GRADE 12 ESSENTIAL
UNIT G – GEOMETRY AND TRIGONOMETRY
GEOMETRY ANGLES WORKSHEET

Angles have relationships, they work with each other.
**Complementary Angles**: Add to make a square corner; a 90° angle.
Linear Pairs: Adjacent angles that add to make a straight line; a 180° angle.
Also called **Supplementary** Angles.
Vertical Angles: Angles across from each other that are formed when two lines intersect. Also commonly called **Opposite Angles**.
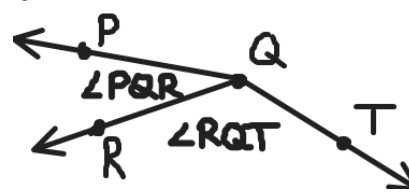**Adjacent Angles**: Angles that are next to each other sharing a common vertex.

1. **Name the relationship**: complementary, linear pair (supplementary), vertical (aka: opposite), or adjacent angles.
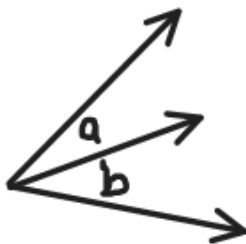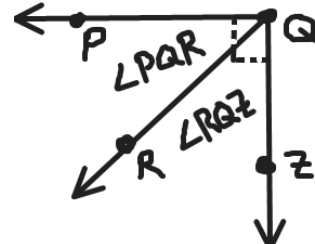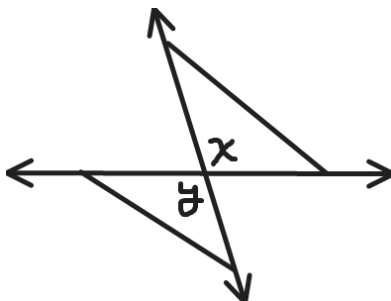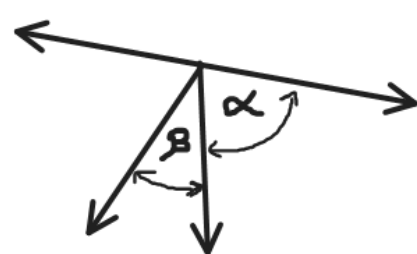
a.
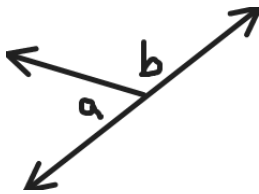
b.

c.

d.
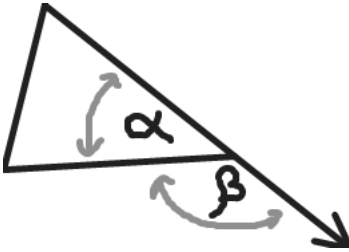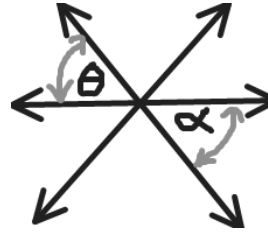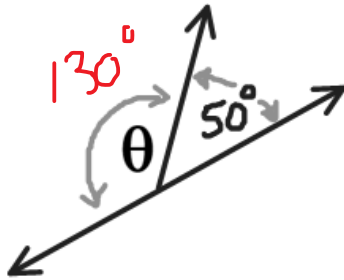
e.

f.

g.

h.

2. Determine the measure of angle θ, state the relationship or rule. (not drawn to scale; a protractor will **not** work)

a.

$130°$
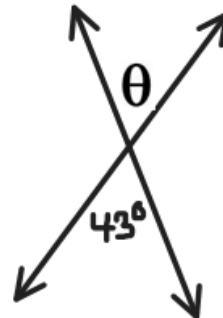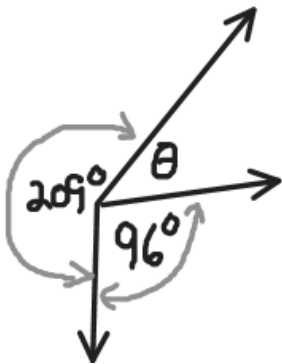
$θ$ $50°$

$θ = 180° - 50°$
$= 130°$
Supplementary
Angle
(linear pair)

b.

$θ$

$43°$

c.

$209°$ $θ$
$96°$

d.

$θ$
$63°$

e.

$35°$
$θ$

f.

$29°$
$θ$

g.

94°

243°

h.

49° θ

i.

90° θ

j.

