GRADE 11 ESSENTIAL
UNIT H – DESIGN MODELING
UNIT NOTES

Name:____________
Date: ____________

You have studied lots of flat two-dimensional (2D) and 3D objects. You have learned how to draw them and name them. You have learned how to make triangles a similar shape using proportions.

In this unit we will learn how to work some more with Three-Dimensional (3D) shapes. We will learn how to:

- make scale models,
- how to draw in three dimensions,
- how to establish what an object looks like when we look at it from different perspectives, and
- ~~how to view its different components~~.

Measurement Conversions

Recall from Grade 10 how to convert measures of length between different units and systems. Some basic conversions factors for metric (SI system) and former Imperial (now American 'customary') units of measure are given here:

Conversions SI Metric for Length and Distance		
1 kilometre [km]	=	1,000 metres [m]
1 meter [m]	=	100 centimetres [cm]
1 centimetre [cm]	=	10 millimetres [mm]
1 metre [m]	=	1,000 millimetres [mm]

Conversions Non-SI (Imperial) for Length		
1 mile [mi]	=	1,760 yards [yd]
1 yard [yd]	=	3 feet [ft]
1 mile [mi]	=	5280 ft
1 foot [ft]	=	12 inches [in]
1 yard [yd]	=	36 inches [in]

Conversions SI to Non-SI Length		
1 metre [m]	=	3.2808 feet [ft]
1 metre [m]	=	39.37 inches [in]
1 kilometre [km]	=	0.6214 miles [mi]
1 mile [mi]	=	1.609 km
1 inch [in]	=	2.54 cm

There are two basic ways to convert: **Proportions Method** and **Unit Factor Method**

Example Proportion Method

Convert 3.4 metres [m] of length into centimetres [cm]of length.
The question is what number of cm are there for 3.4 m?

$\frac{x\ cm}{3.4\ m}$; there is 'some' *x* number of cm for 3.4 m

But we know that there is 100 cm for every metre; $\frac{100\ cm}{1\ m}$

Therefore $\frac{100\ cm}{1\ m} = \frac{x\ cm}{3.4\ m}$; 100 for every 1 is the same as 'what' for every 3.4?

Recall how to solve using the method of proportions and cross multiplication:

$$\frac{100\ cm}{1\ m} = \frac{x\ cm}{3.4\ m}$$

$3.4m \cdot 100m = 1m \cdot x\ cm$

$\frac{3.4m \cdot 100cm}{1m} = x\ cm$; Therefore $x = 340cm$

A length of 3.4 metres is same as 340cm

Practice Solving conversions by proportions.

Solve the following proportions statements:

a. $\frac{x\ cm}{52\ m} = \frac{100\ cm}{1\ m}$ 5,200 cm	b. $\frac{x\ cm}{7.3\ m} = \frac{100\ cm}{1\ m}$ 730 cm
c. $\frac{362\ cm}{x\ m} = \frac{100\ cm}{1\ m}$ 3.62 m	d. $\frac{x\ mm}{5.2\ m} = \frac{1{,}000\ mm}{1\ m}$ 5,200 mm

Notice how we included the units in the above examples. A rather important way to keep track of what you are doing!

Convert the following measurements as indicated by setting up your own proportion statement then solving.

7. 25 m = ______ cm	428 cm = _______ m
355 mm = _________ cm	5.75 km = ___________ m

5.75 km = ________ cm

**Tricky? This one will take a couple steps!*

Whenever you are doing measurements and conversions you should be picturing the actual sizes that are being represented by the measurement. If we are talking about a 2.5 metre length you should be picturing something the length of a folding table or something. If we are doing a problem involving something 8 mm long, you should be picturing a small eraser on the end of your pencil or an ear ring or something. Recall your 'referents' from Grade 10 Measurement.

CONVERSION USING UNIT FACTORS

A factor is something that multiplies another number. A unit means one. So in the **Unit Factor** method we will multiply by a factor of 'one' to convert.

Example: Convert 4.2 metres to a measure in cm.
We know that 100 cm is the same as 1 m. So they are 'one' and the same length! So we can convert units by multiplying by that as follows:

Object → $4.2m \cdot \left(\frac{100\ cm}{1\ m}\right)$ ← UNIT FACTOR

We are not changing the object's length, just the units that we use

$4.2\cancel{m} \cdot \frac{100\ cm}{1\ \cancel{m}}$

The metres 'cancel out' and the length is in the new unit of measure of cm

$4.2\cancel{m} \cdot \frac{100\ cm}{1\ \cancel{m}} = \boxed{420\ cm}$

4.2 m is the same length as 420 cm

The **Unit Factor** method is probably the preferred method, especially if you are going to lots of converting. Of course, knowing that it takes more little units to make some big units is useful too and much more usable!

