurately draw the net of this 3-D Box (actually called a rectangular prism)



25. Accurately draw the net of this 3-D figure (actually called a cube)

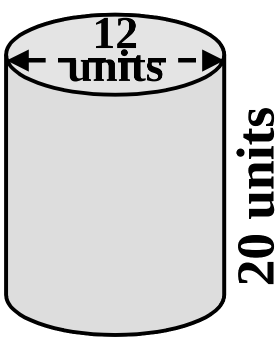


26. Accurately draw the net of this right cylinder on the square grid.

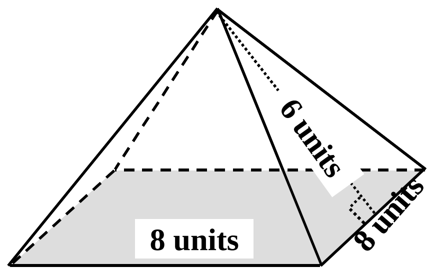


The base length of the rectangle is the circumference of the circle it wraps around

27. Accurately draw the net of this right cylinder. (you will need to '*re-scale*' the grid, count by two's or nickels maybe)



28. Accurately draw the net of this right square pyramid. (you will need to 're-scale' the grid, count by two's or nickels maybe)



30. Accurately draw the net of this.

You do one!!

SURFACE AREA OF 3-D OBJECTS

An important characteristic of 3D solid objects is their surface area. Surface area represents the area of all faces of an object, and can be defined in two ways:

Lateral Surface Area

- Pyramids: Area of all faces (triangles) except the base shape.
- Prisms: Area of all faces (rectangles) except the base shapes.

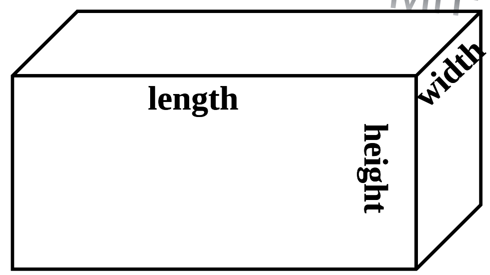
Total Surface Area

- Total Area of all faces including the ends or base(s).

It sometimes helps to at least sketch the 'net' of an object if you can.

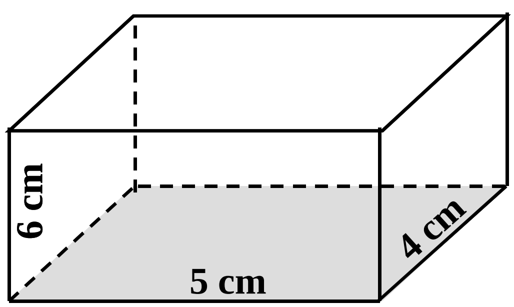
31. **SA** of a rectangular box or cube (called a rectangular '*prism*' really)
(A cube is just a rectangular box but all sides are the same length)

The front and back = l*h*2
The sides = w*h*2
The top and bottom = l*w*2



Total SA: **2*l*h + 2*w*h + 2*l*w**
(Just add the area of all six faces)

32. **Example:** SA of a Rectangular Prism

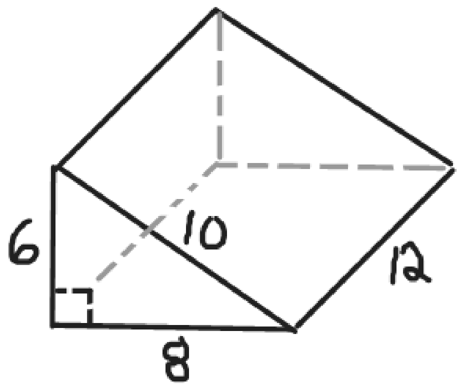


L & R Rectangles	
Bottom & Top	
Front & Back	
Total:	

You will likely find a more simple formula for simple prisms which you might favour too. Explore any fancier formulae on your own.

Check out some of the awesome 3D tools on our webpage!

33. **SURFACE AREA OF** Triangular Prism



There are three rectangular sides and two congruent triangle base shapes.

Just calculate the area of all five faces and add them up!

Sketching the net of the object, helps

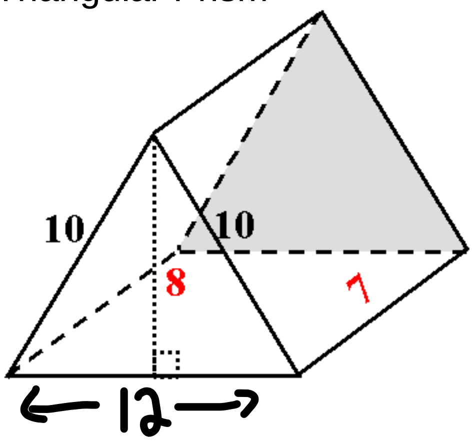
Left Rectangle	
Bottom Rectangle	
Right rectangle	
Front Triangle	
Back Triangle	
Total:	

Note: Even though we can create specific formulae for each object, always use common sense when calculating surface area of an object. The triangular prism, for example, is simply made up of three rectangular *lateral* sides and two triangles as the base shape. Since we know how to calculate the area of rectangles and triangles one should be able to calculate all parts of the triangular prism and add them together. It is not necessary to memorize a specific formula if you know the few basic ones. No need to get fancy unless you do this for a living.

