MA40S APPLIED AND PRE-CALC
PERMUTATIONS AND COMBINATIONS
SUMMARY AND WORD PROBLEMS

FUNDAMENTAL COUNTING PRINCIPLE. If one thing can be done in ***m*** different ways and, when it is done in any one of these ways, a second thing can be done in ***n*** different ways, then the two things in succession can be done in ***m*n*** different ways. For example, if there are 3 candidates for governor and 5 for mayor, then the two offices may be filled in **3*5 = 15** ways.

A PERMUTATION is an arrangement of all or part of a number of things in a definite order. For example, the permutations of the three letters a, b, c taken all at a time are abc, acb, bca, bac, cba, cab. The permutations of the three letters a,b,c taken two at a time are ab, ac, ba, bc, ca, cb.

A COMBINATION is a grouping or selection of all or part of a number of things without reference to the order of the things selected.
Thus the combinations of the three letters a, b, c taken 2 at a time are ab, ac, be.
Note that ab and ba are 1 combination but 2 permutations of the letters a, b.

FACTORIAL NOTATION. The following identities indicate the meaning of factorial n, or n!.
2! = 1*2 = 2, 3! = 1*2*3 = 6, 4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120,
n! = 1*2*3…n,
(r-l)! = l*2*3…(r-l)
Note: 0! = 1 by definition

THE SYMBOL $_nP_r$ represents the number of permutations (arrangements, orders) of n things taken r at a time.
Thus $_8P_3$ denotes the number of permutations of 8 things taken 3 at a time, and $_5P_5$ denotes the number of permutations of 5 things taken 5 at a time.
Note. The notation or symbology P(n,r) having the same meaning as $_nP_r$ is sometimes used.

PERMUTATIONS OF *n* DIFFERENT THINGS TAKEN *r* AT A TIME.

$_nP_r$ = n(n-l)(n-2)...(n-r+1) = $\frac{n!}{(n-r)!}$

When r = n, $_nP_r$ = $_nP_n$ = n(n -1) (n - 2).. . 1 = n!.

Thus $_5P_1$ = 5!, $_5P_2$= 5* 4 = 20, $_5P_3$= 5* 4* 3 = 60, $_5P_4$ = 5* 4*3*2 = 120, $_5P_5$ = 5! = 5*4*3*2*1 = 120, $_{10}P_7$= 10*9*8*7*6*5*4 = 604,800.
For example, the number of ways in which 4 persons can take their places in a cab having 6 seats is $_6P_4$ = 6*5*4*3 = 360 ways

PERMUTATIONSWITH SOME NON-DISTNGUISHABLE THINGS ALIKE, TAKEN ALL AT A TIME. The number of permutations P of n things taken all at a time, of which a quantity of a As are alike and non-distinguishable, b Bs are alike…, c Cs are alike …, etc., is

Permutations= $\frac{n!}{a!b!c!...}$

For example, the number of distinguishable ways 3 dimes and 7 quarters can be distributed among 10 boys, each to receive one coin, is $\frac{10!}{7!3!} = \frac{10*9*8}{3*2*1} = 120$

CIRCULAR PERMUTATIONS. The number of ways of arranging n different objects around a circle is (n -1)! ways.
Thus 10 persons may be seated at a round table in (10 - 1)! = 9! = 362,880 ways.

THE SYMBOL $_n$C$_r$,. represents the number of combinations (selections or groups where the order of selection does not matter) of n things taken r at a time. Thus $_9C_4$ denotes the number of combinations of 9 things taken 4 at a time.
Note. The symbol C(n,r) or $\binom{n}{r}$ having the same meanings as $_nC_r$, are sometimes used.