Solve the following using the unit factor method. They are readily done mentally but show the work using the unit factor method regardless.

7.89 m = _________ cm 789 cm	789 cm = ________ m 7.89 m
4.2 km = ___________ m 4,200 km	137 cm = _________ mm 1,370 mm
1.83 km = ________ cm 183,000 cm	26 years = __________ minutes 13,665,600 min

More practice Converting

Complete the following table. (in your head mentally, since it is all just metric measures and multiples of 10)

1m	100 cm
2 m	________ cm
3 m	________ cm
4.2 m	________ cm
________ m	138 cm
________ m	733 cm

Notice going left to right you multiply by 100, going right to left you divide!

The proportion method and the unit conversion method work equally well when converting the between the old feet, yard, inches, miles that nobody except Americans use anymore.

The proportion method and the unit conversion method work equally well to convert between the old system of feet and inches and the proper metric system of metres and centimetres. (sort of)

Convert the following using a variety of methods and tables provided:

37 feet [ft] = _______ metres [m] 11.28	18 inches [in] = _______ cm 45.72
5 miles = _________ km 8.05	4 feet [ft] = _______ inches [in] 48 in
28 inches = ________ ft _____ in **2 ft 4 in**	16 m = ________ ft _____ in **52 ft 5 ¾ inches**
These last two are examples of why the '*old*' system was a bit silly. You would calculate how many whole feet you had then the remainder would be the inches and possible fractions of inches too! You had to be adept at fractions and know that an inch was 1/12th of a foot for example!	

USING SCALE

Often when we design something we want to have a scale model or diagram of the object. We have to scale it so that the model is 'similar' in shape, just a different size using different dimensions of length.

We had done lots of similar triangles when we studied trigonometry in Grades 10 and 11. We can now apply the idea of scale and scale representation to more aspects.

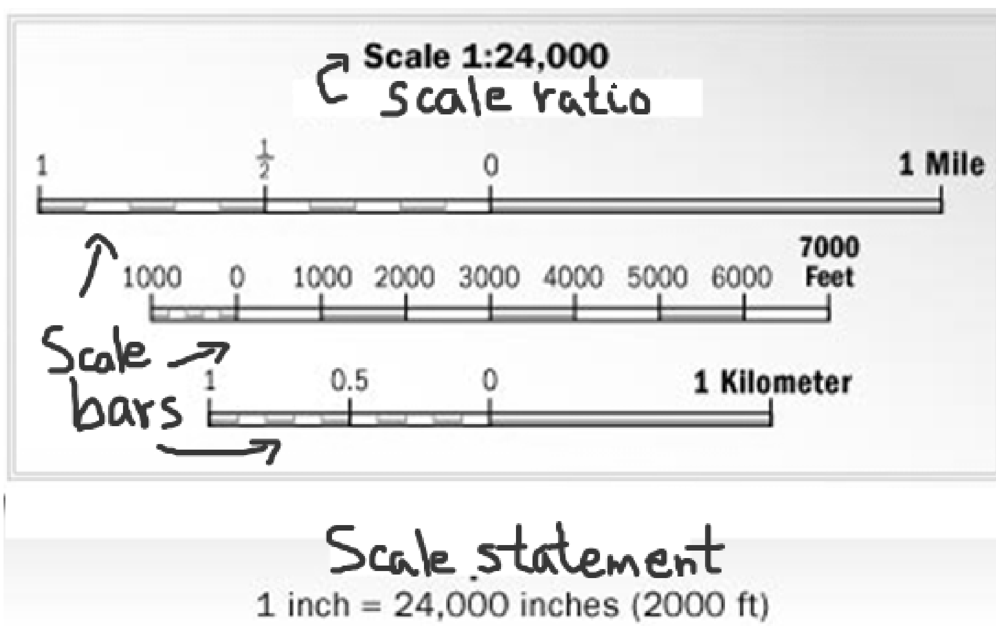
MAPS AND MAP SCALE

The most common usage of scale you are likely familiar with is on your maps App on your device. A map is a small representation of the real world. Scale can be represented by :

