



COUNTING AND PERMUTATIONS FUN (some pretty advanced questions!)

1. Normal Manitoba Licence plates have the form: LLL ###, where each L is an English alphabet letter and each # is any digit, 0-9. [No vanity or custom plates]. For the following separate considerations:

- a. how many possible license plates are there?
- b. how many license plates are there if no number digits on the plate are the same?
- c. how many license plates have **some** of the three number digits the same? (hint: some is the complement of none)
- d. how many licence plates have all three of the number digits the same?
- e. what is the probability that the next car you see (with normal plates) has **some** digits the same?
- f. what is the **probability** the next car you see has all three digits the same?
- g. what is the **probability** the next car you see has exactly two digits the same?

2. **Password.** A website requires users to set up an account that is password protected. If the password format is four letters followed by a single digit number, how many different passwords are possible if:

- a. repeats are allowed
- b. repeats are not allowed

3. **Grad Photos.** Consider the following separate questions regarding arrangements of graduates.

- a. How many ways can we put five grads in a line for a photo?
- b. If there are three boys and two girls how many ways can we put the five grads in a line if it must alternate boy, girl
- c. how many ways can we line up the grads if the two taller boys must be at either end?

4. How many ways can we line up Albert, Bruce, Carla, Dave, Elijah, and Felix (under the separate considerations below). Consider the following separate arrangements:

- a. ways to all line up at the door?
- b. ways to line up at the door if Felix is not allowed at the front?
- c. ways to line up at the door if neither Albert nor Carla is allowed at either end?
- d. ways to line up at the door at the door if Carla and Dave must be together!
- e. ways to line up at the door at the door if Carla and Dave must NOT be together?
- f. what is the probability if all six students randomly line up facing the door that Dave is immediately behind Albert.
- g. what is the probability that Carla is somewhere in line ahead of Albert (hint: will need to add a few different arrangements)

5. Jessica has six family portraits in a box and has room to display some of them in a line on a shelf. Consider the following separate arrangements:
- How many ways can she line up three of the six pictures if aunt Mathilde must be in the centre?
 - How many ways can she line up three of the six pics if Uncle Ed and Aunt Mathilde **must be** up on the shelf side-by-side?
 - if Jessica randomly selects three pics out of the box and arranges them up on the shelf what is the probability that cousin Jen is on the shelf?
 - how many ways can Jessica arrange the three randomly selected pics if cousin Debbie and Debbie's ex, Mike, **cannot** be up on the shelf together, side by side?
 - If Jessica randomly selects three pics and sets the three pics on the shelf, what is the **probability** that Debbie and Jen are on the shelf?

6. Consider the digits 0, 2, 4, 5, 8, 9. Consider the following separate arrangements:
- a) If **repetitions (replacements) are allowed**, how many 3-digit numbers can be formed? (Note: we seldom start a number with a zero as the first digit)
 - b) If **repetitions (replacements) are allowed**, how many 3-digit multiples of 10 can be formed?
 - c) If **repetitions (replacements) are allowed**, how many 3-digit numbers can be formed that are larger than 700 and are also even?
 - d) how many 3-digit numbers that have **no repeating digits** can be formed?

7. At a coffee shop, they offer 10 different drinks and four different desserts.

- a. How many ways can two people order the same drink but a different dessert?
- b. **New**. Determine the probability that two different people randomly end up with the same drink and the same dessert! “Hey, cool! What are the chances we ordered the exact same thing!”

Answers (pretty sure these are correct! lol):

1. a. 17,576,000 b. 12,654,720 c. 4,921,280
d. 175,760 e. 28.00% f. 1.00% g. 27%

2. a. 4,569,760 b. 3,588,000

3. a. 120 b. 12 c. 12

4. a. $6! = {}_6P_6 = 720$ b. 600 or $5 \cdot {}_5P_5$ c. 384
d. $5! \cdot 2$ or ${}_5P_5 \cdot 2 = 240$
e. $720 - 240 = 480$ f. $1/6$ or 16.67%
g. a tough one!!! $120 + 96 + 72 + 48 + 24 = 360$ ways

5.

6. a. 180 b. 36 c. $8 + 12 = 20$ ways d. ${}_6P_3 = 120$

7. 160