COMBINATIONS OF n DIFFERENT THINGS TAKEN r AT A TIME.

$$ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r!)} = \frac{n(n-1)(n-2)(n-3)....(n-r+1)}{r!} $$

For example, the number of handshakes that may be exchanged among a party of 12 students if each student shakes hands once with each other student is

$$ {}_{12}C_2 = \frac{12!}{2!(12-2)!} = \frac{12!}{2!10!} = \frac{12*11}{2*1} = 66 $$

Notice that ${}_{10}C_2 = {}_{10}C_8$ and ${}_7C_5 = {}_7C_2$, etc.

WORD PROBLEMS

1. A student has a choice of 5 foreign languages and 4 sciences. In how many ways can he choose 1 language and 1 science?

   He can choose a language in 5 ways, and with each of these choices there are 4 ways of choosing a science.
   Therefore the required number of ways = 5 *4 = 20 ways

2. In how many ways can 2 different prizes be awarded among 10 contestants if both prizes:

   a) may not be given to the same person,

   b) may be given to the same person?

   a) The first prize can be awarded in 10 different ways and, when it is awarded, the second prize can be given in 9 ways, since both prizes may not be given to the same contestant. Therefore the required number of ways = 10*9 = 90 ways

   b) The first prize can be awarded in 10 ways, and the second prize also in 10 ways, since both prizes may be given to the same contestant. Therefore the required number of ways = 10*10= 100ways

3. In how many ways can 5 letters be mailed if there are 3 mailboxes available?
Each of the 5 letters may be mailed in any of the 3 mail boxes
Therefore the required number of ways = 3*3*3*3*3 = 3 = 243 ways

4. There are 4 candidates for president of a club, 6 for vice-president and 2 for secretary. In how many ways can these three positions be filled?
A president may be selected in 4 ways, a vice-president in 6 ways, and a secretary in 2 ways
Therefore the required number of ways = 4*6*2 = 48 ways

5. In how many different orders may 5 persons be seated in a row?
The first person may take any one of 5 seats, and after the first person is seated, the second person may take any one of the remaining 4 seats, etc.
Therefore the required number of order = 5*4*3*2*1 = 120 orders
Otherwise, Number of orders = number of arrangements of 5 persons taken all at a time = ${}_5P_5 = 5! = 5*4*3*2*1 = 120$ orders

6. It is required to seat 5 men and 4 women in a row so that the women occupy the even places How many such arrangements are possible?

The men may be seated in ${}_5P_5$ ways, and the women in ${}_4P_4$ ways. Each arrangement of the men may be associated with each arrangement of the women. Therefore the required number of arrangements = ${}_5P_5 * {}_4P_4 = 5!$
$4! = 120*24 = 2880$

7. In how many orders can 7 different pictures be hung in a row so that 1 specified picture is

a) at the centre, and

b) at either end?

a) Since 1 given picture is to be at the centre, 6 pictures remain to be arranged in a row
Therefore the number of orders = 6P6 = 6! = 720 orders
b) After the one specified picture is hung in any one of 2 ends, the remaining 6 can be arranged in 6P6 ways. Therefore the number of orders = 2*6P6 = 1440 orders

8. In how many ways can 9 different books be arranged on a shelf so that:

a) 3 of the books are always together

b) 3 of the books are never all 3 together?

a) The specified 3 books can be arranged among themselves in 3P3 ways Since the specified 3books are always together, they may be considered as 1 thing. Then together with the other 6 books (things) we have a total of 7 things which can be arranged in 7P7 ways. Total number of ways = 3P3*7P7 = 3!7! = 6*5040 = 30,240 ways

b) Number of ways in which 9 books can be arranged on a shelf if there are no restrictions = 9! = 362,880 ways. Number of ways in which 9 books can be arranged on a shelf when 3 specified books are always together (from a) above) = 3!7! = 30,240 ways. Therefore the number of ways in which 9 books can be arranged on a shelf so that 3 specified books are never all 3 together = 362,880 - 30,240 = 332,640 ways

9. In how many ways can n men be seated in a row so that 2 particular men will not be next to each other?

With no restrictions, n men may be seated in a row in ${}_nP_n$ ways. If 2 of the n men must always sit next to each other, the number of arrangements = 2! (${}_{n-1}P_{n-1}$). Therefore the number of ways n men can be seated in a row if 2 particular men may never sit together =

${}_nP_n - 2({}_{n-1}P_{n-1}) = n! - 2(n-1)! = n(n-1)! - 2(n-1)! = (n-2)(n-1)!$

10. Six different biology books, 5 different chemistry books and 2 different physics books are to be arranged on a shelf so that the biology books stand together, the chemistry books stand together and the physics books stand together. How many such arrangements are possible?