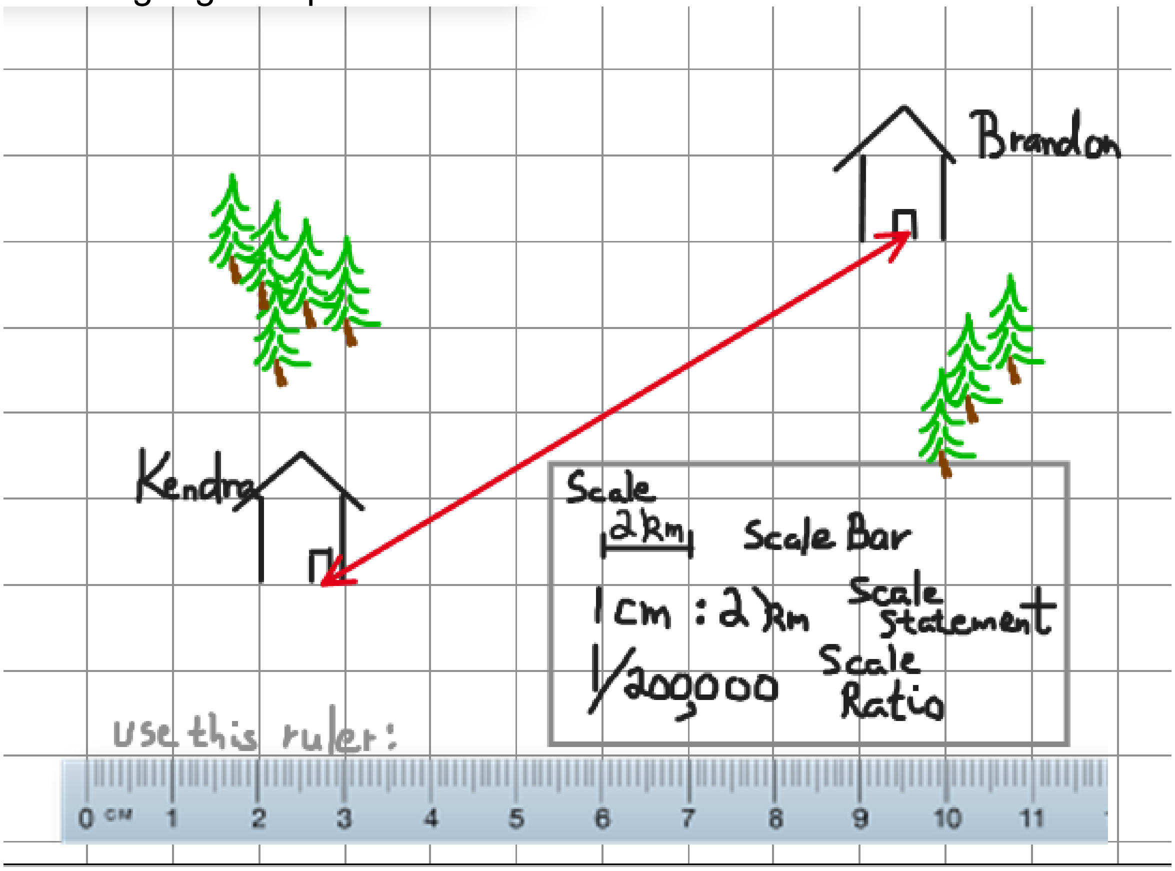
A pure ratio; example 1:50,000
A statement of scale: 1cm = 500 metres
Or a scale bar on the map.

Any map must always have a scale indicated otherwise it is pretty much useless.

Here is a typical way that scale is presented on a map (three methods):



Here is a 'google map':



How far is it from Kendra's to Brandon's house in the actual world?

Using the Scale bar is looks like it would be _______ scale bars.
And since each scale bar is equal to 2km;
the actual distance is: ______ km

Using the scale statement we have a conversion factor of 1cm = 2 km to apply. So using the ruler provided, the map distance is 8 cm, so the actual distance would be: __16__ km.

