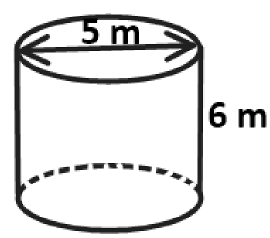
GRADE 12 APPLIED
UNIT G - DESIGN AND MEASUREMENT
CLASS NOTES

1. **Introduction**. In Grades 10 and 11 you learned some powerful measurement skills. But it remains necessary to reinforce the most basic practical type of application, how to determine the material required and the cost of a project. For example: being able to calculate, or at least estimate, the cost to re-decorate a room, paint a community water tank, or build your aunt a nice paving stone patio.

Project Management

2. Project management is the process of **designing** a project, **estimating** the **cost** ($) of the project in **material** and **labour**, setting a **budget**, tracking **expenses**, determining the **duration** of the project (how long it will take), determining **who** is to **do what** and **when** they are to do it; determining what **materials** must be available and **when** and **where** they are available (you don't want your dry wallers showing up to work and not having any dry wall for them to put up because it is still on a truck a hundred miles away), **procuring** the materials and **contractors**, signing **contracts**, making **reports**, **negotiating** with workers and suppliers, and lots more! If you find you enjoy this type of activity then being a Project Manager is for you. Project management is an entire year-long course all by itself though but hopefully we will get a chance to examine it rather briefly. If time permits we will do simple exercise in the '**Critical Path Method**' of Project Management.

3. **Example**. Your community has a large cylindrical water tank. It is **6** metres high and has a diameter of **5** metres. The council is afraid the tank will rust soon if not painted so they contractors to prepare an estimate of how much it will cost to paint the tank.



3. **Solution Approach**. You will need to calculate the '*surface area*' of the tank to paint, find out how much the paint will cost to paint it, and perhaps estimate the cost of the labour to actually apply the paint the tank. And, of course the council will likely want to know how long it will take too. And further of course you want to be paid for your design work and coming up with the plan and buying the materials and actually having the work done. You will also need to provide the council a neat comprehensible **written estimate** explaining your calculations.

4. You try now to make a neat written estimate of how much it will cost the council. A suggested possible template is given. A four litre can of pain costs $31.65 plus taxes and each can covers 300 sq ft (ft^2). You will need to remove the old paint. You will need to transport the material. There will be other factors to consider too. You will prepare a neat written estimate so you know how much material you need and what it will cost.

Joe's Eco Paint	Estimate for Council Water tank painting Subject to condition report, weather
Surface Area to Prepare and Paint	**Labour**
SA = $1\pi r^2 + 2\pi rh$ *not painting bottom* SA =	Estimated Time to prepare the tank (sand blast) Estimated Time to paint
245 lbs of Silica Blasting Sand at $2.95/kg. Silica Sand Cost:	Cost of Labour at $80 / hr. Cost of material transport: $900 Cost of scaffolding and ladder rentals: $1,800
Cans of Paint Required (for one coat): Paint Cost: (1 Coat only)	Cost of travel and accommodation for two employees. Design and planning costs (taxes at 12%): 10 hrs @ $65/ hr = $650 * 1.12 = $728.00
Total Cost Material:	**Total Labour and Support:**
Total ESTIMATED Cost of project (Material , Labour, and Support).	

Approximate Estimate (Before the full formal written estimate)

7. Being able to *estimate* is an important skill, being able to look at a room and in less than one minute get a close answer (*estimate*) to the area of the floor or of the walls for example just by using your eyeball! There are lots of 'trades people' who can walk into a room and in 30 seconds tell you *approximately* ('eyeball') how much a certain *renovation* will cost just by looking around. It requires lots of experience though to be accurate!

a. Estimate the area of the floor in the entire classroom you are in right now! Your estimate: ___________________ m^3. (Your solution:)

b. If you are to carpet the floor of this room at $48.33 per square metre for carpet and $24.50 per hour for labour, how much will it cost (estimate)? ______________. (Your solution:)

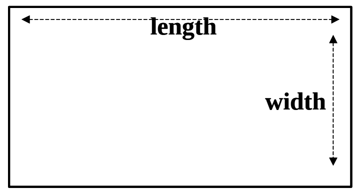
c. What about taxes?__________ . What about delivery of supplies? ___________. What other costs might you still need to consider?