θ 73°
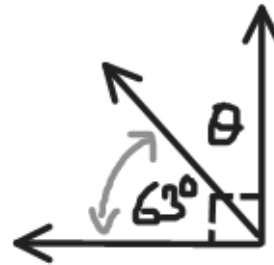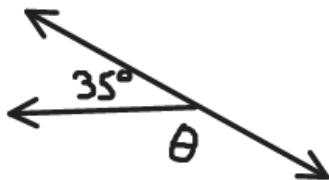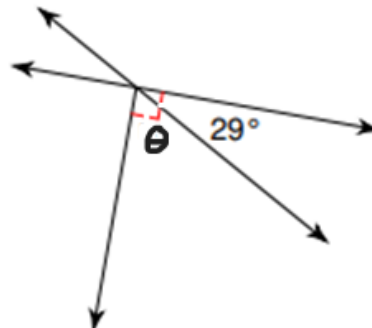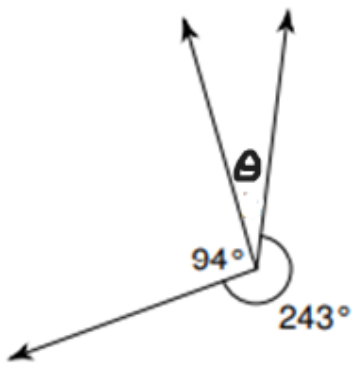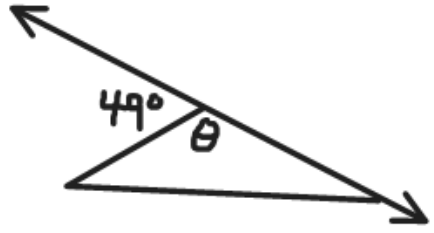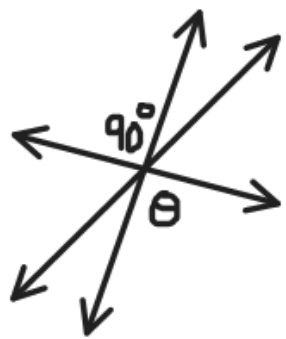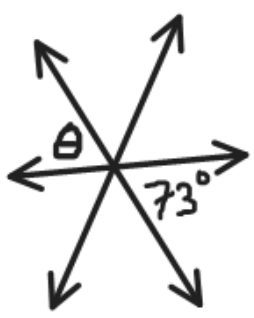
ANGLES IN A TRIANGLE

A useful law of triangles is the **Triangle Sum Theorem**:

"**The sum of the interior angles of triangle equals 180°**"

A

100°

50° 30° B

C

$\angle A + \angle B + \angle C = 180°$
$100° + 30° + 50° = 180°$
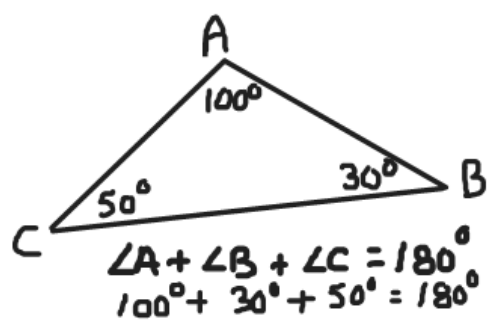
3. Determine the measure of each indicated unknown angle. Use the triangles sum theorem, the supplementary angles rule, and the opposite (vertical) angles rule.

a.

65° α

57°

b.

40°

α

c.

20°

130° θ

d.

∠A = _____

85°

50°

A

e.

137°

102°   θ

f.

B = 135°

Supplementary angle B

opposite angle
35

45° triangle
Sum
rule

100°

35°

g.

P

R

30°   Q

20°

m∠PQR

= _____

h.

155°

60°

α

i.

39° θ

20°   40°

j.

35°

65°

50°

60°   α
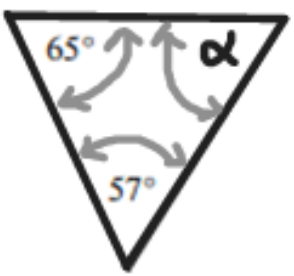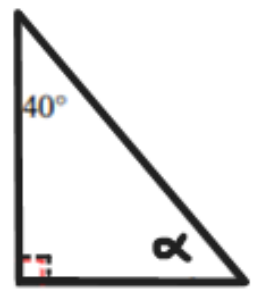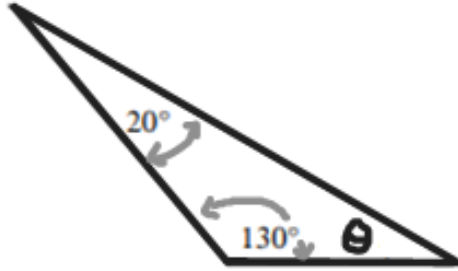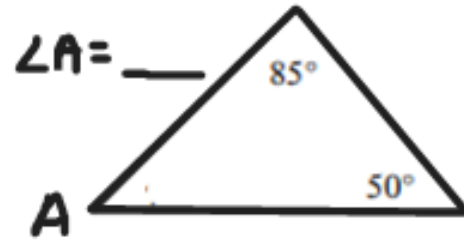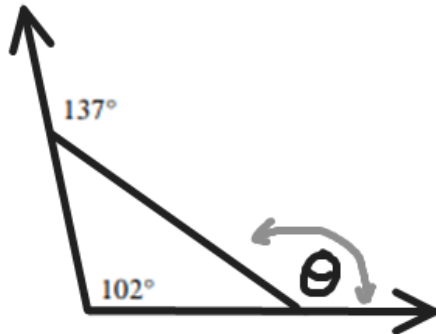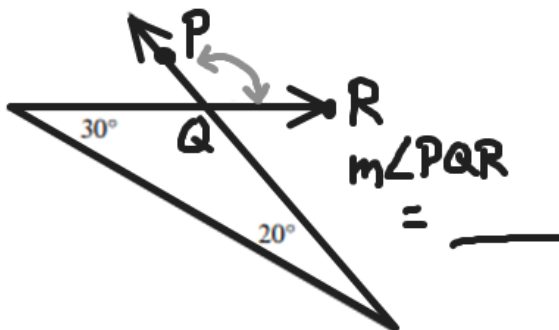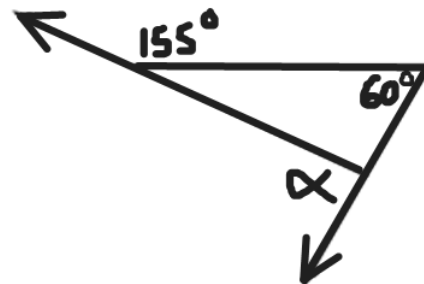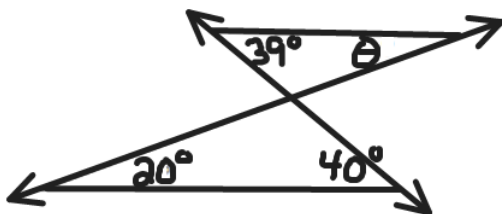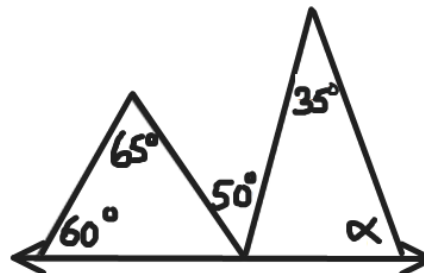
k.
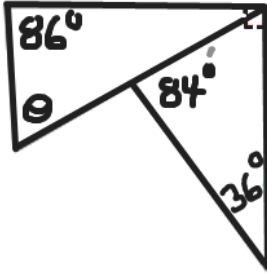
l.