We use the scale ratio to indicate that the real world is 200,000 times bigger than the map. So 8 cm X 200,000 is 1,600,000 cm which is 16,000 m which is 16 km

**We have to use the ruler I put on the map since images get shrunk in making these notes*
***Notice also only the ground distance is 'to scale' otherwise the buildings and trees would be several km in height!*
**** You may have noticed we could have used the square grid and a bit of Pythagoras too!*

SCALE FACTOR FORMULA

The scale factor for maps and models is always expressed as:

$$Scale\ Factor = \frac{model\ (or\ map)\ distance}{actual\ distance}$$

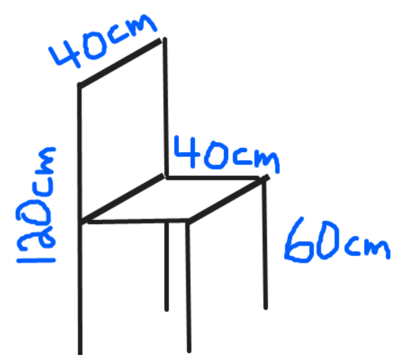
Or sometimes a ratio expressed this way with a colon (**:**)

Scale = model length **:** actual length

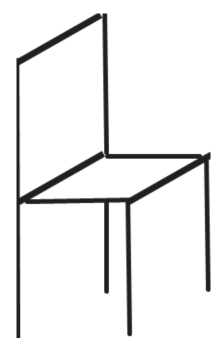
Units are **always** included if they are different units being compared

Below is a picture of a chair. It has actual dimensions of: height of entire back with legs 120 cm, height of seat 60 cm, and a square seat 40 cm on each edge. Calculate the dimensions of a 1/40 (1:40) scale model for a doll house.

Actual Chair dimensions:



Label the 1/40 Scale model dimensions from calculations in table below



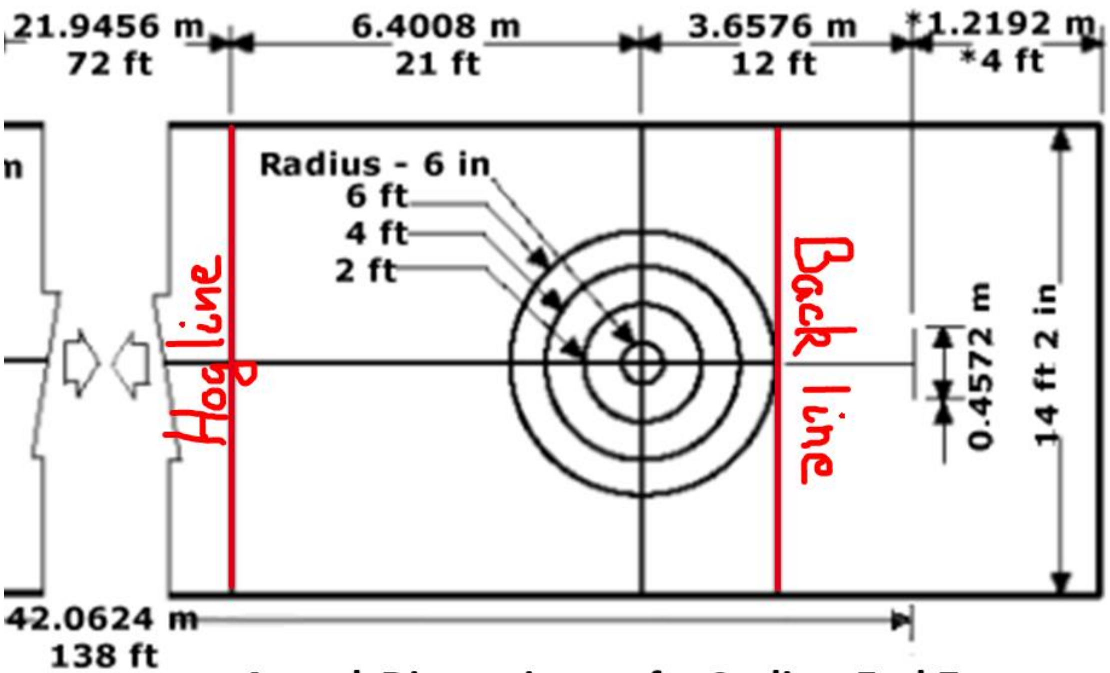
	Scale ratio	Scale factor	Back height	Front height	Seat width & length
model	**1**	**1 cm**			
actual	**40**	**40 cm**	**120 cm**	**60 cm**	**40 cm**

Example. Calculate height of the model chair along entire back:

$$\frac{model}{actual} = \frac{1}{40} = \frac{x}{120\ cm} \ ; \ x = \frac{1 \cdot 120cm}{40} = \boxed{3cm}$$

