GRADE12 APPLIED

UNIT C FUNCTIONS

WORKBOOK 2

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EXPONENTIAL APPLICATIONS

1- 1. The population of a small town is 750 in 2012. The population of the town is estimated to triple every 5 years.

a) State an equation that models the situation.
b) What will the population be in 2017? 2022?
c) What will the population be in 2015?
d) When the town reaches a population of 10 000, it will build a community centre. In what year will the community centre be built?
e) What percent is the population increasing by each year?

1-2. The value of a particular brand of car is said to decrease by a quarter of its value every 9 months. The initial value of the car is $28,500.

a) State an equation that models the situation.
b) What will the price of the car be after 1.5 years?
c) What will the price of the car be after 4 years?
d) A customer is looking to sell the car when it is worth less than $5000. After how long should they sell it?

1-3. The average rent of a 1-bedroom apartment in a major city is said to be $575. Experts predict the rent cost will increase 7.5% each year for the foreseeable future.

a) State an equation that models the situation.
b) What will the rent for a 1-bedroom apartment be in 10 years?
c) A man is paying a $4500 monthly mortgage on an apartment complex with 10 1-bedroom units, each renting for $575 monthly. After how many years will he be making a profit of $4000/month on his apartments?

1-4. The number of fans that attend a town's new hockey team is said to increase by 30% each game. At the first game, there were 200 fans.

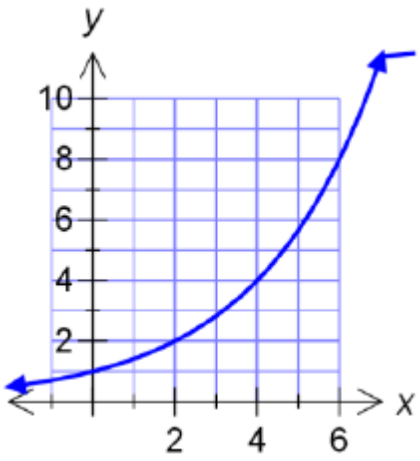
a) State an equation that models the situation.
b) What will attendance be after 5 games?
c) How many games will it take for the 2500 seat arena to sell out?

1-5. A scientist has 100 mg of iodine-131 which has a half-life of 8 days.

a) State an equation that models the situation.
b) Graph the amount of iodine-131 remaining after each of the first five
half-lives.
c) When will the scientist have less than 1 mg of iodine-131 remaining?

EXPONENTIAL GRAPHS

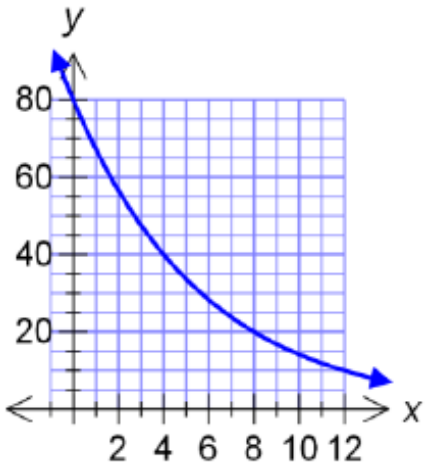
2-1. Consider the graph to the right; it has the general form $y = f(x) = ab^x$:



a) Is the graph increasing or decreasing?
b) What is the y-intercept?
c) State the domain and the range.
d) State the equation of the function.
e) If the value of the 'a' coefficient changed to 3, state the new y-intercept and domain & range.

Workspace

2-2. Consider the graph of f(x) to the right:



a) What is the equation of the function?
b) Find the value of y if x is 2.5 [ie: find f(2.5)]
c) At what value(s) of x is y equal to 1.5? [ie: solve for f(x) = 1.5]

Workspace

2-3. Respond to the following:

a) Graph a decreasing exponential function that has an initial value of 100 and passes through the point (1, 50).
b) What is the equation of the exponential function?

2-4. The number of daily customers at a new store is recorded in the table below:

Day	1	2	4	6	8
Customers	8	10	18	40	65

a) Find the exponential equation of best fit.
b) Comment on how good the fit is.
c) Use your equation to estimate the number of customers that will come on the 10th day.
d) Explain any limitations on the model.