The biology books can be arranged among themselves in 6! ways, the chemistry books in 5! ways, the physics books in 2! ways, and the three groups in 3! ways. Required number of arrangements = 6!5!2!3! = 1,036,800.

11. Determine the number of different words of 5 letters each that can be formed with the letters
of the word chromate:

a) if each letter is used not more than once,

b) if each letter may be repeated in any arrangement. (These words need not have meaning)

a) Number of words = arrangements of 8 different letters taken 5 at a time = ${}_8P_5$=8*7*6*5*4 = 6720 words
b) Number of words = 8*8*8*8*8 = 8^5 = 32,768 words.

12. How many numbers may be formed by using 4 out of the 5 digits 1,2,3,4,5

a) if the digits must not be repeated in any number,

b) if they may be repeated? If the digits must not be repeated, how many of the 4-digit numbers

c) begin with 2,

d) end with 25?

a) Numbers formed = 5P4 = 5*4*3*2= 120numbers

b) Numbers formed = 5*5*5*5 = 5 = 625numbers

c) Since the first digit of each number is specified, there remain 4 digits to be arranged in 3 places. Numbers formed = ${}_4P_3$ = 4*3*2 = 24 numbers

d) Since the last two digits of every number are specified, there remain 3 digits to be arranged in 2 places. Numbers formed = 3P2 = 3*2 = 6 numbers

13. How many 4-digit numbers may be formed with the 10 digits 0,1,2,3,…,9 if:

a) each digit is used only once in each number?

b) How many of these numbers are odd?

a) The first place may be filled by anyone of the 10 digits except 0, ie: by anyone of 9 digits. The 9 digits remaining may be arranged in the 3 other places in ${}_9P_3$ ways. Numbers formed = $9*{}_9P3 = 9*(9*8*7) = 4536$ numbers

b) The last place may be filled by any one of the 5 odd digits, 1,3,5,7,9. The first place may be filled by any one of the 8 digits, ie: by the remaining 4 odd digits and the even digits, 2,4,6,8. The 8 remaining digits may be arranged in the 2 middle positions in ${}_8P_2$ ways. Numbers formed = $5*8*{}_8P_2 = 5*8*8*7 = 2240$ odd numbers.

14.

a) How many 5-digit numbers can be formed from the 10 digits 0,1,2,3,…,9 with repetitions allowed?
How many of these numbers
b) begin with 40,
c) are even,
d) are divisible by 5?

a) The first place may be filled by any one of 9 digits (*any of the 10 except 0*). Each of the other 4 places may be filled by any one of the 10 digits whatever. Numbers formed = $9*10*10*10*10 = 9*10^4$ = 90,000 numbers

b) The first 2 places may be filled in 1 way, by 40- The other 3 places may be filled by any one of the 10 digits whatever. Numbers formed = $1*10*10*10 = 10^3 = 1000$ numbers

c) The first place may be filled in 9 ways, and the last place in 5 ways (0,2,4,6,8)
of the other 3 places may be filled by any one of the 10 digits whatever.
Even numbers = 9*10*10*10*5 = 45,000 numbers
d) The first place may be filled in 9 ways, the last place in 2 ways (0,5), and the other 3
places in 10 ways each. Numbers divisible by 5 = 9*10*10*10*2 = 18,000 numbers

15. How many numbers between 3000 and 5000 can be formed by using the 7 digits 0,1,2,3,4,5,6 if each digit must not be repeated in any number?