8. Some refresher from prior grades on some formulas and unit conversion techniques ↓.

FORMULAE FOR CALCULATING VARIOUS LENGTHS, AREAS AND VOLUMES OF SELECTED SHAPES
[**A comprehensive list of Geometric Formulae is Appended at the end of these notes**]

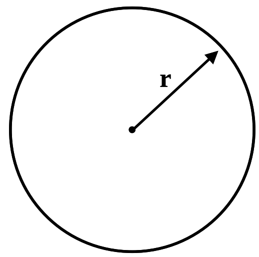
12. **Lengths (Perimeters, Circumferences)**:

a. Distance around a rectangle (Perimeter)
Add the length of sides together
Perimeter = 2w + 2l where w = width and
l = length (all measured in the same units)



Of course if it is a square (a special rectangle) then it is 4 * the length of a side

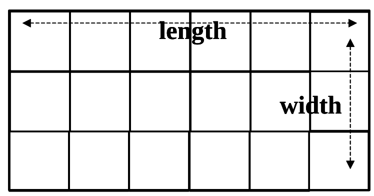
b. Distance around a circle (Circumference)
Circumference is really a perimeter, but a different word is used for circles.



C = 2πr; where r is the radius measurement
or since 2*r is a diameter ; or
C = πd where d is the diameter measurement

13. **Areas**: How many 'squares' would fit onto a surface.

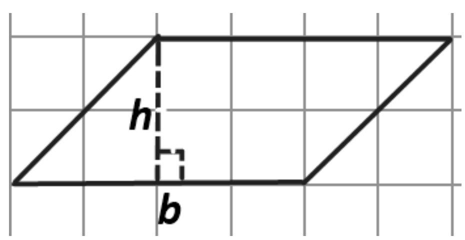
Rectangles and squares



Area = l * w *or* b*h

This area is 6 units* 3 units = 18 square units or **18 units^2**

Parallelograms. A parallelogram is just a leaning rectangle. The perimeter will not change but the area will depending on much it is leaning!



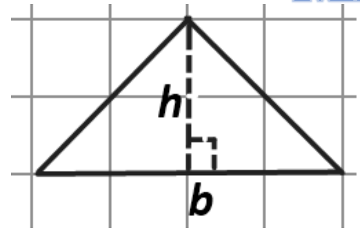
You can turn a parallelogram into a rectangle by just chopping off a triangle and moving it to the other side!

Area = base * height = b*h

****Height must be measured perpendicular to the base.****

The area of this parallelogram is (in square units):

Triangles



Notice that any triangle is just a parallelogram that has been sliced in half corner to corner!

Area = ½ * base * height

Note that 'height', h , is always measured perpendicular to the base! When you measure your kid's height they stand up straight I hope, square to the floor!

What is the area of triangle if it is 2 units high and has a base length of 4 units

A$_\Delta$ =

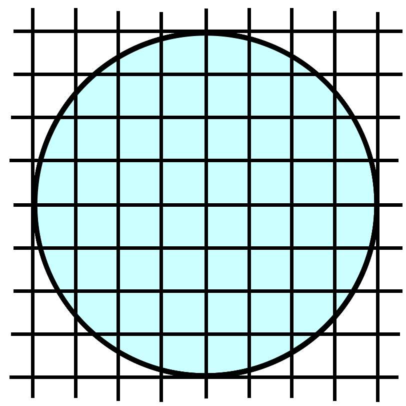
Circles

Area = πr^2

Where **r** is the radius (or half the diameter)

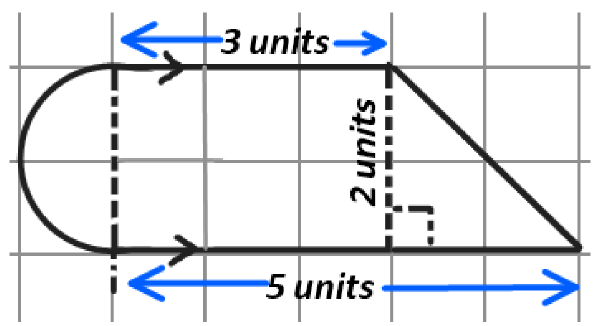
How many square cm are in a circle of radius **4 cm**? _______________

count how many 'squares' are in here →



Pretend each unit square here is 1cm^2

Irregular Shapes. Often shapes are a combination of others!! Determine the area of this irregular shape.