2-5. The average annual salary of a city employee in Exponentville is given in the table below:

Year	2000	2003	2006	2009	2012
Salary ($)	41 000	46 000	52 000	58 500	69 000

a) Find the exponential equation of best fit.
b) Comment on how good the fit is.
c) Use your equation to find how long it will take for the average salary to reach $100 000.

LOGARITHMIC REGRESSIONS

3-1. Write each as a logarithmic expression.

a) $4^3 = 64$

b) $6^4 = 1296$

3-2. Find the value of each of the following logarithms (*b & d need Change of Base Formula*).

a) log100

c) log50

b) $\log_2 32$

d) $\log_2 60$

3-3. The number of social networking website friends someone has is said to fit a logarithmic model. Consider the data below for a recent new user of the website:

# Days	1	4	9	15	25
Friends	2	19	28	35	41

a) Find the logarithmic regression equation that matches the data.
b) How many friends will this person have after 2 months?
c) How long will it take the user to earn 65 friends?

3-4. A bowling alley offers bowling lessons to anyone interested. They share the following scores for a recent customer of theirs:

Game #	2	5	9	25	40
Score	125	155	175	215	230

The bowling alley claims the data above proves that you can bowl a game of 200 in 15 games or less. Using the logarithmic regression equation, find the validity of that claim.

EXPONENTIAL AND LOGARITHMIC REVIEW

4-1. The number of listeners to a particular podcast is said to model an exponential function. The number of current listeners is 1200. Every15 days, the number of listeners doubles.

a) State an equation that models the situation.

b) How many listeners will there be in 30 days?

c) How many listeners will there be in 50 days?

d) If the number of listeners reaches 50 000, a radio station will give the podcast airtime. How long will it take for the radio station to give the podcast airtime?

e) What percent is the number of listeners increasing by each day?

4-2. There is 50 g of a certain radioactive substance. The half-life of the substance is 18 days.

a) State an equation that models the situation.

b) Graph the amount of the substance remaining after each of the first five half-lives.

c) How much material will there be left after 50 days?

d) After how many days will there be less than 5 g left?

4-3. The average cost of a baseball ticket for a particular minor league team is $20. History suggests that the price of a ticket increases by 5% each year.

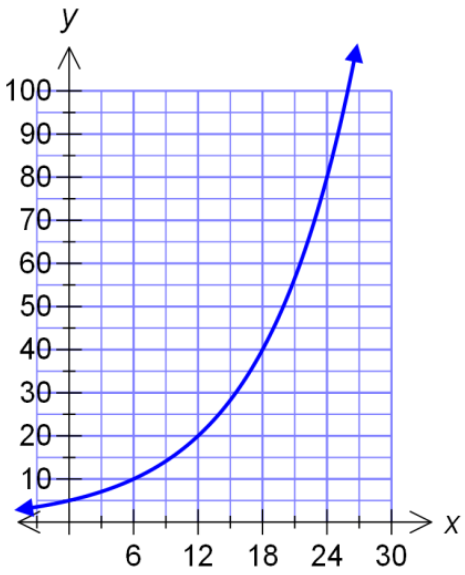
a) State an equation that models the situation.
b) What will be the cost of a ticket in 8 years?
c) When will the price of a ticket reach $45?

4-4. The number of customers of dial-up internet in a small town is currently 500. Once high speed internet is introduced to the town, the number of customers with dial-up decreases by 24% each month.