KISS Keep It Simple Silly!

36. **Example**: Calculate the lateral surface area and total surface area for the following triangular prism

SURFACE AREA OF
Triangular Prism



There are three rectangular sides and two congruent triangular triangle sides.

Just calculate the area of all five faces and add them up!

Sketch the net of this object, helps

Left Rectangle	
Bottom Rectangle	
Right rectangle	
Front Triangular Base Shape	
Back Triangle	
Total:	

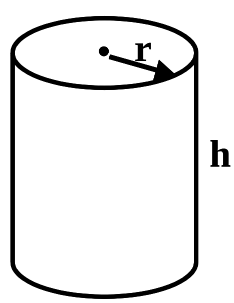
SURFACE AREA OF CYLINDERS

40. A cylinder is simply a base shape of circles and a single rectangle wrapped around it!

As usual, sketching out the net is often useful.

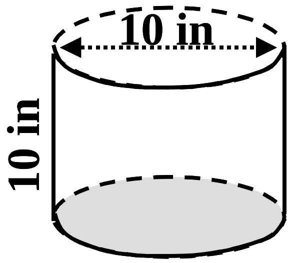
41. **SA** of a cylinder
The surface area of a cylinder if you were to cover its outside in squares of some size would be:

Top and Bottom: $\pi r^2 * 2$
Lateral sides: $2\pi r*h$



Total: $2\pi rh + 2\pi r^2$.

42. Example Cylinder Calculation.



Lateral (Rectangle)
Area
(the 'tube')

Bottom and Top
Circles

Total:

43. **Surface Area of a Sphere**. A neat thing! Four circles of the same diameter as the sphere will cover a sphere exactly!

SA = 4 πr^2

What is the Surface Area (SA) of this sphere of diameter 10 cm?

SA = 4*π***r^2 =** ______________

Ans: 314.16 cm^2

44. There exist more formulas that are readily explained for other shapes in your Geometric Formula Sheet in the Appendix at the end of these notes. You are not expected to memorize the formulae, you will always have access to them on the course for tests and the exam.

VOLUME OF 3-D OBJECTS

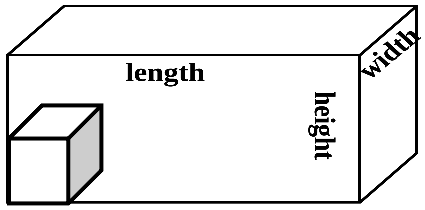
44. **Volume**. Volume is how many '***cubes***' of some size you could fit inside a three-dimensional object. Picture how many ***sugar cubes*** could fit into your shoe for example.

a. Volume of a rectangular prism or cube (a box)

V = Base Area * height

V = B *h_{prism}

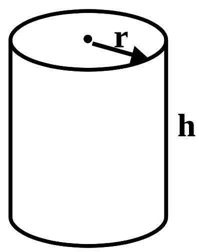
V = (l*w) *h_{prism}



If your 'rectangular prism' is :
(**2cm * 3cm) * 4 cm**
your volume is 24 cubic cm or 24 cm^3

The Volume of ***any*** prism is its Base shape area, **B**, times the **height** the base shape is stacked up

b. Volume of a cylinder:



V_{cyl} = area of circle base * height

= (πr^2)*h_{cyl}

Determine the volume of a cylinder of radius **8 cm** and height **125 mm**?
Your Solution:

Ans: 2513.27 cm^3

c. There is a formula for the volume of a sphere:

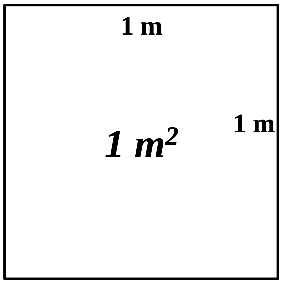
$$V_{sphere} = \frac{4}{3}\pi r^3$$

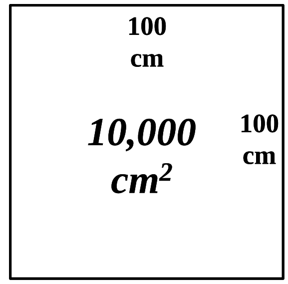
So what is the volume of sphere of radius 4 cm? ___________. (Your Solution:)

Ans: 268.08 cm^3

Converting Areas and Volumes using Square and Cube Dimensions

45. Be very careful when computing areas and volumes in **square** and **cube** types of units. For example; **1 m^2** is **not the same** as **100 cm^2;** it is *actually* another **100** times that or **10,000 cm^2**.