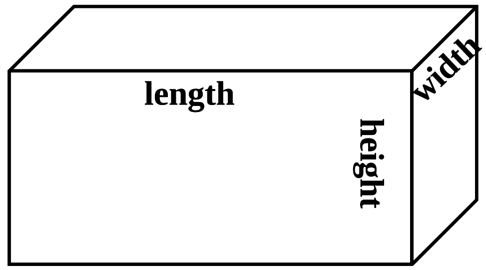
15. **Memorize.** All of these formulae to this point are so basic and fundamental that is important they be memorized for life. Combined with a couple basic trigonometry formulae and you will have all the basics of shape and design to last you a life-time.

16. **Rhombuses, Trapezoids, Parallelograms**: You would want to consult other references and prior studies for these formulae and for other uncommon shapes.

Review of Three Dimensional (3-D) Shapes

17. Surface Area (**SA**). The area of all the surfaces of a three-dimensional object if you covered the outside in squares of square units (m^2, ft^2, etc.). It is usually simpler to just add the area of each individual face of the object.

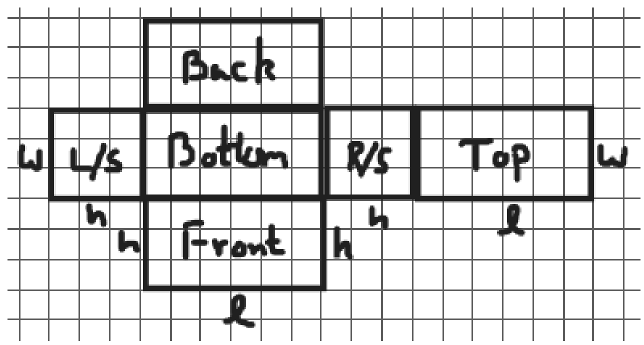
a. SA of a rectangular box or cube (called a rectangular '*prism*' really)
(A cube is just a rectangular box but all sides are the same length)
The front and back = l*h*2
The sides = w*h*2
The top and bottom = l*w*2
Total: 2*l*h + 2*w*h + 2*l*w



NET. Often if you unfold the object it is easier to see all its 'faces' and areas.

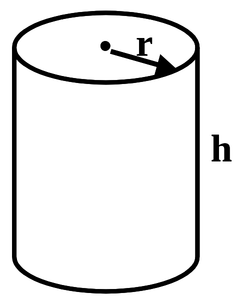
The rectangular prism above would look like this as a '**net**' →

How many square units on its surface?



b. SA of a cylinder
The surface area of a cylinder if you were to cover its outside in squares of some size would be:

Top and Bottom: $\pi r^2 * 2$
Lateral side (tube): $2\pi r * h$
Total: $2\pi h + 2\pi r^2$.

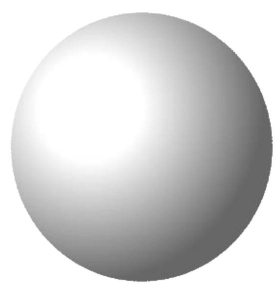


c. SA of a sphere (a ball)

The surface area of a sphere if you were to cover its outside in squares is:

$$\mathbf{SA_{sphere} = 4\pi r^2}$$

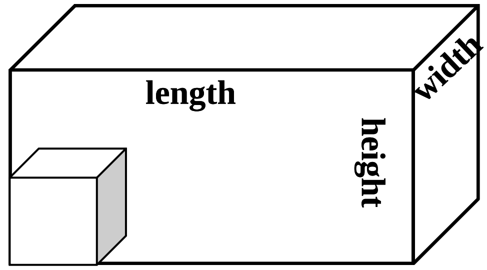
This is really remarkable! Why?



What is the surface area of half a sphere (a 'hemisphere')? (like the top 'half' of a cylindrical grain silo with a radius of 4 metres?)