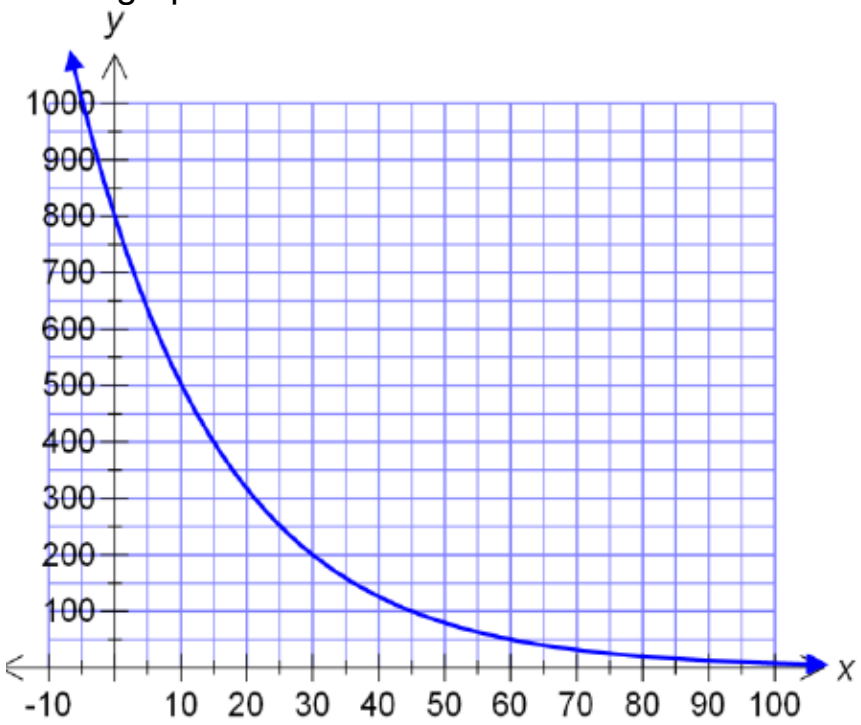
a) State an equation that models the situation.
b) How many dial-up customers will the town have in 6 months?

4-5. Consider the graph at right:

a) Is the graph increasing or decreasing?
b) What is the y-intercept?
c) State the domain and the range.
d) State the equation of the function.



4-6. Consider the graph below:



a) What is the equation of the function, f(x)?
b) Find the value of y if x is 10; f(10).
c) At what value(s) of x is f(x) equal to 280?

4-7. Complete the following:

a) Graph an increasing exponential function that has an initial value of 75 that doubles every 6 x-values.
b) What is the equation of the exponential function?

4-8. The average hourly wage of employees at an office building is expected to increase over time, in years, according to an exponential model:

Year	0	2	5	8	12	15	20
Average Wage ($)	18.00	19.30	21.40	23.90	27.20	30.35	35.00

Let 1992 be year 0, 1993 be year 1, ...

a) Find the exponential equation of best fit.
b) Comment on how good the fit is.
c) Use your equation to estimate the hourly wage in 2020.
d) Explain any limitations on the model.

4-9. Pat & Rick decide to sell their popcorn locally and around Manitoba at various conventions. The number of daily sales they have is said to follow an exponential model. Some data is shown below:

Day	3	5	10	15	20	25	30
Sales	15	18	32	60	105	190	340

a) Find the exponential equation of best fit.
b) Comment on how good the fit is.
c) Use your equation find out how many customers they expect on the 12th day.
d) After how many days will Pat & Rick have 1000 daily sales?

4-10. Write each as a logarithmic expression.

a) $3^5 = 243$ b) $a^4 = 25$

4-11. Find the value of each of the following logarithms.

a) log10,000 c) log500

b) $\log_5 125$ d) $\log_2 360$

4-12. The population of fish in a certain lake after fishing season begins is given in the table below:

Days	1	4	8	15	30
# of Fish	100	70	60	45	32

a) Find the logarithmic regression equation that matches the data.
b) How many fish remain after 20 days?
c) How long will it take the number of fish to drop below 25?

ANSWERS

It is possible there is an occasional error! And accuracy of answers will be within fractions of a percentage subject to the quality of the calculations, the accuracy of the calculator. The base of a logs should be carried to at least 4 decimal places, since a small change in a log base can make a very large difference in the answer.

1-1

a. Two equivalent possibilities:
$y = 750*3^{(t/5)}$ from context or $750*1.246^{x}$ from TI-83 regression

b. In 2017: 2250 people; in 2022: 6750 people

c. In 2015: 1450 people

d. 11.79 years

1-2

a. Two equivalent possibilities:
Value = $28{,}500 * (0.75)^{(12x/9)}$ or Value = $28{,}500 * 0.681^{x}$ from TI-83 if x measured in years
Value = $28{,}500\ (0.75)^{\mathbf{x/9}}$ or $28{,}500 * 0.9685^{\mathbf{x}}$ if x measured in months

b. Value = 16,016.42 after 1.5 years (18 months)

c. $6144 after 4 years (48 months)

d. 4.54 years it is worth $5,000

1-3

a. $y = 575*(1.075)^{x}$ where x is years.