46. **Example**: Convert **5.2 m^2** into **ft^2**. (Given that 3.28 feet is the same length as one meter)

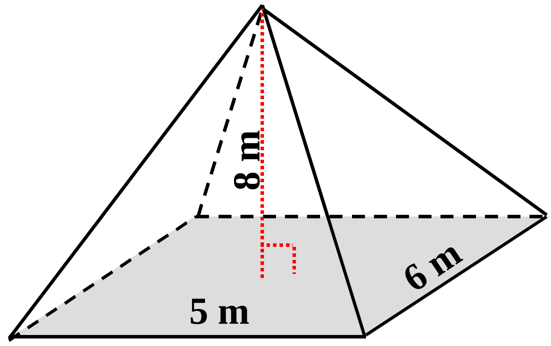
$$5.2m^2 * \frac{3.28ft}{1m} * \frac{3.28ft}{1m} = 55.9ft^2$$

47. **Example**: Your gas bill shows you used **200 cubic meters** [**m^3**] of gas to heat your house, but your meter is in **cubic feet [ft^3]**. So *how many* cubic feet of gas did you use if you want to check the company's readings.

Ans: 7057.51 ft^3 (depending on the accuracy of the conversion factor you used)

Volume of a Rectangular Pyramid

48. The neat thing about pyramids is that they have ***exactly*** **one-third** the volume of their equivalent prism shape.



The pyramid at right would fit inside a **prism** of volume 5 * 6 * 8 = 240 m^3.

Determine the volume of the rectangular pyramid above:

The pyramid of the same BASE and height is one third that volume:

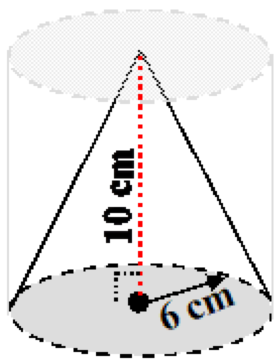
Vol = $\frac{1}{3} * Base_{area} * height_{pyramid}$

$= \frac{1}{3} * l_{base} * w_{base} * h_{pyramid}$

Ans: 80 m^3 (cubic metres)

Volume of a Cone

49. A cone that fits into a cylinder of the same base and height is exactly one-third the volume of the cylinder volume.

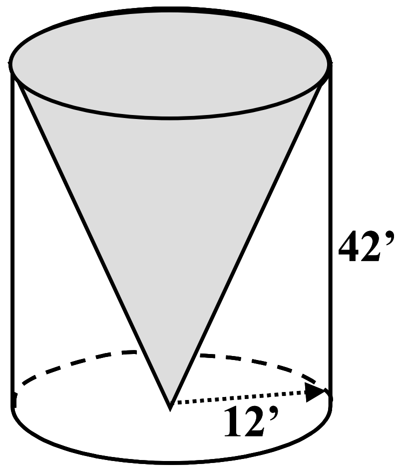


Determine the volume of this Cone:

$V_{cone} = \frac{1}{3}\pi r^2 h$

Ans: 376.99 cm^3

50. If a cone inserted within a cylinder is filled with water, what is the volume of air left in the cylinder?



Solution

Volume of Cylinder:

$V = Bh$ (B = circular base of cylinder = πr^2)

$= \pi r^2 h$

$= \pi(12)^2(42)$

≈ 19000.4 ft^3

Volume of cone:

$V = \frac{1}{3}Bh$ (B = circular base of cone if it were turned over)

$= \frac{1}{3}\pi(12)^2(42)$

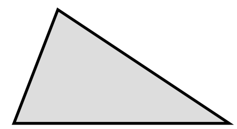
$= 6333.5$ ft^3

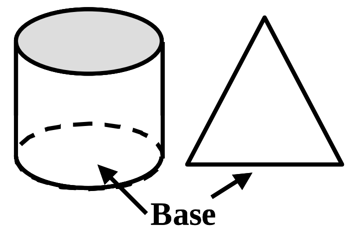
$\therefore$ Volume of air left in cylinder

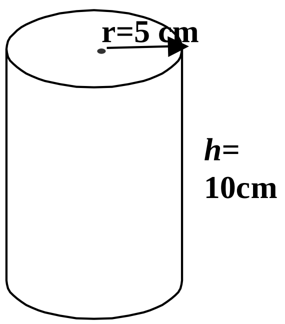
V = 19000.4 – 6333.5

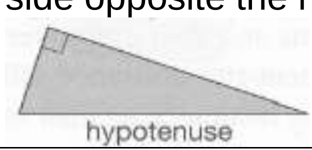
= 12666.9 ft^3

GLOSSARY
GRADE 11 ESSENTIAL UNIT C
MEASUREMENT

accuracy	how close a measurement is to what is believed to be the true value.
acute angle	an angle measuring less than 90°
acute triangle	a triangle with three acute angles all less than 90° 
altitude	the perpendicular distance from the base length of a figure to the highest point of the figure. Also, the height of an object above the earth's surface.
angle of depression	the angle between the horizon and the line of sight to an object that is above the horizon.
angle of elevation	the angle between the horizon and the line of sight to an object that is below the horizon.
approximation	a number close to the exact value of a measurement or quantity. Symbols such as ≈ or ≅ or are used to represent approximate values.
area	the number of square units needed to cover a region

base:	(1) the side of a polygon, or the face of a solid, from which the height is measured.  (2) the factor repeated in a power. Eg: In the expression of a power 5^3, the 5 is the base, the **3** is the exponent.
caliper	an instrument used to make precision measurements. Some common calipers may be used to measure to a precision of 0.05 mm.
capacity	the volume of a liquid that can be poured into a container. Two units of capacity are litre and gallon.
complementary angles:	two angles whose sum is 90°
cone	a solid formed by a region and all line segments joining points on the boundary of the region to a point not in the region. 
congruent:	figures that have the same size and shape, but not necessarily the same orientation
corresponding angles	in similar triangles two angles, one in each triangle, that are equal.