____

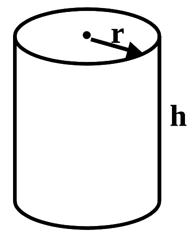
18. Volume. Volume is how many 'cubes' of some size you could fit inside a three-dimensional object. Picture how many sugar cubes could fit into your shoe for example.



a. Volume of a rectangular prism or cube (a box)

V = Base Area * h_{prism} = (l*w)*h

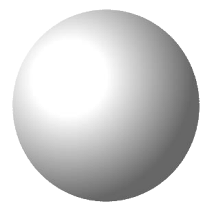
If your 'rectangular prism' is :
2cm * 3cm * 4 cm
your volume is 24 cubic cm or 24 cm^3



b. Volume of a cylinder:

V_{cyl} = Base area * height = (πr^2) * h

What is the volume of a cylinder of radius **8 cm** and height **125 mm**? Your Solution:



c. There is a formula for the volume of a sphere:

$$V_{sphere} = \frac{4}{3}\pi r^3$$

What is the radius of sphere that holds a volume of 1 litre?

19. So what is the volume of sphere of radius 4 cm? ___________. (Your Solution:)

Units of measurement and conversion.

20. It is often necessary to convert between units of measure. If the can of paint says on the label it will cover 90 square feet but you had measured the area in square meters, knowing how to *convert* square feet to square meters is essential. Or if you are buying the material from another country knowing how to convert currency from US Dollars to Canadian Dollars, etc.

CONVERSIONS BETWEEN UNITS

See the Conversion Tables attached at the end of these Notes

21. There are several ways to do conversions between units. But the best, and that you will need in science, is the conversion factor method. Converting between units is a fundamental and essential skill for everyday life.

23. Experience and familiarity with the feel for different measures is important also. If you cannot roughly approximate a metre or have a sense of what a Kilogram weighs or if you do not know how many millimetres are in a metre then you will need to elevate your experience with these units of measure.

CONVERSION FACTOR METHOD

24. A **factor** is simply a number that multiplies another number. Use factors to convert between units of measure

23. Example, how many inches are there in $3\frac{1}{2}$ feet? Hopefully you have a rough picture of the answer! A foot is longer than an inch so you are expecting a larger number of inches as the answer.

25. Tables or experience will tell you that 1 foot is 12 inches. So to convert $3\frac{1}{2}$ feet to inches first:

a. write down what you are trying to convert including the units!

$3\frac{1}{2}$feet

b. make a ratio, a fraction, of the known conversion : $\frac{12 inches}{1 foot}$. Ensure you keep the units. Ensure the new unit that you want is in the numerator (top).

c. Multiply the given measure by the conversion factor ensuring that the former units cancel and leave you with your new desired unit of measure:

$$3\frac{1}{2}\cancel{feet} * \frac{12 inches}{1\ \cancel{foot}} = 42\ inches$$

26. **Examples**. Convert: (see tables at the back if you need them)

a. 25 centimetres (cm) to metres (m): ______________________________

b. 25 kilometres to meters: ____

c. 5.6 Kilograms to grams: _____________________________

d. 17 pounds (lb) to Kilograms (Kg): ___________________________

e. 4 Imperial Gallon to litres. _______________________________
(caution there are two different gallons, American is smaller that the British Imperial gallon)

f. 8.2 miles to Kilometres: ___________________________________

g. 355 millilitres (ml) to litres ('l'): __________________________________

h. 12 cubic centimetres (cc) to ml: ______________________________

i. 74 inches (in) to centimetres (cm): _____________________________

j. 160 acres to hectares: ________________________________-

k. 12 ounces (oz) (of volume) to ml: ____________________________
Caution there is also different ounces to measure weight too! Confusing!

Converting Areas and Volumes using Square and Cube Dimensions

27. Be **very careful** when computing areas and volumes in **square** and **cube** types of units. For example; **1 m^2** is **not the same** as **100 cm^2;** it is *actually* another **100** times that or **10,000 cm^2**.

Example: Convert **5.2 m^2** into **ft^2**. (Given that 3.28 feet is the same length as one meter)

$$5.2m^2 * \frac{3.28ft}{1m} * \frac{3.28ft}{1m} = 55.9ft^2$$

Or: $$5.2m^2 * \left(\frac{3.28ft}{1m}\right)^2 = 55.9ft^2$$

Example: Your gas bill shows you used 200 cubic meters of gas to heat your house (by law Canadians use metric units) , but your meter in the basement is in units of cubic feet. So how many cubic feet of gas did you use if you want to check the company's readings.