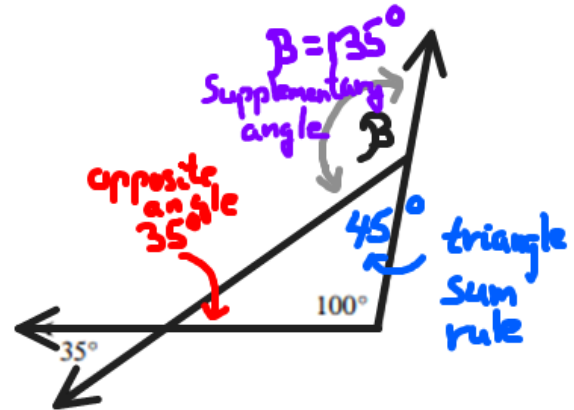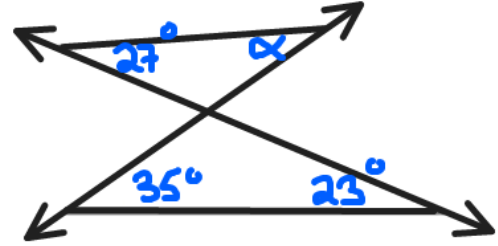## PARALLEL LINES AND TRANSVERSALS

Parallel lines are lines going the exact same direction, they never meet (ie: they never intersect). A **transversal** is a line that cuts two other lines. Some interesting and self-evident attributes of angles occur when a transversal cuts parallel lines.

Corresponding Angles. ∠x and ∠y are 'matching'; corresponding and congruent (their measures are equal). ∠α and ∠ω are also corresponding and congruent angles.
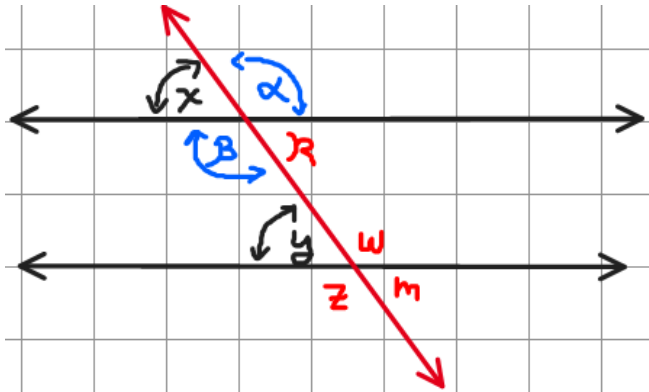
Exterior Angles. Exterior angles are those on the outside of the parallel lines! (eg. ∠x and ∠m)

Interior Angles. Interior angles are those on the inside of the lines.

Alternate angles. Alternate angles are on opposite sides of a transversal. ∠β and ∠k *or* ∠m and ∠z for example.

Alternate Interior and Alternate Exterior Angles. You likely figured out that **alternate interior** angles are on the inside of the parallel lines *and* on opposite sides of the transversal. Eg: ∠y and ∠k. There is a law:
Alternate interior angles are congruent;
their angular measure is equal.

You likely figured out that **alternate exterior** angles are on the outside of the parallel lines *and* on opposite sides of the transversal. Eg: ∠x and ∠m. There is a law:
**Alternate exterior angles are congruent**; their angular measure is equal.

Consecutive Angles are angles that are 'next in order' as you move along the transversal, often called 'same side'. ∠β and ∠y, for example, are **consecutive interior** angles. There is an (obvious) law:
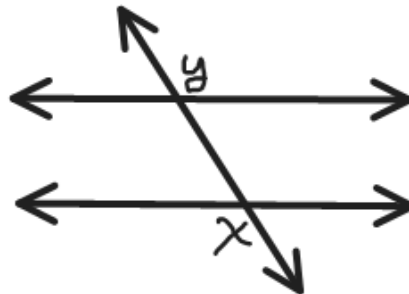
Consecutive Interior angles are supplementary.