b. 1185.09

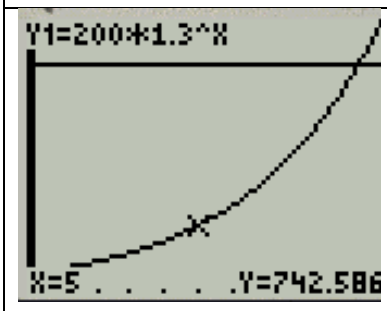
c. needs to get $850 gross from each apartment, so 5.40 years till he starts making reasonable money

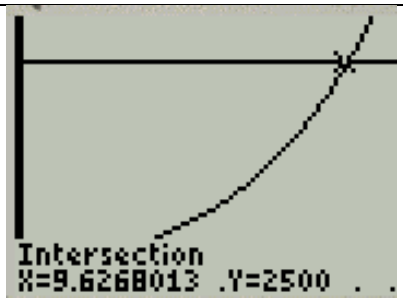
1-4

a. From context and TI-83 regression: y= 200(1.3)^x

b. after 5 games : 742 people (rounded down)

c. 9.63 games, so need to wait for game 10!



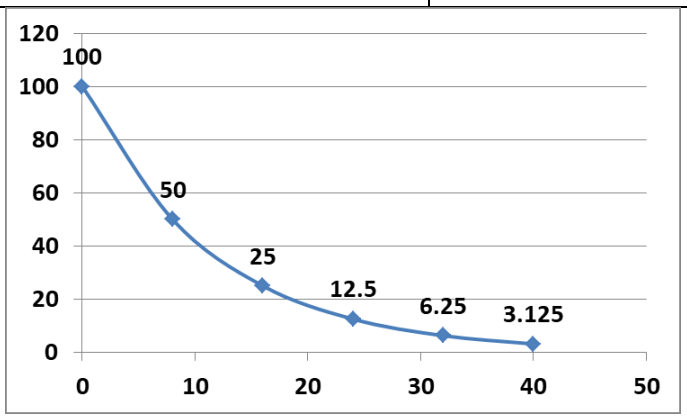


1-5

a. y= 100*(0.9170)^x or

$y = 100 * 0.5^{x/8}$

b. 53 days



2-1

a. Increasing to the right

b. y-intercept : (0 , 1)

c. domain: All Real x; or $\infty<x<\infty$, or $(-\infty,\infty)$

Range: $0<y<\infty$ or $(0,\infty)$

d. $y = 2^{x/2}$ or $y=1.414^x$

e. if $y = 3*2^{x/2}$; then y-intercept is (0, 3) and domain no change but range $3<y<\infty$ or $(3,\infty)$

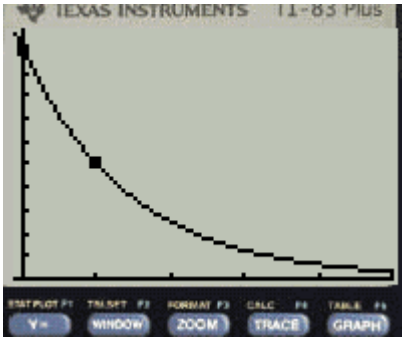
2-2

a. function : 80 * 0.5 ^(x/4) or 80 * 0.8409^x

b. 51.87

c. x= 22.95 , y=1.5

2-3

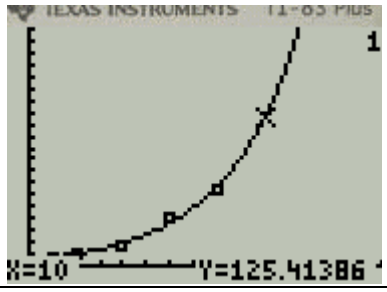



```
ExpReg
 y=a*b^x
 a=100
 b=.5
 r²=1
 r=-1
```

2-4

```
ExpReg
 y=a*b^x
 a=5.586964824
 b=1.364950682
 r²=.9934195478
 r=.9967043432
```

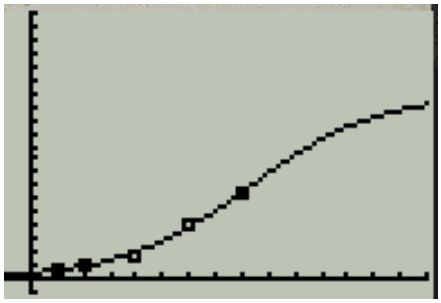
125 customers



X	Y1
4	19.393
5	26.47
6	36.131
7	49.317
8	67.315
9	91.882
10	125.41

X=10

Limitations: Clearly your customers cannot grow exponentially forever, your store would not hold them all. Eventually it would have to level off.