Calculate curling rink dimensions for a scale drawing:

Here are the actual dimensions of the end zone of a curling rink:



Actual Dimensions of a Curling End Zone

If we are to make a separate 1 cm = 1.5 ft scale drawing then determine the scale drawing sizes to the nearest mm (ie 0.1 cm) of:

a. the radii of the four concentric rings
6 in= _____ cm; 2 ft = ____ cm; 4 ft = _____ cm; 6 ft = _____ cm

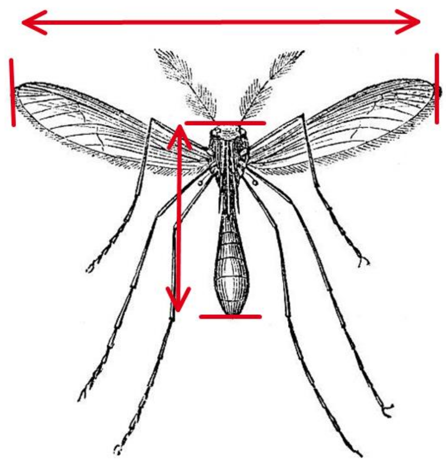
b. the distance hog line to back line. _______ cm

c. width of the curling sheet; 14 ft 2in = _______ cm

Ans: a. 0.3, 1.3, 2.7, 4 cm; b. 18 cm; c. 9.4 cm

ENLARGED (BLOW UP) MODELS

Occasionally we make the model or drawing larger. At left is an actual size mosquito, at right is the enlargement.



If the actual sizes of the real mosquito is: wing tip to wing tip 5 mm; tail to nose 3 mm.

Use a proper ruler to measure the enlarged picture above.

a. Determine the scale ratio of the model that you used to make the enlarged diagram ______ **:** _____

b. Determine a scale statement using 1 cm = ________ mm

Ans: a. 12:1 (approx) b. 1 cm = 0.85 mm (approx). Depending on how accurately you read the ruler.

Rounding and Significant Digits

It is generally sufficient to round scales to two *significant digits*. So for example if a result is 1**:**17,612 then rounding it to the two highest place values is generally sufficient: 1:18,000.

Convert Scale Ratios and Scale Statements

A ***ratio*** is a comparison of numbers with **no units** necessary. My income is half his income. My dog is 1.5 times taller than yours. The units in these comparisons were un-necessary. Whether you measured the income in $ or in ¢ the ratio is still the same; whether you measured and compared the two dogs in inches or in cm would not matter.

So a 1:50,000 scale map is the same as 1 cm : 50,000 cm **or** 1 inch for 50,000 inches or 1 foot for 50,000 feet.

Or the equivalent scale statements of 1 cm : 500 m
or possibly 1 cm = 0.5 km

Convert the following: (round to two significant digits)

a. Ratio: **1/25**
Statements: 1 cm = ____ cm; 1 mm: _____ mm; 1 cm = _____ inch

Example:

$1/25 \equiv 1cm/25cm \cong 1cm/9.84in$

b. Ratio: **1: 400**
Statements: 1 cm = ____ cm; 1 cm: _____ m; 1 cm = _____ inch

c. Ratio: **25 : 1** (enlargement, notice the model is bigger than the actual!)
Statements: 1 m = ____ cm; 1 ft: _____ in; 1 in = _____ mm

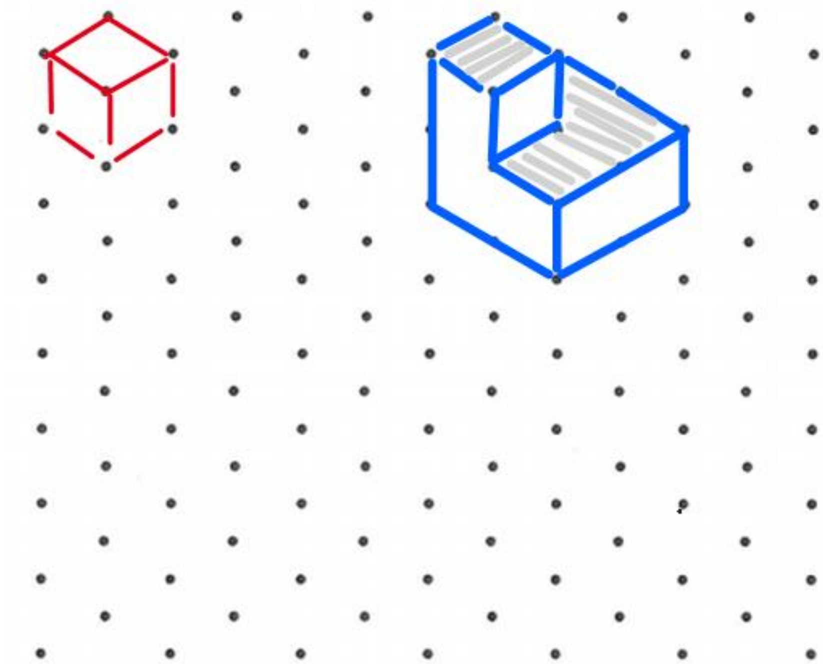
Ans: a. 25 cm, 250 mm, 9.8 in; b. 400 cm, 4 m, 160 in; c. 4 cm, 0.48 in, 1 mm