4. For the parallel lines below being cut by a transversal, identify each indicated pair of angles as corresponding, alternate interior, alternate exterior, or consecutive interior.
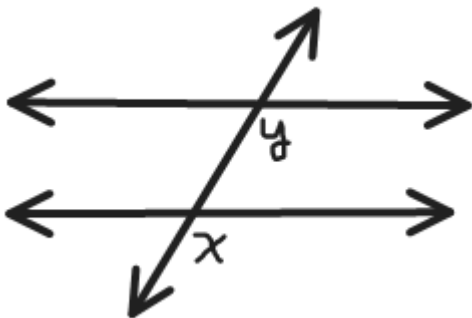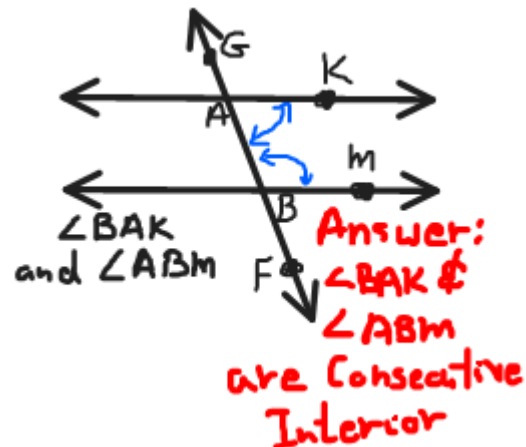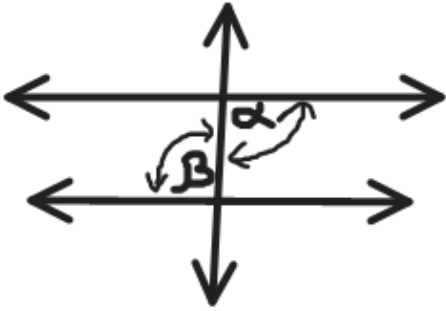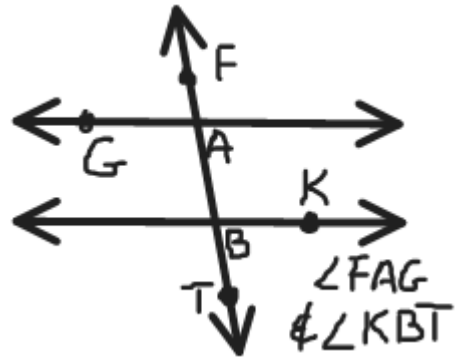
a.

Ans:
∠y an∠x are

_____   _____

b.

c.

d.

∠BAK
and ∠ABM

Answer:
∠BAK &
∠ABM
are Conseative
Interior

e.

∠FAG
≰∠KBT
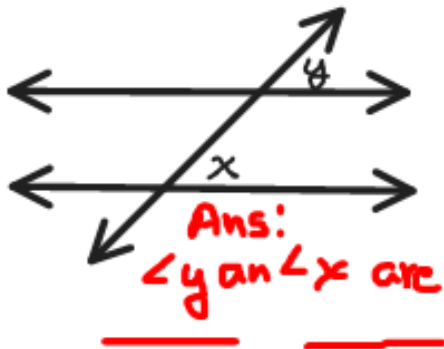
g.
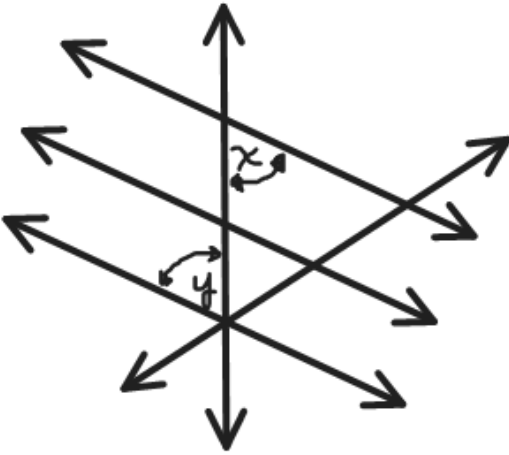
h.
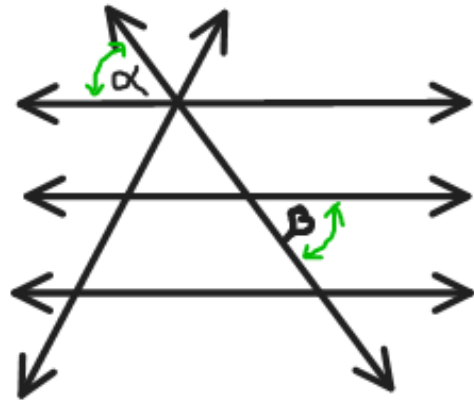
i.

j.

5. State the measure of each angle indicated.

a.

$\angle x = \underline{120^\circ}$

b.

$\angle x = \underline{\hspace{2cm}}$

85°

x

c.

100°

x

d.