Since the numbers are between 3000 and 5000, they consist of 4 digits. The first place may be filled in 2 ways, *ie* by digits 3,4. Then the remaining 6 digits may be arranged in the 3 other places in $_6P_3$ ways. Numbers formed = 2* $_6P_3$ = 2(6*5*4) = 240 numbers.

16. From 11 novels and 3 dictionaries, 4 novels and 1 dictionary are to be selected and arranged on a shelf so that the dictionary is always in the middle. How many such arrangements are possible?

The dictionary may be chosen in 3 ways. The number of arrangements of 11 novels taken 4 at a time is $_{11}P_4$. Required number of arrangements = 3* $_{11}P_4$. = 3(11*10*9*8) = 23,760

17. How many naval signals can be made with 5 different flags by raising them any number at a time?

Signals may be made by raising the flags 1,2,3,4, and 5 at a time - Therefore the total number of signals is $_5P_1 + _5P_2 + _5P_3 + _5P_4 + _5P_5$ = 5 + 20 + 60 + 120 + 120 = 325 signals.

18. Compute the sum of all the 4-digit numbers which can be formed with the four digits 2,5,3,8 if each digit is used only once in each arrangement.

The number of arrangements, or numbers, is $_4P_4$ = 4! = 4*3*2*1 = 24
The sum of the digits = 2 + 5 + 3 + 8 = 18, and each digit will occur 24/4 = 6 times each in the units', tens', hundreds', and thousands' positions. Therefore the sum of all the numbers formed is:
1(6*18) + 10(6*18) + 100(6*18) + 1000(6*18) = 119,988

19.

a) How many arrangements can be made from the letters of the word 'cooperator' when all are taken at a time?

How many of such arrangements

b) have the three o's together,

c) begin with the two r's ?

a) The word cooperator consists of 10 letters: 3 o's, 2 r's, and 5 different letters.

Number of arrangements = $\frac{10!}{3!2!} = \frac{10*9*8*7*6*5*4*3*2*1}{(3*2*1)(2*1)} = 302,400$

b) Consider the 3 o's as 1 letter or 1 place. Then we have 8 letters of which 2 r's are alike. Number of arrangements = $\frac{8!}{2!} = 20,160$

c) The number of arrangements of the remaining 8 letters, of which 3 o's are alike, =8!/3! = 6720

20. There are 3 copies each of 4 different books. In how many different ways can they be arranged on a shelf?

There are 3*4 = 12 books of which 3 are alike, 3 others alike, etc

$\frac{12!}{3!3!3!3!} = 369,600$

21.

a) In how many ways can 5 persons be seated at a round table?

b) In how many ways can 8 persons be seated at a round table if 2 particular persons must always sit together?

a) Let 1 of them be seated anywhere. Then the 4 persons remaining can be seated in 4! Ways. Therefore there are 4! = 24 ways of arranging 5 persons in a circle.

b) Consider the two particular persons as one person. Since there are 2! ways of arranging 2 persons among themselves and 6! ways of arranging 7 persons in a circle, the required number of ways = 2! 6! = 2*720 = 1440 ways

22. In how many ways can 4 men and 4 women be seated at a round table if each woman is to be between two men?

Consider that the men are seated first. Then the men can be arranged in 3! ways, and the women in 4! ways. Required number of circular arrangements = 3!4! = 144

23. By stringing together 9 differently coloured beads, how many different bracelets can be made?

There are 8! arrangements of the beads on the bracelet, but half of these can be obtained from the other half simply by turning the bracelet over.

Therefore there $\frac{1}{2}(8!) = 20{,}160$ bracelets

24. How many different sets of 4 students can be chosen out of 17 qualified students to represent a school in a mathematics contest?