Drawing in Three Dimensions

Being able to draw in three dimensions is a handy skill if you are to design things. You probably already '*sketched*' lots of prisms and pyramids and cylinders in a previous unit of study.

A handy device to draw in three dimensions is Isometric Dot paper.
Below are two simple shapes drawn on isomeric dot paper.

You copy them beneath the shapes.



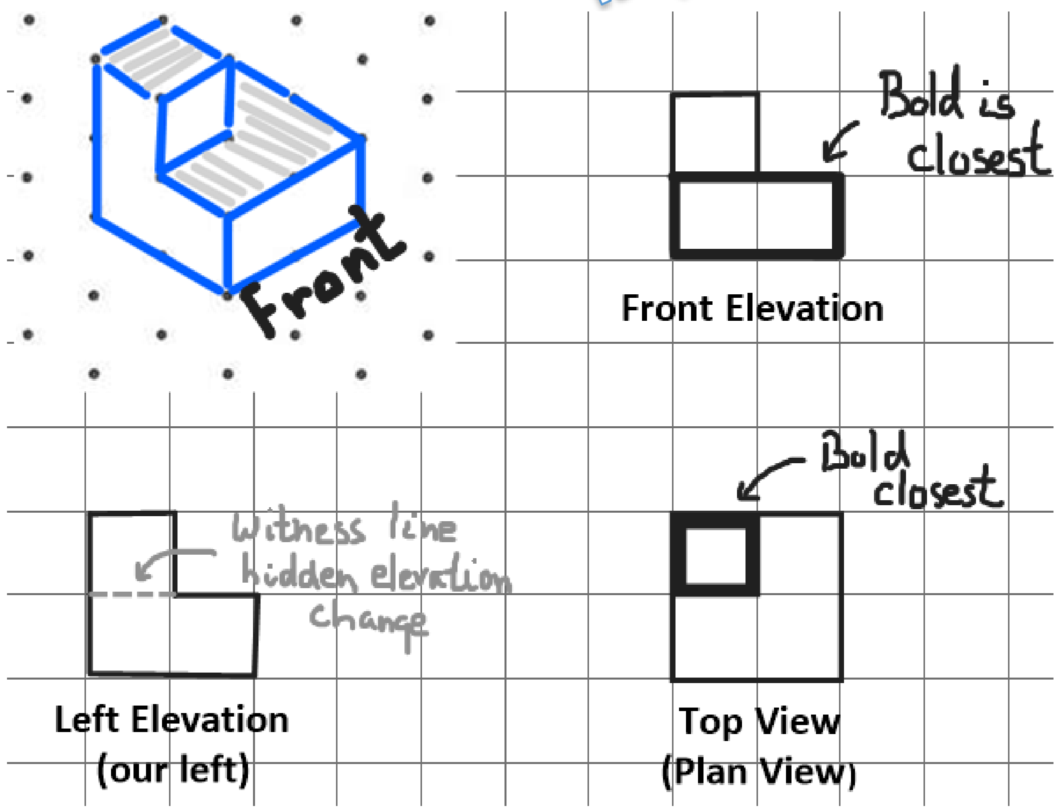
Shading the tops is often useful

Some isometric Dot Paper is enclosed at the end of these notes. Children love it too!