114°
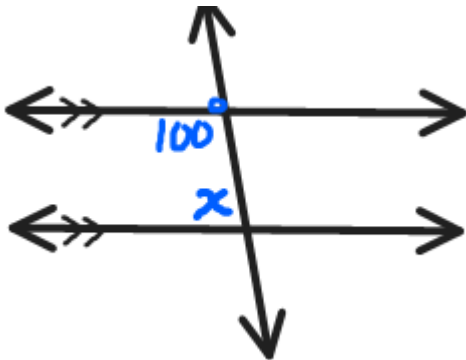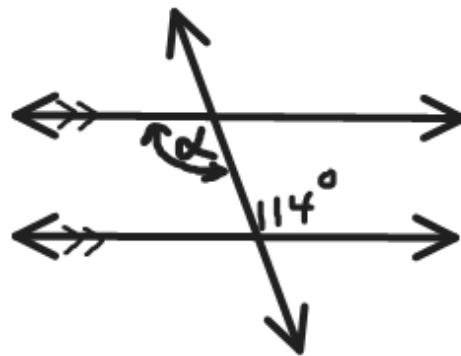
You may see crazy questions like this; you need algebra (or good guessing skills). Determine the value of x.

e.

60°

8x+4

$60 = 8x + 4$

Eg: $56 = 8x$; $\boxed{x = 7}$

f.

x+139

132°
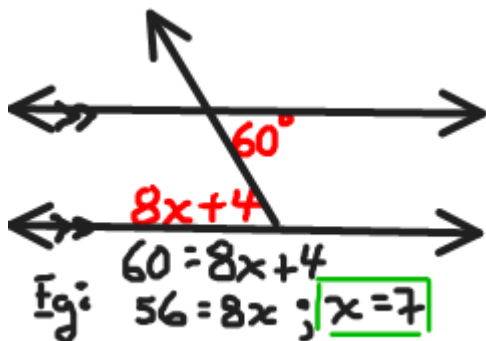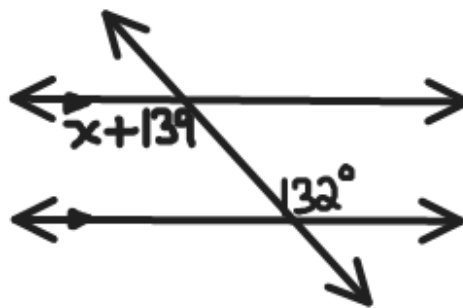
6. Angles in Quadrilaterals

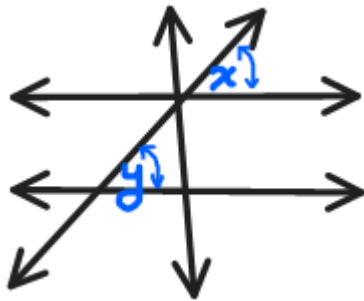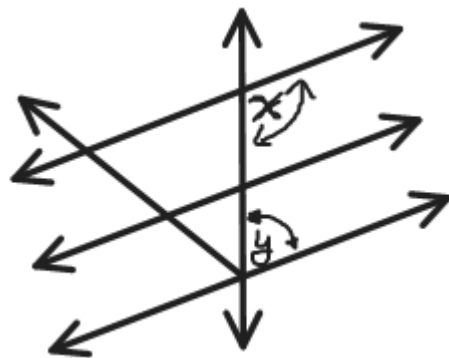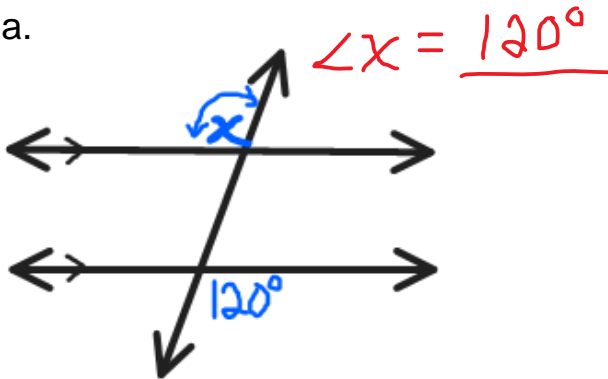As an extension of the Triangle Sum theorem, any quadrilateral (4-sided polygon) is really just two triangles. So; the sum of the corners of a quadrilateral is 360°.

> **Law:** *The sum of the interior angles of a quadrilateral is 360°.*

a, c, e, b, d, f
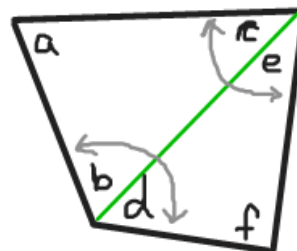
$\angle a + \angle b + \angle c = 180^\circ$
$+ (\angle f + \angle d + \angle e = 180^\circ)$
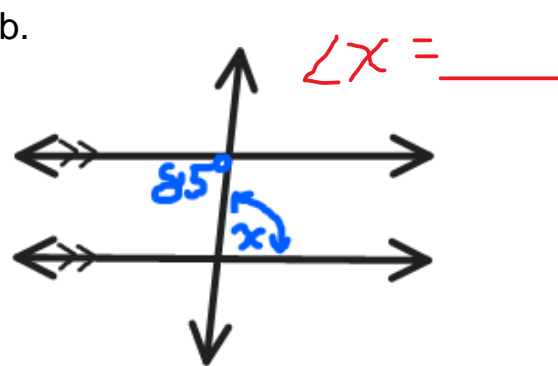$\angle a + \angle f + (\angle b + \angle d) + (\angle c + \angle e) = 360^\circ$

7. Determine the measure of each angle indicated:

a.