Number of sets = number of combinations of 4 out of 17 students = ${}_{17}C_4$= 2380 sets of 4 students

25. In how many ways can 5 styles be selected out of 8 styles?
Number of ways = number of combinations of 5 out of 8 styles

$${}_8C_5 = \frac{8*7*6}{3*2*1} = 56 \text{ ways}$$

26. In how many ways can 12 books be divided between Cathy and David so that one may get 9 and the other 3 books?

In each separation of 12 books into 9 and 3, Cathy may get the 9 and David the 3, or Cathy may get the 3 and David the 9.
Therefore the number of ways = 2 * 12C9 = 440 ways

27. Determine the number of different triangles which can be formed by joining the six vertices of a hexagon, the vertices of each triangle being on the hexagon

Number of triangles = number of combinations of 3 out of 6 points.
=6C3=20 triangles

28. How many angles less than 180° are formed by 12 straight 1ines which terminate in a point, if no two of them are in the same straight line?

Number of angles = number of combinations of 2 out of 12 lines
= 12C2 = 66 angles

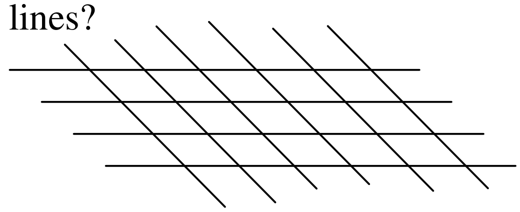
29. How many diagonals has an octagon?

Lines formed = number of combinations of 2 out of 8 corners (points)

$${}_8C_2 = \frac{8*7}{2} = 28$$

Since 8 of these 28 lines are the sides of the octagon, the number of diagonals =20

30. How many parallelograms are formed by a set of 4 parallel lines intersecting another set of 7 parallel lines?



Each combination of 2 lines out of 4 can intersect each combination of 2 lines out of 7 to form a parallelogram
Number of parallelograms = ${}_4C_2 * {}_7C_2$= 6*21 = 126 parallelograms

31. There are 10 points in a plane. No three of these points are in a straight line (not co-linear) , except 4 points which are all in the same straight line (co-linear). How many straight lines can be formed by joining the 10 points?

Number of lines formed if no 3 of the 10 points were in a straight line
$= {}_{10}C_2 = 45$
Number of lines formed by 4 points, no 3 of which-are co-linear = ${}_4C_2 = 6$
Since the 4 points are co-linear, they form 1 line instead of 6 lines.
Number of lines = 45 - 6 + 1 = 40 lines

32. In how many ways can 3 men be selected out of 15 men
a) if 1 of the men is to be included in every selection,
b) if 2 of the men are to be excluded from every selection,
c) if 1 is always included and 2 are always excluded?

a) Since 1 man is always included, we must select only 2 out of 14 men.
Therefore the number of ways = ${}_{14}C_2 = 91$ ways
b) Since 2 men are always excluded, we must select 3 out of 13 men
${}_{13}C_3 = 286$ ways
c) Number of ways = ${}_{15-1-2}C_{3-1} = {}_{12}C_2 = 66$ ways.

33. An organization has 25 members, 4 of whom are doctors, In how many ways can a committee of 3 members be selected so as to include at least 1 doctor?

Total number of ways in which 3 can be selected out of 25 = ${}_{25}C_3$.
Number of ways in which 3 can be selected so that no doctor is included = ${}_{25-4}C_3 = {}_{21}C_3$
Then the number of ways in which 3 members can be selected so that at least 1 doctor is ${}_{25}C_3 - {}_{21}C_3 = \frac{25*24*23}{3!} - \frac{21*20*19}{3!} = 970$ ways

34. From 6 chemists and 5 biologists, a committee of 7 is to be chosen so as to include 4 chemists. In how many ways can this be done?

Each selection of 4 out of 6 chemists can be associated with each selection of 3 out of 5 biologists. Therefore the number of ways = ${}_6C_4 * {}_5C_3 = 15*10$ = 150 ways.

35. Given 8 different consonants and 4 different vowels, how many 5-letter words can be formed, each word consisting of 3 different consonants and 2 different vowels?