cosine.	for an acute ∠*A* in a right triangle, the ratio of the length of the side adjacent to ∠*A,* to the length of the hypotenuse. $cos(\angle A) = \frac{LengthofOppositeSide}{LengthofHypotensue}$
cube	a solid with six congruent, square faces 
cubic units	units that measure volume.
cylinder:  	a solid with two parallel, congruent, circular bases connected on their edges by one wrapped rectangle. $Volume_{cyl} = base*height$ $= \pi*r^2*h$ $= \pi*5^2*10 = 785cm^3$ $SurfaceArea =$ $2*\pi*r^2 + 2*\pi*r*h$ $= 471cm^2$
decagon	a polygon with ten sides.
denominator	the term below the line in a fraction
dodecahedron	a polyhedron with twelve faces.
equiangular polygon	a polygon where all the angles have the same measure.

evaluate	substitute a value for each of the variables in an expression and simplify the result. (as opposed to solve)
formula	a rule that is expressed as an equation.
Heron's formula	a formula for the area of a triangle $$A = \sqrt{(s)(s-a)(s-b)(s-c)}$$ where **a**, **b**, and **c** are the lengths of the sides of the triangle, and '**s'** is ***half*** the perimeter.
hectare	a unit of area that is equal to 10 000 m^2. A square 100m by 100m would be a hectare. Roughly the amount of surface in a football field if you included end zones and team areas. One hectare = 2.47 acres.
heptagon	a polygon with seven sides.
hexagon	a polygon with six sides.
hexahedron	a polyhedron with six faces. A regular hexahedron is a cube.
hypotenuse	the longest side of a right-angle triangle. The side opposite the right angle in a right triangle. 

Imperial system:

a system of measures that was used in Canada prior to 1976; a variation is still used in the U.S.A.
Measuring devices using this system often have each unit subdivided by halving, then halving the subdivisions, etc. Eg:½ inch, ¼ inch, two pints to a quart, four quarts to a gallon, etc.

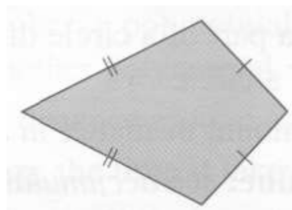
Selected conversions	
Imperial to Imperial	**Imperial to Metric (or SI)**
Length	
1 mile=1760 yards	1 mile=1.609 km
1 yard = 3 feet	1 yard = 0.9144 m
1 foot = 12 inches	1 inch =2.54 cm
Capacity (volume)	
1 gallon = 4 quarts	1 Gallon = 4.546 l
1 quart = 2 pints	
Mass (weight)	
1 ton = 2000 lbs	1 pound = 0.454 kg
1 pound = 16 ounces	1 ounce = 28.35 g
Caution: US gallons and quarts are different capacities than Imperial	

irrational number

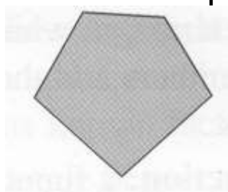
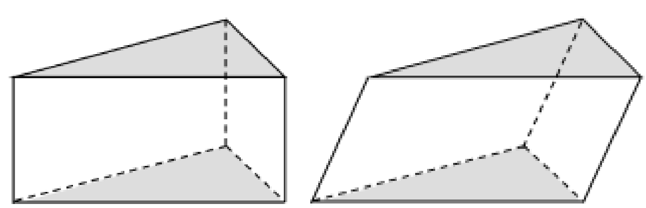
a number that **cannot** be written in the form ***m/n*** where *m* and *n* are integers ($n \neq 0$). Irrational numbers cannot be written as decimals (decimals are really fractions anyway). Examples of irrational numbers: $\pi, \sqrt{2}, \sqrt[3]{5}, \ldots$

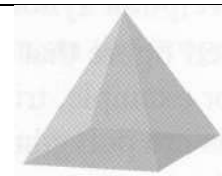
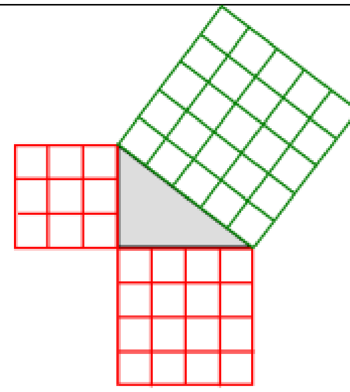
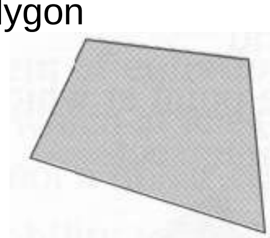
isosceles acute triangle:

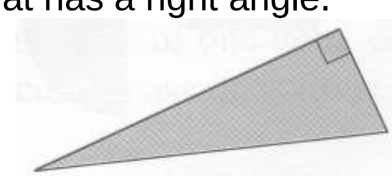
a triangle with two equal sides and all angles less than 90°

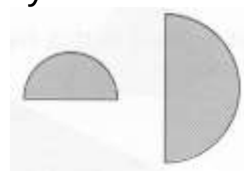
isosceles obtuse triangle:	a triangle with two equal sides and one angle greater than 90°
isosceles right triangle:	a triangle with two equal sides and a 90° angle
isosceles triangle:	a triangle with two equal sides
kite	a quadrilateral with two pairs of equal adjacent sides 
least common denominator	the least common denominator of two fractions, a/b and c/d, is the smallest number that contains both b and d as factors.
least common multiple	the least common multiple of two numbers, a and b, is the smallest number that contains both a and b as factors.
legs	the sides of a right triangle that form the right angle.
line	an infinitely long path that has no thickness and no curves.
mass	the amount of matter in an object. Mass is measured in units such as grams or kilograms.

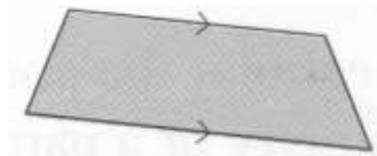
metric system:	also called the **SI** (Système International) system; based on a decimal system, with each unit subdivided into tenths and prefixes showing the relation of a unit to the base unit; commonly used base units are: Metre (m) for length Gram (g) for mass Litre (l) for capacity Second (s) for time The prefixes include: Mega-: Million, Kilo-: Thousand; centi-:1/100; milli-: 1/1000
micrometer	an instrument used for precision measurement.
natural numbers	the counting numbers. The set of numbers that includes {1, 2, 3, 4, ·,· , ·}
numerator	the top number in a fraction or rational number.
octagon	a polygon with 8 sides.  A ‘regular’ octagon has all sides and angles the same.
octahedron	a polyhedron with 8 faces.
parallelogram	a quadrilateral with opposite sides parallel.

pentagon	a five-sided polygon. 
perpendicular	two lines are perpendicular if the angle between them is 90 degrees.
polygon	the union of three or more line segments that are joined together so as to completely enclose an area.
polyhedron	a solid that is bounded by plane polygons.
precision	the size of the smallest measurement unit used when doing or reporting a measurement. For example, a measurement of 23.27 cm is more precise than 23.3 cm because 23.27 cm is measured to the nearest hundredth of a centimetre and 23.3 cm to the nearest tenth. Your alarm clock only displays minutes, so although it may be accurate it is not precise in its readings.
prism	a solid that has two congruent and parallel faces (the *bases),* and other faces that are rectangles (or parallelograms if it is 'leaning' and not 'right') 
protractor	an instrument for measuring angles.

pyramid	a solid with a polygon for a base, and all other sides being triangles that meet at a point (the vertex). 
Pythagorean Theorem	for any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. the theorem that relates the three side lengths of a right triangle: $\mathbf{a^2 + b^2 = c^2}$; where **c** is the hypotenuse 
Pythagorean triple	three natural numbers that satisfy the Pythagorean theorem. One example is: 3, 4 and 5. 5, 12, 13 also works.
quadrilateral:	a four-sided polygon 
rectangle:	a quadrilateral that has four right angles
rectangular prism	a prism that has rectangular faces 

rational number	a number that can be expressed exactly as the ratio of two integers. The letter Q (for quotient) is frequently used to represent the set of rational numbers.
rectangle	a quadrilateral with four 90 degree angles.
rectangular pyramid	a pyramid with a rectangular base
regular polygon	a polygon in which all the angles have the same measure and all of the sides are equal in length.
regular polyhedron	a polyhedron whose faces are congruent, regular polygons.
repeating decimal	a decimal in which the digits endlessly repeat a pattern. A repeating decimal may be rewritten in rational form.
rhombus:	a parallelogram with four equal sides
rhombus	a quadrilateral with four equal sides.
right angle	an angle whose measure is 90 degrees.
right circular cone	a cone whose base is a circle located so that the line connecting the vertex to the centre of the circle is perpendicular to the plane containing the circle.
right circular cylinder	a cylinder whose bases are circles and whose axis is perpendicular (right angles) to its bases.
right triangle	a triangle that has a right angle. 

similar figures	figures with the same shape, but not necessarily the same size 
sine	in a right triangle, the length of a side opposite an angle divided by the length of the hypotenuse of the triangle. The formula may be written as: $\sin\theta = \frac{opposite}{hypotenuse}$
solid	a three dimensional object that occupies or encloses space (i.e. has a volume).
sphere	the set of all points in space that are a fixed distance from a given point. A ball is an example of a sphere.
square root	of a number, x, is the number that, when multiplied by itself gives the number, x. for example, $\sqrt{9} = 3$ because $3^2 = 9$.
tangent	in a right triangle, the length of a side opposite an angle divided by the length of the side adjacent to the angle. The formula may be written as: $\tan\theta = \frac{opposite}{adjacent}$
tetrahedron	a polyhedron with four faces. Could also be called a triangular prism. 