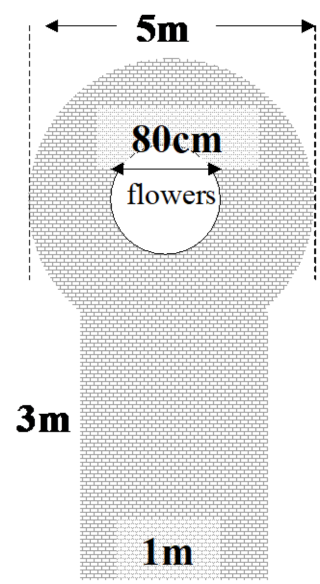
28. **Calculating Retail Taxes.** Various levels of government collect taxes upon sale of items and services. What is taxed and not is complicated, but pretty much everything; material and services is taxed by all levels. Presently (in calendar year 2020) Manitoba Taxes are 7% (PST) of the price and Federal taxes (GST) are 5% **of** the price.

Total cost of an item with taxes is then: **P**rice + **P**rice * 7% + **P**rice * 5%
Total Cost = P + P*7/100 + P * 5/100 = P * (1 + 0.07 + 0.5) = **P *(1.12)**

So, the best way to calculate the total cost with taxes is to multiply the price by 1.12 in this case.

Example - Project Flower garden design

29. Your aunt wants to improve her back yard. She wants a paving stone path, and a nice little circle at the end with a circular flower bed in the middle of the circle. You assist her in her design and offer to do this project!



Can you prepare a nice neat written estimate of what it will cost?

EXPANDED CONVERSION TABLES

SI (Metric) System Conversions

Conversions SI Metric – Length and Distance		
1 kilometre km	=	1,000 metres m
1 meter m	=	100 centimetres cm
1 centimetre	=	10 millimetres mm

Conversions SI Metric – Mass		
1 tonne	=	1,000 kg
1 kilogram kg	=	1,000 grams [g]
1 gram g	=	1,000 milligrams [mg]

Conversions SI Metric – Volume		
1 litre l	=	1,000 millilitres ml
1 litre l	=	100 centilitres cl
1 litre l	=	1,000 cc (or 1,000 cm^3)
1 millilitre ml		1 cc (or 1 cm^3)

'cc' stands for cubic centimetre which is really just $cm^{3.}$
Notice also that a cube of dimensions 10cm by 10 cm by 10 cm is a litre

Conversions SI Metric – Area		
1 square metre	=	10,000 cm^2
1 hectare	=	10,000 m^2

So a square 100 m by 100 m is a hectare. Used for measuring land area.

For a hectare, picture perhaps a football field with the side and end zones included.

Non-SI (American, Conventional, Imperial) System Conversions

Conversions Non-SI (Imperial) – Length		
1 mile [mi]	=	1,760 yards yd
1 yard [yd]	=	3 feet [ft]
1 foot [ft]	=	12 inches [in]

Conversions Non-SI Imperial – Mass		
1 ton	=	2,000 pounds lb
1 pound [lb]	=	16 ounces oz

Conversions Non-SI Imperial – Volume (English)		
1 gallon	=	0.125 bushels
8 bushels	=	1 gallon [gal]
1 gallon	=	160 ounces [oz]
1 pint	=	0.125 gallons
1 quart [qt]	=	0.25 gallons [gal]
4 quarts	=	1 gal
1 pint	=	0.5 quarts

Conversions Non-SI Imperial – Volume (USA)		
1 gallon (US)	=	0.832 gallons (English)
1 gallon (US)	=	128 ounces oz (US)
Really gets confusing with two different volumes depending on your country! Americans had made **their** gallon smaller to cheat the British.		

Caution Ounces of weight are different from ounces of volume.
If you do science you will discover lots of other units:
Pascals, Dynes, Newtons, millibars, farads, amps, ohms, volts, ….

Conversions Non-SI Imperial – Area		
1 acre	=	43,560 ft^2
1 acre	=	4,840 yd^2
1 sq ft	=	144 in^2
1 square mile	=	640 acres

So a square having sides of 208 feet would be an acre.
An acre originally was supposed to be the amount of land a horse could plow in one day, so it depended on how good your horse was!