The 3 different consonants can be selected in ${}_8C_3$ ways, the 2 different vowels in ${}_4C_2$ ways, and the 5 different letters (3 consonants, 2 vowels) can be arranged among themselves in ${}_5P_5 = 5!$ ways
Therefore the number of words = ${}_8C_3 * {}_4C_2 * 5! = 40,320$

36. From 7 capitals, 3 vowels and 5 consonants, how many words of 4 letters each can be formed if each word begins with a capital and contains at least 1 vowel, all the letters of each word being different?

The first letter, or capital, may be selected in 7 ways
The remaining 3 letters may be
a) 1 vowel and 2 consonants, which may be selected in ${}_3C_1 * {}_5C_2$ ways
b) 2 vowels and 1 consonant, which may be selected in ${}_3C_2 * {}_5C_1$ ways
c) 3 vowels, which may be selected in ${}_3C_3 = 1$ way
Each of these selections of 3 letters may be arranged among themselves in ${}_3P_3$ arrangements.
Therefore the number of words = $7*3!({}_3C_1 * {}_5C_2 + {}_3C_2 * {}_5C_1 + 1)$
$= 7*6(3*10 + 3*5 + 1) = 1932$ words.

37. A has 3 maps and B has 9 maps. Determine the number of ways in which they can exchange maps if each keeps his initial number of maps.

A can exchange 1 map with B in ${}_3C_1 * {}_9C_1 = 3*9 = 27$ ways.
A can exchange 2 maps with B in ${}_3C_2 * {}_9C_2 = 3*36 = 108$ ways.
A can exchange 3 maps with Bin ${}_3C_3 * {}_9C_3 = 1*84 = 84$ ways.
Total number of ways = 27 + 108 + 84 = 219 ways.

Or

Another Method: Consider that A and B put their maps together. Then the problem is to find the number of ways A can select 3 maps out of 12 not including that A selects his original 3 maps
Therefore, ${}_{12}C_3 - 1 = 219$ ways

38.

a) In how many ways can 12 books be divided among 3 students so that each receives 4 books?
b) In how many ways can 12 books be divided into 3 groups of 4 each?

a) The first student can select 4 out of 12 books in ${}_{12}C_4$ ways.
The second student can select 4 of the remaining 8 books in ${}_{8}C_4$ ways
The third student can select 4 of the remaining 4 books in 1 way
Number of ways ${}_{12}C_4 * {}_{8}C_4 * {}_{4}C_4 = 34{,}650$ ways
b) The 3 groups could be distributed among the students in 3! = 6 ways
Therefore the number of groups = $\frac{34{,}650}{3!}$ = 5775 groups

39. In how many ways can a person choose 1 or more of 4 electrical appliances?

Each appliance may be dealt with in 2 ways, as it can be chosen or not chosen. Since each of the 2 ways of dealing with an appliance is associated with 2 ways of dealing with each of the other appliances, the number of ways of dealing with the 4 appliances $=2*2*2*2 = 2^4$ ways. But 2^4 ways includes the case in which no appliance is chosen. Therefore the required number of ways $= 2^4 - 1 = 16 - 1 = 15$ ways.
OR
Another Method: The appliances may be chosen singly, in twos, etc,
Therefore the required number of ways $= {}_{4}C_1 + {}_{4}C_2 + {}_{4}C_3 + {}_{4}C_4 = 4 + 6 + 4 + 1 = 15$ ways

40. How many different sums of money can be drawn from a wallet containing one bill each of 1, 2, 5, 10, 20 and 50 dollars?
Number of sums $= 2^6 - 1 = 63$ sums

41. In how many ways can 2 or more ties be selected out of 8 ties?

One or more ties may be selected in $(2^8 - 1)$ ways. But since 2 or more must be chosen, the required number of ways $= 2^8 - 1 - 8 = 247$ ways
Or
Another Method: 2, 3 ,4, 5, 6, 7, or 8 ties maybe selected in
${}_{8}C_2 + {}_{8}C_3 + {}_{8}C_4 + {}_{8}C_5 + {}_{8}C_6 + {}_{8}C_7 + {}_{8}C_8$
$= 28 + 56 + 70 + 56 + 28 + 8 + 1 = 247$ ways

42. There are available 5 different green dyes, 4 different blue dyes, and 3 different red dyes. How many selections of dyes can be made, taking at least 1 green and 1 blue dye?