theorem	a statement that has been proven.
three-dimensional	having length, width, and depth or height
transversal	a line that intersects two other lines.
trapezoid	a quadrilateral with one pair of opposite sides parallel but not equal in length. 
triangle	a three-sided polygon.
trigonometric function	a function that involves the sin, cos, or tan of the independent variable. One example is $y = \sin x + 1$.
trigonometry	the study of triangles and the relations between the side lengths and the angle measures.
Vernier scale	calipers and micrometers are frequently equipped with a Vernier scale which is required to make precision measurements.
volume	the amount of space occupied by an object. One unit of volume measure is the cubic metre, or m^3.
whole numbers	the set of numbers that includes zero and all of the natural numbers. W = {0, 1, 2, 3, 4, ·, ·, · }

GRADE 11 ESSENTIAL
UNIT C – 3-D GEOMETRY
APPENDIX - MEASUREMENT CONVERSION FACTORS

Système Internationale (SI) Metric System Unit Ratio Conversions
Metric to Metric Conversions

Conversions SI Metric for Length and Distance		
1 kilometre [km]	=	1,000 metres [m]
1 meter [m]	=	100 centimetres [cm]
1 centimetre [cm]	=	10 millimetres [mm]

Conversions SI Metric for Mass		
1 tonne [t]	=	1,000 kg
1 kilogram [kg]	=	1,000 grams [g]
1 gram [g]	=	1,000 milligrams [mg]

Conversions SI Metric for Volume		
1 litre [L]	=	1,000 millilitre [mL]
1 litre [L]	=	100 centilitres [cL]
1 litre [L]	=	1,000 cc (or 1,000 cm^3)
1 millilitre [mL]	=	1 cc (or 1 cm^3)

'cc' stands for cubic centimetre which is really just cm^3. Notice also that a cube of dimensions 10cm by 10 cm by 10 cm is a litre

Conversions SI Metric for Area		
1 square metre	=	10,000 cm^2
1 hectare	=	10,000 m^2
1 cm^2	=	100 mm^2

So a square 100 m by 100 m is a hectare. Used for measuring land area.

Imperial to Imperial (and American) Conversions

Conversions Non-SI (Imperial) for Length

1 mile [mi]	=	1,760 yards [yd]
1 yard [yd]	=	3 feet [ft]
1 mile [mi]	=	5280 ft
1 foot [ft]	=	12 inches [in]
1 yard [yd]	=	36 inches [in]

Conversions Non-SI Imperial for Mass

1 ton [t]	=	2,000 pounds lb
1 pound [lb]	=	16 ounces oz

Note : No such thing as 2.75 pounds,
it is 2 lbs 12 oz

Conversions Non-SI (Imperial) for Volume

8 gallon [gal]	=	1 bushel
4 quarts [qt]	=	1 gallon [gal]
or 1 quart [qt]	=	0.25 gal
2 pints	=	1 quart [gt]
8 pints	=	1 gallon [gal]
20 ounces [oz]	=	1 pint

Conversions Non-SI Imperial for Volume (USA)

1 gallon (US)	=	0.832 gallons (English)
1 gallon (US)	=	128 ounces oz (US)

Really gets confusing with two different gallons depending on your country!

Caution Ounces of weight are different from ounces of volume.

Conversions Non-SI Imperial for Area

1 acre	=	43,560 ft^2
1 acre	=	4,840 yd^2
1 square foot [ft^2]	=	144 square inches [in^2]
1 square yard [yd^2]	=	9 ft^2
1 square mile	=	640 acres

So a square having sides of 208 feet would be an acre. An acre originally was supposed to be the amount of farmland a man could work in one day, so it depended on how strong the man was! Making a standard measure became important eventually.

Converting between systems (Imperial ↔ Metric [SI])

Conversions SI to Non-SI Length		
1 metre [m]	=	3.2808 feet [ft]
1 metre [m]	=	39.37 inches [in]
1 kilometre [km]	=	0.6214 miles [mi]
1 mile [mi]	=	1.609 km
1 inch [in]	=	2.54 cm

Conversions Non-SI Imperial – Mass		
1 kilogram kg	=	2.205 pounds lb
1 tonne	=	1.1 ton

Conversions SI to Non-SI Volume		
1 gallon (English)	=	4.546 litres
1 gallon (US)	=	3.785 litres
1 gallon (English)	=	4,546 cc^3
1 gallon (US)	=	3,785 cc^3

Conversions SI to Non-SI Area		
1 sq mile	=	259 hectares
1 sq mile	=	2,589,988 m^2
1 square metre	=	10.76 ft^2
1 square metre	=	1,550 in^2

CONVERT BY PROPORTIONS:

6 feet is how many inches??

$\frac{12\,in}{1\,ft} = \frac{x}{6\,ft}$; so: $\frac{12*6}{1} = x$ so: **x = 72 inches**

Just like grandma's secret recipe for muffins

16 lbs is how many kg?