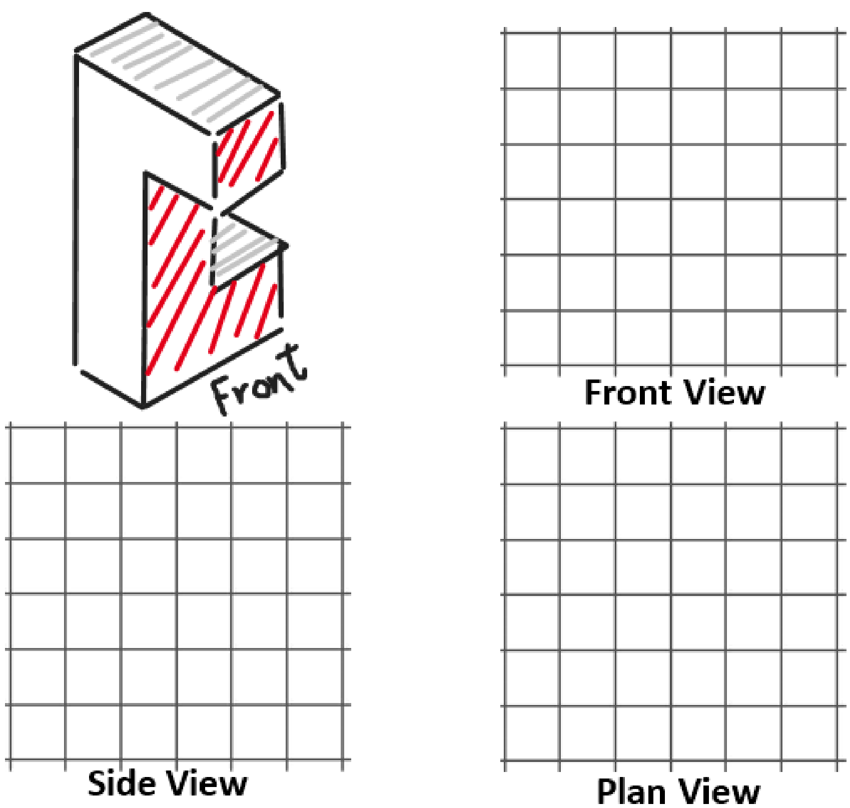
ORTHOGRAPHIC DRAWINGS

The definition of an orthographic projection is a two-dimensional drawing of a three-dimensional object, using two or more additional drawings to show additional views of the object.

‘Ortho’ means correct, ‘-graphic’ means draw. An orthodontist corrects your teeth.



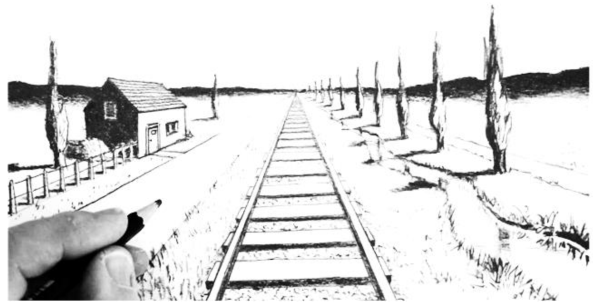
You try: (Some facings are shaded to help you see it better) (Hint: it is 4 blocks tall)