The green dyes can be chosen in $(2^5 - 1)$ ways, the blue dyes in $(2^4 - 1)$ ways, and the red dyes in 2_3 ways
Number of selections = $(2^5 - 1)(2^4 - 1)(2^3) = 31*15*8 = 3720$ selections

SUPPLEMENTARY PROBLEMS

PERMUTATIONS

50. Evaluate: ${}_{16}P_3$, ${}_{7}P_4$ ${}_{5}P_5$, ${}_{12}P_1$

51. In how many ways can six people be seated on a bench?

52. With four signal flags of different colours, how many different signals can be made by displaying two flags one above the other?

53. With six signal flags of different colours, how many different signals can be made by displaying three flags one above the other?

54. In how many ways can a club consisting of 12 members choose a president, a secretary and a treasurer?

55. If no two books are alike, in how-many ways can 2 red, 3 green and 4 blue books be arranged on a shelf so that all the books of the same colour are together?

56. There are 4 hooks on a wall. In how many ways can 3 coats be hung on them, one coat on a hook?

57. How many two digit numbers can be formed with the digits 0.3,5,7 if no repetition in any of the numbers is allowed?

58. How many even numbers of two different digits can be formed from the digits 3,4,5,6,8?

59. How many three digit numbers can be formed from the digits 1,2.3,4,5 if no digit is repeated in any number?

60. How many numbers of three digits each can be written with the digits 1,2,...,9 if no digit is repeated in any number?

61. How many three digit numbers can be formed from the digits 3,4,5,6,7 if digits are allowed to be repeated?

62. How many odd numbers of three digits each can be formed without the repetition of any digit in a number, from the digits a) 1,2,3,4, b) 1,2,4,6,8?

63. How many even numbers of four different digits each can be formed from the digits 3,5,6,7,9?

64, How many different numbers of 5 digits each can be formed from the digits 2,3,5,7,9 if no digit is repeated?

65. How many integers are there between 100 and 1000in which no digit is repeated?

66. How many integers greater than 300 and less than 1000 can be made with the digits 1,2,3,4,5 if no digit is repeated in any number?

67. How many numbers between 100 and 1000 can be written with the digits 0,1,2,3,4 if no digit is repeated in any number?

68. How many four digit numbers greater than 2000 can be formed with the digits 1,2,3,4 if repetitions

a) are not allowed,

b) are allowed?

69. How many of the arrangements of the letters of the word logarithm begin with a vowel and end with a consonant?

70. In a telephone system four different letters P,R,S,T and the four digits 3,5,7,8 are used. Find the maximum number of "telephone numbers" the system can have if each consists of a letter followed by a four digit number in which the digits may be repeated.

71. In how many ways can 3 girls and 3 boys be seated in a row, if no two girls and no two boys are to occupy adjacent seats?

72. How many telegraphic characters could be made by using three dots and two dashes in each character?

73. In how many ways can three dice fall?

74. How many fraternities can be named with the 24 letters of the Greek alphabet if each has three letters and none is repeated in any name?

75. How many signals can be shown with 8 flags of which 2 are red, 3 white and 3 blue, if they are all strung up on a vertical pole at once?

76. In how many ways can 4 men and 4 women sit at a round table so that no two men are adjacent?

77. How many different arrangements are possible with the factors of the term a2b~c~ written at full length?

78. In how many ways can 9 different prizes be awarded to two students so that one receives 3 and the other 6?

79. How many different radio stations can be named with 3 different letters of the alphabet? How many with 4 different letters in which W must come first?