$\frac{x\,kg}{16\,lb} = \frac{1\,kg}{2.205\,lb}$; so: $\frac{16}{2.205} = x$ so: **x = 7.26 kg**

CONVERT BY UNIT FACTORS:

5 years is how many hours?

$$5\,yr * \frac{365\,days}{1\,yr} * \frac{24\,hr}{1\,day} = \frac{5*365*24}{1*1} = 43{,}800\,hours$$

Units in top and bottom cancel until you are left with what you want!

$$units\ not\ want * \frac{units\ want}{units\ not\ want}$$

16 lbs is how many kg?

$$16\ lb * \frac{1\ kg}{2.205\ lb} = \frac{16}{2.205} kg = 7.26\ kg$$

GRADE 11 ESSENTIAL
UNIT C – 3-D GEOMETRY
APPENDIX - GEOMETRIC FORMULAE

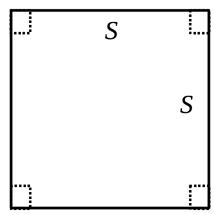
Shape	Diagram	Formulae

FLAT OBJECTS
2 DIMENSIONAL

Square

(all four sides same length, 90°corners)

(a rectangle with all sides same length)



Perimeter, P:

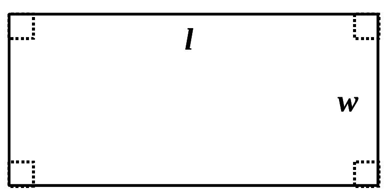
P = s + s + s + s = 4*s

Area, A:

A = s * s = s^2

Rectangle

(Four sides, square corners)



Perimeter, P:

P = *l* + *w* + *l* + *w* = 2*l* + 2*w*

Area, A:

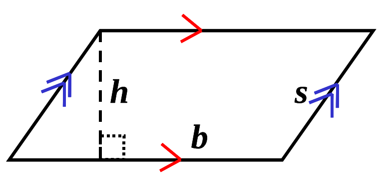
A = *l* * *w*

Parallelogram and Rhombus

(leaning rectangle/leaning square)

Note

b is always ⊥ to h



Perimeter; P:

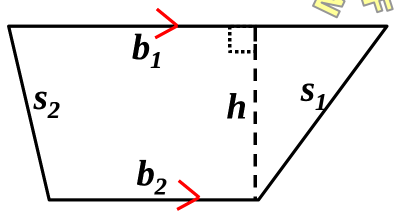
P = 2b + 2s

Area; A:

A = b * h

Trapezoid

(Four sides, only two sides parallel { || })



Perimeter; P:

P = $b_1 + s_1 + b_2 + s_2$

Area; A:

A = $b_{average}$ *h

$$= \frac{1}{2}(b_1 + b_2) * h$$

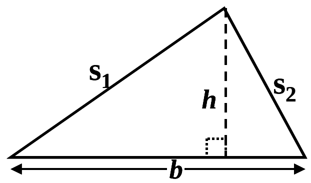
Note

b is always ⊥ to h

Triangle

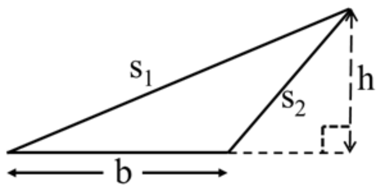
(three sides)

(half a parallelogram or rectangle)
(acute, obtuse, or right)
(scalene, isosceles, equilateral)



Perimeter; P:

P = $s_1 + s_2 + b$



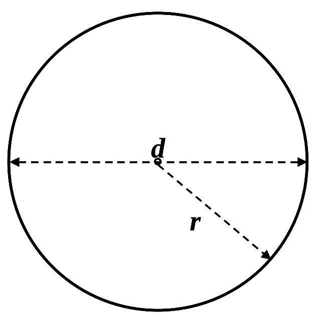
Area; A:

A = $\frac{1}{2} * b * h$

Note

b is always ⊥ to h

Circle



Circumference; C:

C = $\pi d = 2\pi r$

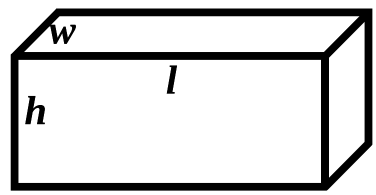
Area; A

A = πr^2

SOLID OBJECTS
3 DIMENSIONAL

Rectangular Prism

(Two congruent rectangles connected at edges by rectangles)



Surface Area; SA

SA = Add area of all faces; *or* **SA= 2*lw* + 2*hl* + 2*hw***

Volume; V:

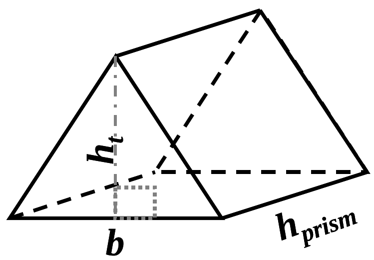
V = $Base_{area}$ * h_{prism}

V = (l * w) * h

Triangular Prism

(Two congruent triangles connected at edges by rectangles)

Gets confusing using height for the triangle, h_{Δ}, and height for the prism, h_{prism}.



Surface Area; SA

SA = Add area of all faces; the net is two triangles and three rectangles.