Converting between systems

Conversions SI to Non-SI Length		
1 metre m	≅	3.2808 feet ft
1 metre m	≅	39.370 inches in
1 kilometer km	≅	0.6214 miles mi
2.54 cm	≅	1 inch

Conversions Non-SI Imperial – Mass		
1 kilogram kg	≅	2.205 pounds lb
1 tonne	≅	1.1 ton

Conversions SI to Non-SI Volume		
1 gallon (English)	≅	4.546 litres
1 gallon (US)	≅	3.785 litres
1 gallon (English)	≅	4,546 cc^3
1 gallon (US)	≅	3,785 cc^3

Conversions SI to Non-SI Area		
1 sq mile	≅	259 hectares
1 sq mile	≅	2,589,988 m^2
1 hectare [ha]	≅	2.47 acres
1 square metre	≅	10.76 ft^2
1 square metre	≅	1,550 in^2

Examples:

a. To convert 3 miles to kilometres:

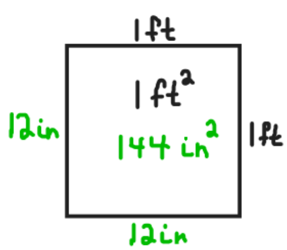
$$3 miles = \underline{????}\, km? \qquad 3\ \cancel{mi} * \frac{1\ \boldsymbol{km}}{0.6214\ \cancel{mi}} = 4.83\ \boldsymbol{km}$$

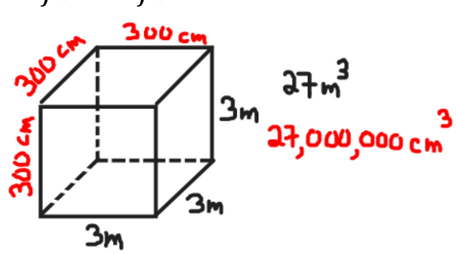
b. Don’t forget!: If the conversion factors you want aren’t here you can always apply several different factors to make a combination conversion. Example:
Eg: To convert 1 ton to kilograms:

$$1\ \cancel{ton} * \frac{2000\ \cancel{lb}}{1\ \cancel{ton}} * \frac{1\ \boldsymbol{kg}}{2.205\ \cancel{lb}} = 907\ \boldsymbol{kg}$$

c. To convert square feet to square inches (notice you apply the conversion factor twice!):

$$1ft^2 = 1\cancel{ft^2} * \frac{12\ in}{1\cancel{ft}} * \frac{12\ in}{1\cancel{ft}} = 144\ in^2$$





GRADE 11 ESSENTIAL
UNIT C – 3-D GEOMETRY
APPENDIX - GEOMETRIC FORMULAE

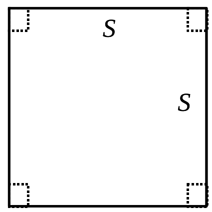
Shape	Diagram	Formulae

FLAT OBJECTS
2 DIMENSIONAL

Square

(all four sides same length, 90°corners)

(a rectangle with all sides same length)

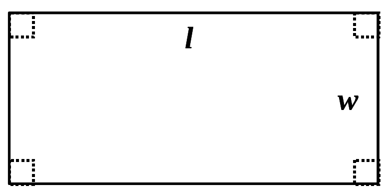


Perimeter, P:
P = s + s + s + s = 4*s

Area, A:
A = s * s = s²

Rectangle

(Four sides, square corners)



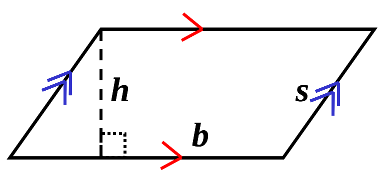
Perimeter, P:
P = *l* + *w* + *l* + *w* = 2*l* + 2*w*

Area, A:
A = *l* * *w*

Parallelogram and Rhombus

(leaning rectangle or leaning square)

Note
b is always ⊥ to h



Perimeter; P:
P = 2b + 2s

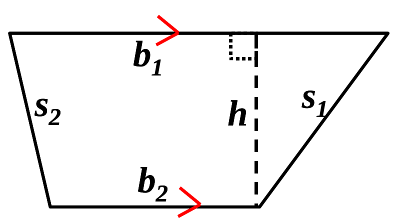
Area; A:
A = b * h

[b and h; perpendicular; at 90°]