COMBINATIONS

80. In each case find n:

a) $4*{}_{n}C_{2} = {}_{n+2}C_{3}$ b) ${}_{n+2}C_{n} = 45$ c) ${}_{n}C_{12} = {}_{n}C_{8}$

81. If $5*{}_{n}P_{s} = 24*{}_{n}C_{4}$, find n.

82. Evaluate:

a) ${}_{7}C_{7}$ b) ${}_{5}C_{3}$, c) ${}_{7}C_{2}$,
d) ${}_{7}C_{5}$, e) ${}_{7}C_{6}$, f) ${}_{8}C_{7}$
g) ${}_{8}C_{5}$, h) ${}_{100}C_{98}$

83. How many straight lines are determined by a) 6, b) n points, no three of which lie in the same straight line?

84. How many chords are determined by seven points on a circle? ,

85. A student is allowed to choose 5 questions out of 9. In how many ways can he choose them?

86. How many different sums of money can be formed by taking two of the following: a cent, a nickel, a dime, a quarter, a half-dollar?

87. How many different sums of money can be formed from the coins of Problem 86?

88. A baseball league is made up of 6 teams. If each team is to play each of the other teams a) twice, b) three times, how many games will be played?

89. How many different committees of two men and one woman can be formed from a) 7 men and 4 women, b) 5 men and 3 women?

90. In how many ways can 5 colours be selected out of 8 different colours including red, blue and green

a) if blue and green are always to be included,
b) if red is always excluded,
c) if red and blue are always included but green excluded?

91. From 5 physicists, 4 chemists and 3 mathematicians a committee of 6 is to be chosen so as to include 3 physicists, 2 chemists and 1 mathematician. In how many ways can this be done?

92. In Problem 91, in how many ways can the committee of 6 be chosen so that

a) 2 members of the committee are mathematicians,
b) at least 3 members of the committee are physicists?

93. How many words of 2 vowels and 3 consonants may be formed (considering any set a word) from the letters of the word a) stenographic, b) facetious?

94. In how many ways can a picture be collared if 7 different colours are available for use?

95. In how many ways can 8 women form a committee if at least 3 women are to be on the committee.

96. A box contains 7 red cards, 6 white cards and 4 blue cards. How many selections of three cards can be made so that

a) all three are red,
b) none are red?

97. How many baseball teams of nine nines can be chosen from 13 candidates if A, B, C, D are the only candidates for two positions and can play no other position?

98. How many different committees including 3 Conservatives and 2 Liberals can be chosen from 8 Conservatives and 10 Liberals?

99. At a meeting,. after everyone had shaken hands once with everyone else, it was found that 45 handshakes were exchanged. How many were at the meeting?

100. Find the number of
a) combinations, and
b) permutations
of four letters each that can be made from the letters of the word TENNESSEE.

Answers

50.	3360, 840, 120, 12			51.	720	52.	12
53.	120	54.	1320	55.	1728	56.	24
57.	9	58.	12	59.	60	60.	504
61.	125	62.	a) 12, b) 12	63.	24	64.	120
65.	648	66.	36	67.	48	68.	a) 18, b) 192
69.	90,720	70.	1024	71.	72	72.	10
73.	216	74.	12,144	75.	560	76.	144
77.	1260	78.	168	79.	15,600; 13,800	80.	a) 2, 7 b) 8 c) 20
81.	8	82.	a) 1 , b) 10 c) 21, d) 21 e) 7, f) 8 g) 56, h) 4950	83.	a) 15 b) $\frac{n(n-1)}{2}$	84.	21

85.	126	86.	10	87.	31		
88.	a) 30, b) 45	89.	a) 84, b) 30	90.	a) 20, b) 21, c) 10	91.	180
92.	a) 378, b) 462	93.	a) 40,320, b)4800	94.	127	95.	219
96.	a) 35, b) 120	97.	216	98.	3360	99.	10
100 .	a) 17, b) 163						