SA = $P_{base}h_{prism}+bh_{\Delta}$
(fancy)

Volume; V:

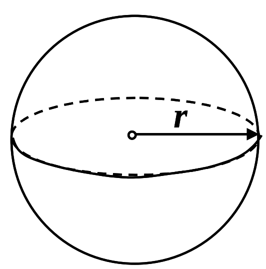
V = $Base_{area}$ * h_{prism}

V = $\frac{1}{2}(bh_{\Delta})*h_{prism}$

Sphere

All the points in space that are equidistant from a single centre point

(Ball)



Surface Area; SA

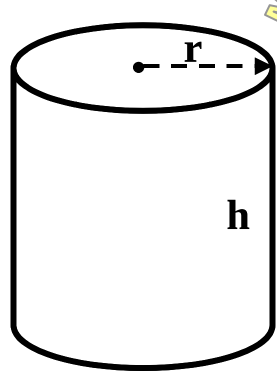
SA = $4\pi r^2$

Volume; V:

V = $\frac{4}{3}\pi r^3$

Cylinder

(Two congruent circles connected with a rectangle wrapped around circumference)



Surface Area; SA top & bottom / lateral side

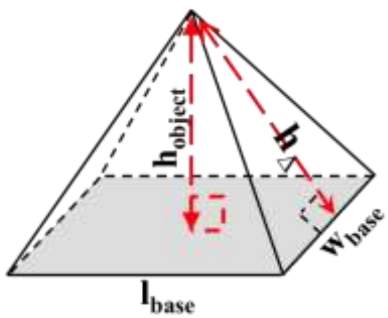
SA = $2\pi r^2 + 2\pi rh$ (tube)

Volume; V:

V = $Base_{area}$ * h
= A * h
= $\pi r^2 h$

Rectangular Pyramid or Square Pyramid

(A rectangle connected to an apex point by triangles on its edges)



caution the pyramid has a height, and the triangular faces each have a height

Surface Area; SA

SA = add up area of all the faces (Base area plus four triangles)

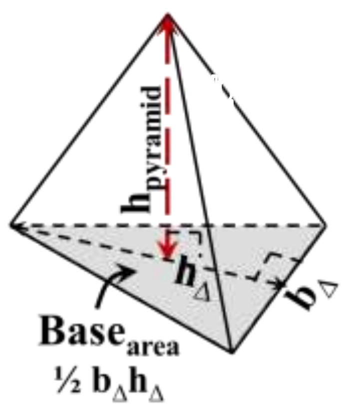
Volume; V:

$V=\frac{1}{3} * Base_{area} * h_{pyramid}$

$= \frac{1}{3} * (l * w) * h_{pyramid}$

Triangular Pyramid

(A triangle base connected to an apex point by triangles on its edges)



caution the pyramid has a height h_{object}, and the triangular faces have a height, h_Δ

Surface Area; SA

SA = add up area of each of the four triangular faces.

Volume; V:

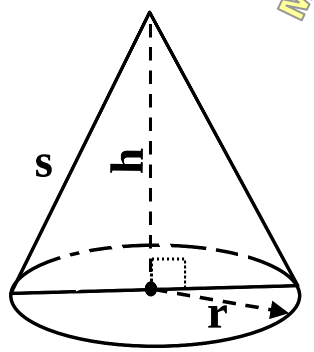
$V = \frac{1}{3} * Base_{area} * h_{pyramid}$

$\frac{1}{3} * \left(\frac{1}{2} * b_\Delta * h_\Delta\right) * h_{object}$

$= \frac{1}{6} * b_\Delta * h_\Delta * h_{object}$

Cone

(The arc of a circular sector of a circle connected to a smaller circle base and coming to an apex point)



Surface Area; SA

$$SA = \pi r^2 + \pi rs$$

*('**s**' here is 'slant range' along the side of the cone)*

Volume; V:

$$V = \frac{1}{3} * Base_{area} * h_{cone}$$

$$V = \frac{1}{3} * (\pi r^2) * h_{cone}$$

Letter Abbreviations:

r ≡ radius, d ≡ diameter; h ≡ height; A ≡ area; l ≡ length; w ≡ width; B = $Base_{area}$

s ≡ *side* or *sometimes* slant range; ⊥ ≡ perpendicular

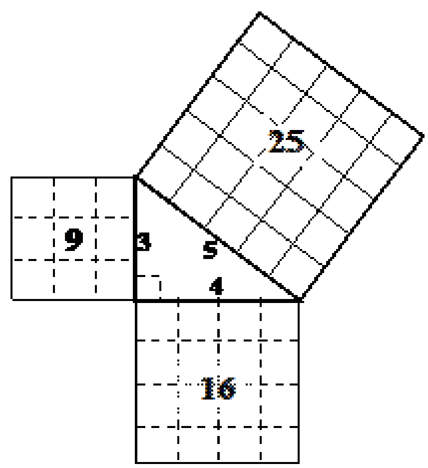
And do not forget Pythagoras!

Pythagoras

"The square on the longest [hypotenuse] side of a right triangle equals the sum of the squares on the shorter two sides"

$$c^2 = a^2 + b^2$$

*where **c** is the length of the **hypotenuse** and **a** and **b** are the lengths of the **shorter** two sides*



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