Trapezoid

(Four sides, only two sides parallel { '||' })

Note
b is always ⊥ to h
[perpendicular; at 90°]



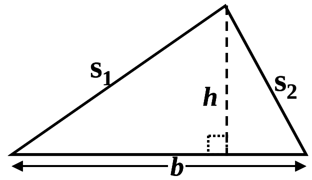
Perimeter; P:
$P = b_1 + s_1 + b_2 + s_2$

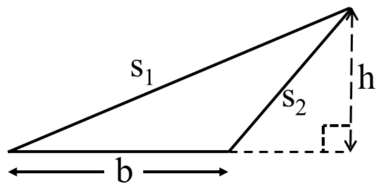
Area; A:
$$A = b_{average} * h$$
$$= \frac{1}{2}(b_1+b_2)*h$$

Triangle
(three sides)

(half a parallelogram or rectangle)
(acute, obtuse, or right)
(scalene, isosceles, equilateral)

Note
b is always ⊥ to h





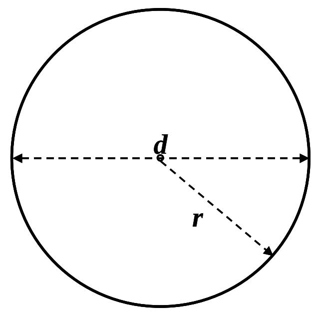
Perimeter; P:

$$P = s_1 + s_2 + b$$

Area; A:

$$A = \frac{1}{2} * b * h$$

Circle



Circumference; C:

$$C = \pi d = 2\pi r$$

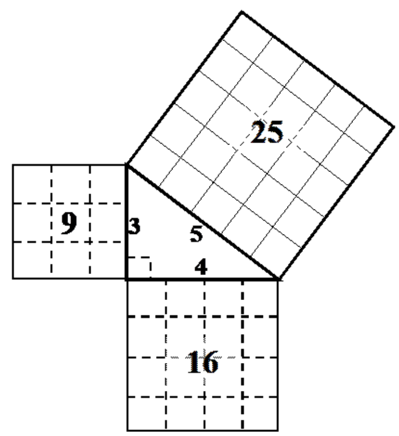
Area; A

$$A = \pi r^2$$

Pythagoras

$$c^2 = a^2 + b^2$$

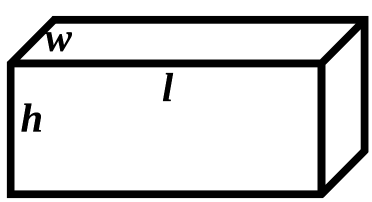
where ***c*** *is the length of the* ***hypotenuse*** *and* ***a*** *and* ***b*** *are the lengths of the shorter two sides*



SOLID OBJECTS
3 DIMENSIONAL

Rectangular Prism

(Two congruent rectangles connected at edges by rectangles)



Surface Area; SA

SA = Add area of all faces; *or*
SA= $2lw + 2hl + 2hw$

Volume; V:

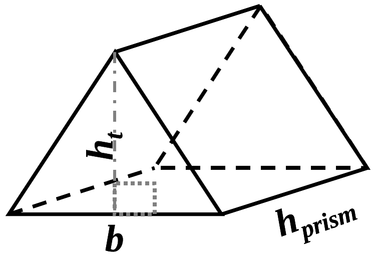
$$V = Base_{area} * h$$

$$V = (l * w) * h$$

Triangular Prism

(Two congruent triangles connected at edges by rectangles)

*Gets confusing using height for the triangle, **h_t**, and height for the prism, **h_{prism}**.*



Surface Area; SA

SA = Add area of all faces; the net is two triangles and three rectangles.