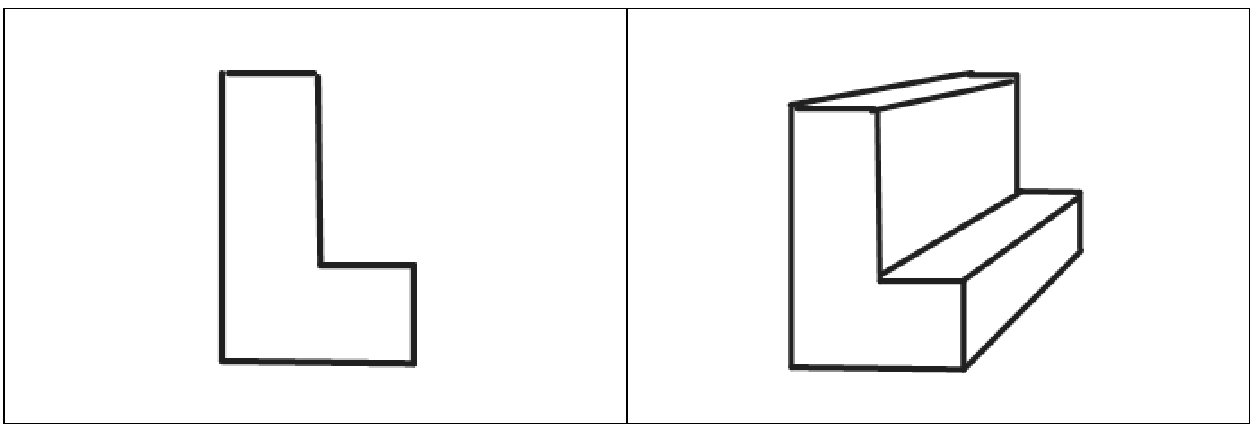
Drawing One-Point Perspective

You are familiar with one-point perspective →

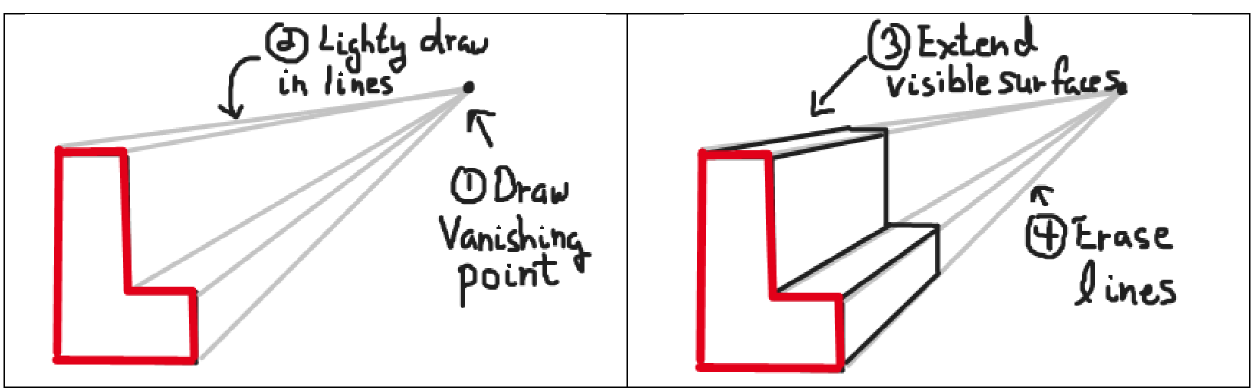
If looking at objects or scenes with very long lengths, the further portions look smaller! And even vanish into the distance.



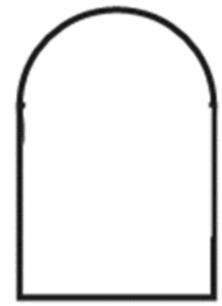
Example:
Take this simple block letter L and make it look more three-dimensional by extending one dimension off to a perceptible distance.



Here is the method:

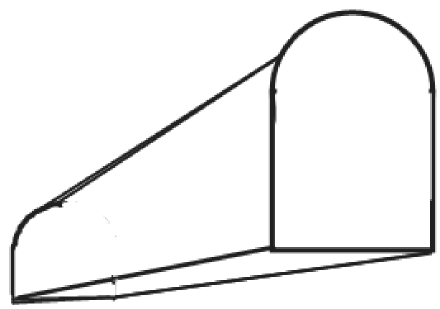


You try. Extend the shape off to a vanishing point to the lower left

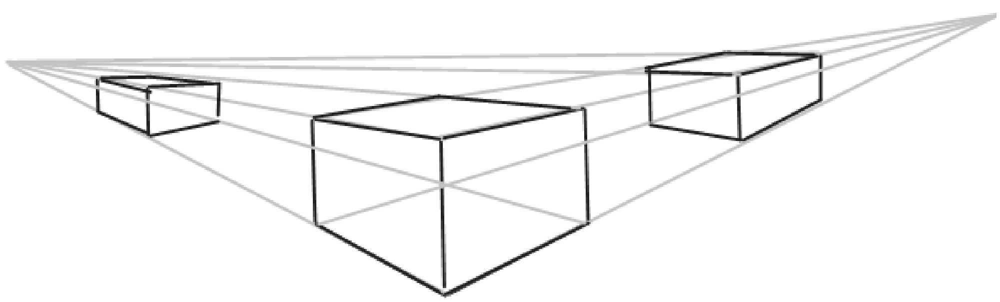


This one if difficult. Use a very sharp pencil, and a straight edge, draw *very light* guide lines, be ready to *erase lots*!

Here is one solution



Of course objects do not just disappear off to only one direction, they disappear in all directions. But you will learn that in Art Class!



Drawing Exploded Views

Omitted in these notes for now.