W ...... T
? ...... 100°
100° ...... 80°
V ...... U

$\angle W = 360° - (100 + 100 + 80)$
$= 80°$

b.

D
?
C
115°
95° ...... 80°
B ...... A

c.

F ...... E
65° ...... 100°
87°
? ...... D
G

d.

M L
?
J ...... K

e.

K L
96° ...... ?
96°
68° ...... M
J

f.

T ...... W
114°
86° ...... ?
U V

g.

T
?
108°
Q
104° 80°
S R

h.

X
95°
Y
?
85° 85°
W Z
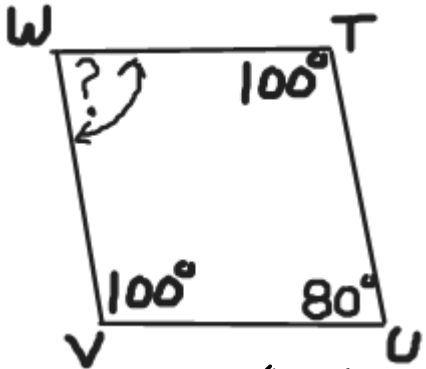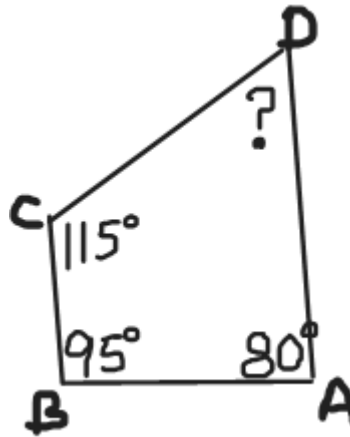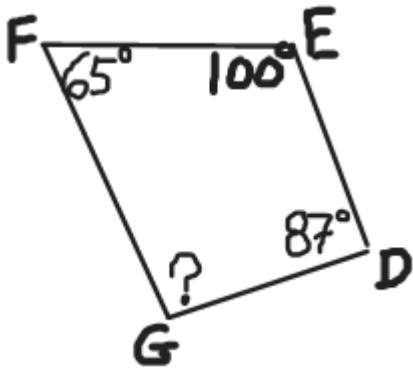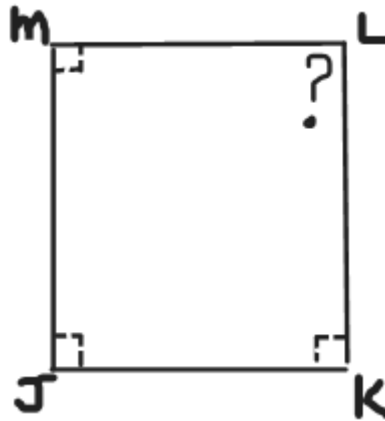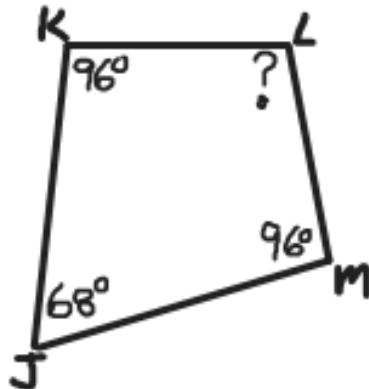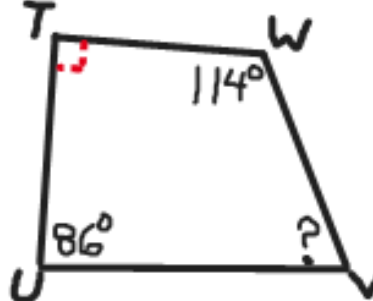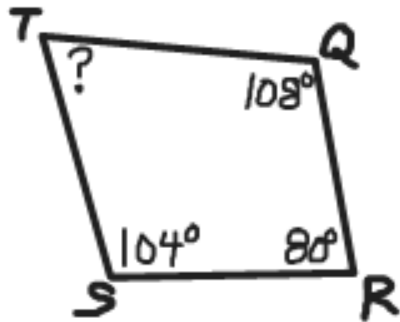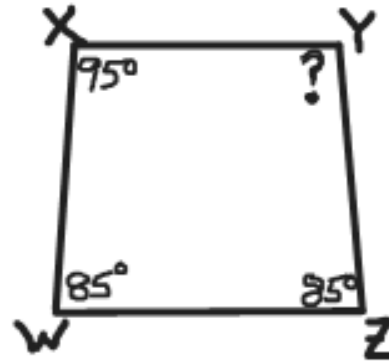
FYI. Here is a curious 'law' we figured out a couple thousand years ago; if two opposite corners of a quadrilateral are supplementary then the quadrilateral fits exactly into a circle! And vice versa. (Grade 9 Geometry)

X 95° Y ? 180° 85° 85° W Z
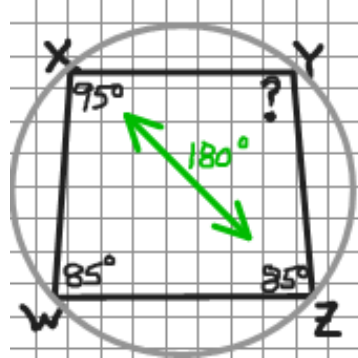
You will often see problems like this too just to practice your algebra; solve for x:

i.