SA = $P_{base}h_{prism}+bh_t$ *(fancy)*

Volume; V:

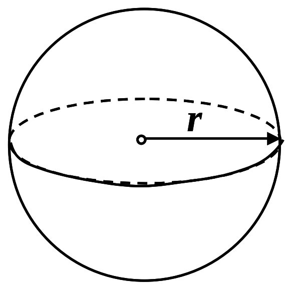
V = Base$_{area}$ * h

$$V = \frac{1}{2} bh_{triangle} * h_{prism}$$

Sphere

All the points in space that are equidistant from a single centre point

(Ball)



Surface Area; SA

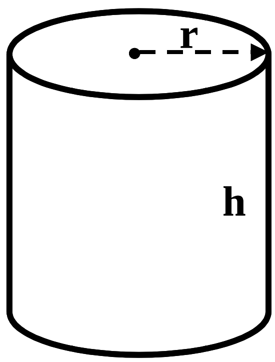
SA = $4\pi r^2$

Volume; V:

$$V = \frac{4}{3}\pi r^3$$

Cylinder

(Two congruent circles connected with a rectangle wrapped around circumference)



Surface Area; SA

top & bottom lateral side

SA = $2\pi r^2 + \underbrace{2\pi rh}_{\text{tube}}$

Volume; V:

V = Base$_{area}$ * h

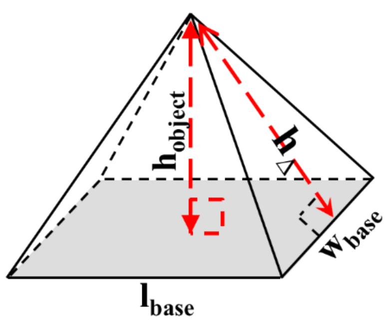
= A * h

= $\pi r^2 h$

Rectangular Pyramid or Square Pyramid

(A rectangle connected to an apex point by triangles on its edges)

*****caution the pyramid has a height,** and the **triangular faces each have a height*****



Surface Area; SA

SA = add up area of all the faces (Base area plus four triangles)

Volume; V:

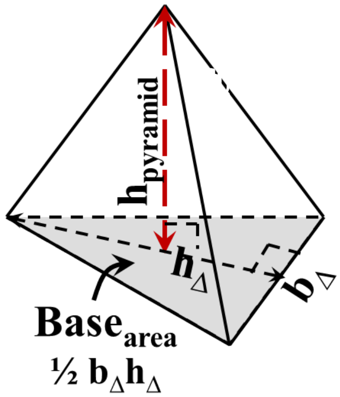
$$V = \frac{1}{3} * Base_{area} * h_{pyramid}$$

$$= \frac{\mathbf{1}}{\mathbf{3}} * (\boldsymbol{l} * \boldsymbol{w}) * \boldsymbol{h_{pyramid}}$$

Triangular Pyramid

(A triangle base connected to an apex point by triangles on its edges)

caution the pyramid has a height h_{object}, and the triangular faces have a height, h_Δ



Surface Area; SA

SA = add up area of each of the four triangular faces.

Volume; V:

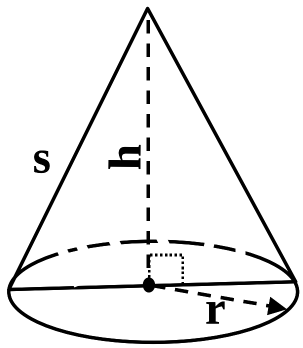
$$V = \frac{1}{3} * Base_{area} * h_{pyramid}$$

$$\frac{1}{3} * \left(\frac{1}{2} * b_\Delta * h_\Delta\right) * h_{object}$$

$$= \frac{1}{6} * b_\Delta * h_\Delta * h_{object}$$

Cone

(The arc of a circular sector of a circle connected to a smaller circle base and coming to an apex point)



Surface Area; SA

SA = $\pi r^2 + \pi rs$

('s' here is 'slant range' along the side of the cone)

Volume; V:

$$V = \frac{1}{3} * Base_{area} * h_{cone}$$

$$V = \frac{1}{3} * (\pi r^2) * h_{cone}$$

Letter Abbreviations:

r ≡ radius, d ≡ diameter; h ≡ height; A ≡ area; l ≡ length; w ≡ width; B = $Base_{area}$

s ≡ *side* or *sometimes* slant range; ⊥ ≡ perpendicular

And do not forget Pythagoras!

Pythagoras

$$c^2 = a^2 + b^2$$

*where **c** is the length of the **hypotenuse** and **a** and **b** are the lengths of the **shorter** two sides*