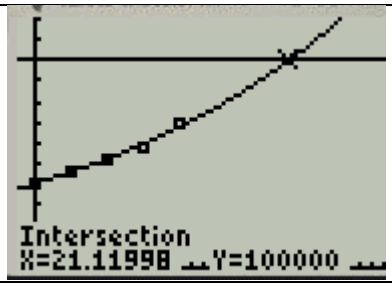
2-5

```
ExpReg
 y=a*b^x
 a=3.208515E-33
 b=1.043640562
 r²=.9947608189
 r=.9973769693
```

$y = 0 + 1.0436^x$
be careful rounding bases, give them 4 decimal places!

```
ExpReg
 y=a*b^x
 a=40569.93364
 b=1.043640562
 r²=.9947608189
 r=.9973769693
```

Start counting years at zero

Very reliable fit to data 21.12 years till workers make $100K	

3-1

a. If $4^3 = 64$ then saying it backwards $\log_4 64=3$. The exponent you put on a base of 4 to get 64 is 3.

b. If $6^4 = 1296$ then $\log_4 1296 = 4$. The exponent you put on a base of 6 to get 1,296 is 4.

3-2

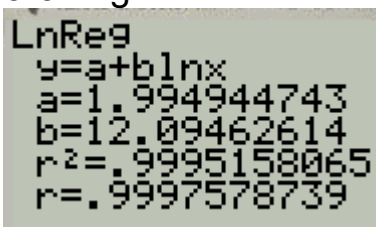
a. log100 means $\log_{10}100$ by default. So the log base 10 of 100 is the exponent you put on a 10 to get 100; namely: 2 . Thus: $\log_{10}100=2$. The **log** button on your calculator does base 10 logs.

c. log(50) means $\log_{10}(50)$ by default. So the base 10 logarithm of 50 is the exponent you put on a 10 to get 50; namely: 1.6990. Thus: $\log_{10}50=1.6990$

b. $\log_2 32$ means the exponent you put on a base of 2 to get 32. Namely 5; since 2^5 = 32 if you wanted to check. So $\log_2 32= 5$. You have no base 2 log button on your calculator though! So could always use the change of base law if you had too! $\boldsymbol{log_2 32 = \frac{log_{10}32}{log_{10}2} = 5}$

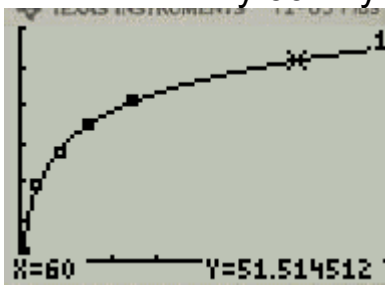
d. $\log_2 60$ means the exponent you put on a base of 2 to get 60. Namely 5.91; since 2^5.91 = 60 if you wanted to check. So **$\log_2 60= 5.91$**. You have no base 2 log button on your calculator though! So could always use the change of base law if you had too! $\boldsymbol{log_2 60 = \frac{log_{10}60}{log_{10}2} = 5.91}$

3-3 Log model

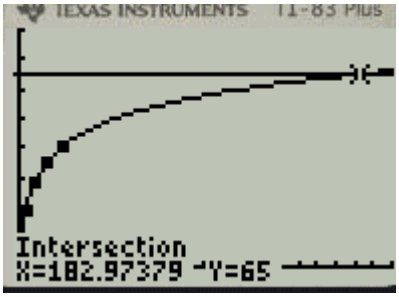


y = 1.99* 12.04 ln(x)

2 months is notionally 60 days!



So 51 friends after 60 days



182 days to get to 65 friends.

The log function starts to tail off , never does level off though! But the slope is increasingly small as you go further right.

3-4 Bowling alley

The graph shows you should need 17.18 games to break 200. So 18 games is required. So the bowling alley is wrong.

Unless of course you are really that shitty, lol