F E
86° 94°
x = _____
94° 11x−2
G D

j.

m J
17x+8 66°
74° 110°
L K
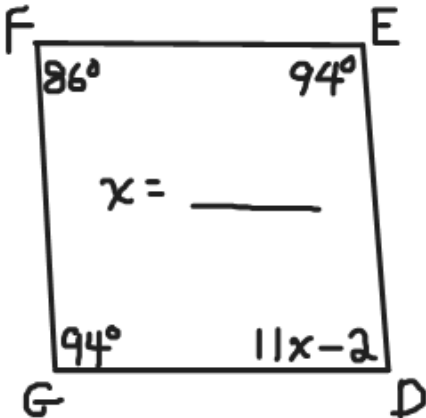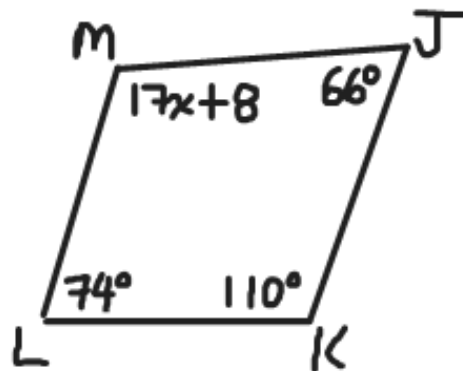
Properties of Parallelograms

8. Parallelograms are just a special case of a quadrilateral. Opposite sides are parallel.

Can you see that if you were to extend the sides that opposite corners would be congruent (equal)

Parallelogram

MrF
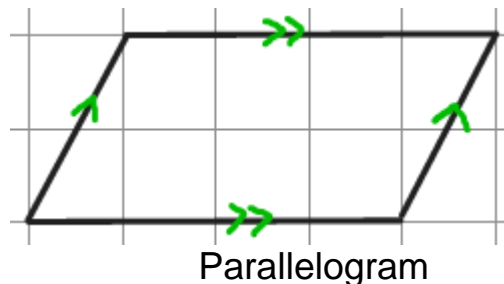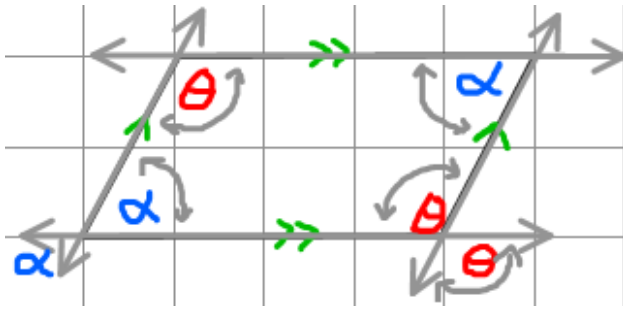
Our law about transversals and parallel lines proves that:

"opposite corners of a parallelogram are congruent."

A 'sub law' of a first law is called a corollary

9. State the measurement indicated in each parallelogram.

a.

$\angle R =$ _____

?

$138°$

b.

?

$70°$

c.

? $80°$

d.

?

$99°$
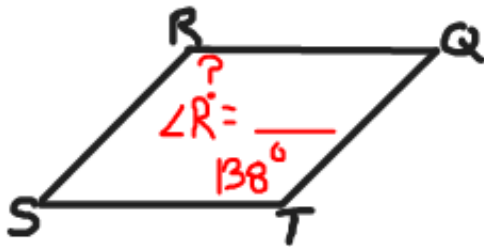
10. LAW: Opposite Sides of a Parallelogram are congruent

Sort of obvious?

Can you use a bit of logic to prove that statement?

A α B θ
β
β
θ α
C D

$\triangle ABD \sim \triangle ACD \leftarrow$ they are **Similar** from transversal laws since matching corners are congruent
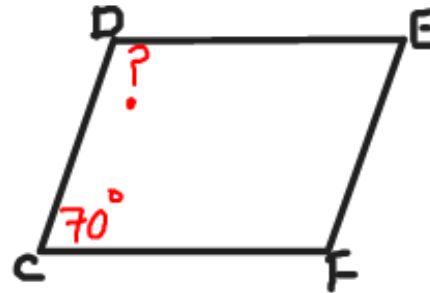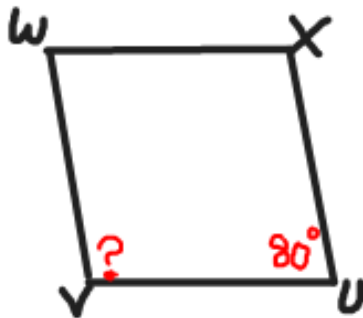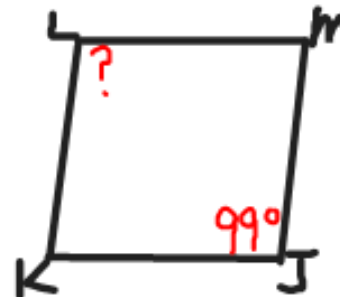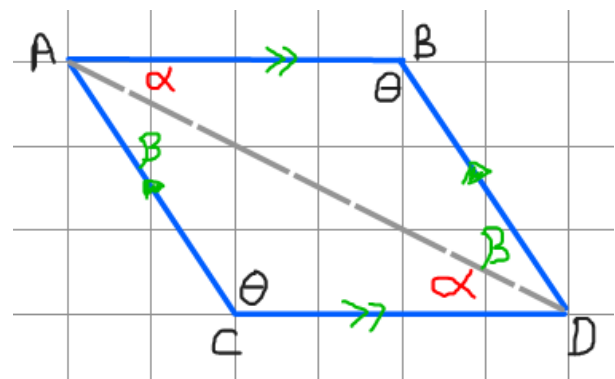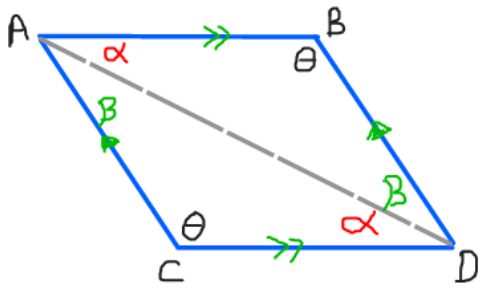
& length $\overline{AD}$ is the same for both $\triangle s$. Therefore all the corresponding sides are also congruent

Therefore $\overline{AB} = \overline{CD}$ and $\overline{AC} = \overline{BD}$

If logic and truth and facts and proof are not your thing, then just memorize the law.

Opposite sides of a parallelogram are congruent.

Laws that have stood up for thousands of years.

a.

$\overline{RT} = 19.8$

$\overline{RP} = \underline{\ ?\ }$

Hint:
$\triangle STR \cong \triangle QTP$

R Q
S P

b.

Y V
$\angle VXW = \underline{\qquad}$
52
?
60
X W

c.

Z Y
83
?
W X
$\angle YXZ = \underline{\qquad}$

d.

E D
35 ?
44
$\angle EDF = \underline{\qquad}$
F C
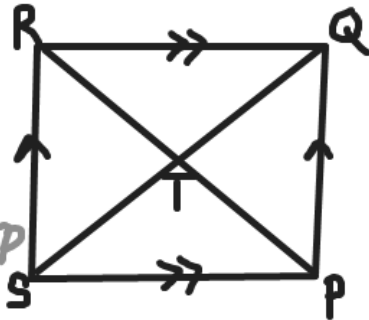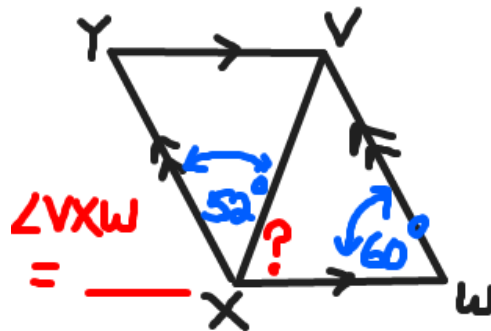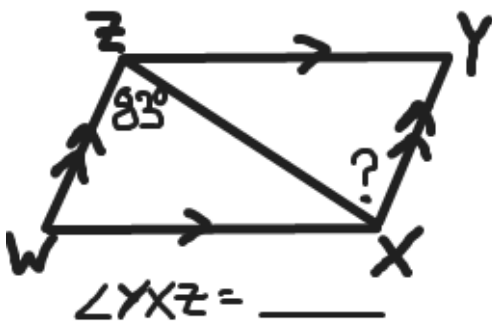
All of the above is just a very, very small piece of ancient mathematics. Notice numbers were not really necessary!

Check out a Book written 2,500 years ago by Euclid! Google it.

ANSWERS:

1a) Linear Pair (Supplementary) b) Adjacent c) adjacent
d) complementary; e) opposite (or vertical) angles f) adjacent angles
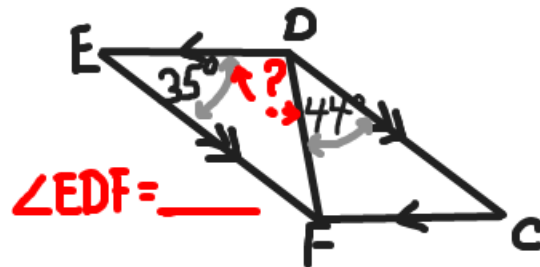g) supplementary (linear pair) h) opposite (vertical) angles

2a) 130 b) 43 (opposite angles) c) 60 (360 in a circle) d) 27
(complementary) e) 145 (supplementary) f) 61 (complementary) g) 23
h) 131 (supplementary)

3a) 58; b) 50 c) 30 d) 45 e) 145 f) 135 g) 130 h) 85 i) 21 j) 70
k) 34 l) 31

4a) corresponding b) alternate exterior c) corresponding
d) consecutive interior e) alternate interior f) alternate exterior
g) alternate interior h) alt ext i) corresponding j) consecutive interior

5a) 120 b) 85 c) 80 d) 114 e) x = 7 f) x = -7

7a) 80 b) 70 c) 108 d) 90 e) 100 f) 70 g) 68 h) 95
i) x = 8 j) x = 6

9. a) 138 b) 110 c) 100 d) 99

10. Advanced. Some of these will take a few steps of logic!

a) 19.8 b) 58 c) 83 d) 101