

**Grade 11 Essential
Unit F - Relations and Patterns
Work Book**

Part 2

SOLUTION

**GRADE 11 ESSENTIAL
UNIT F - RELATIONS AND PATTERNS
WORK BOOK**

Name: _____

Date: _____

1. A relationship between two different variables can readily be graphed to show any pattern. A pattern allows us to make predictions. Where we have been, where we are going.

Relations and patterns occur every day of your life, multiple times! Hopefully you notice them! Lots of books in your backpack, it is heavier. The bus is slow, it will take longer to get to school. Study better, better mark. What goes up, must come down.

Calculating Slope of a Line

The line has a slope; steep, shallow, flat, ...

Formulae for calculating slope :

$$\text{slope} \equiv m \equiv \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise up}}{\text{run right}}$$



every time you go
tobogganing with the family

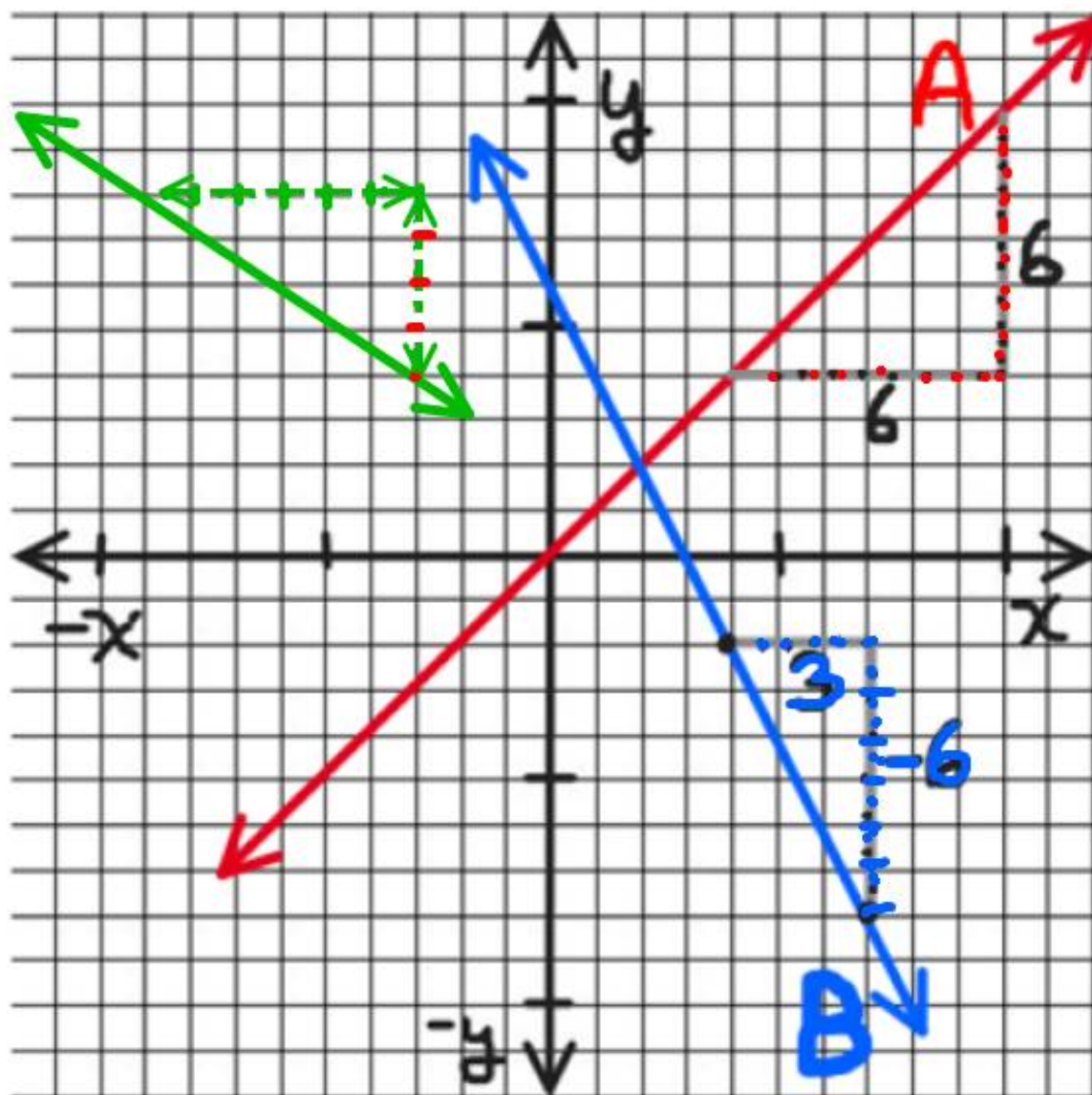
10. Determine the slope of lines A and B.

$$\text{slope}_A = \frac{\text{rise}}{\text{run right}} = \frac{6}{6} = 1$$

$$\text{slope}_B = \frac{\text{rise}}{\text{run right}} = \frac{-6}{3} = -2$$

You make one and calculate it!

$$\begin{aligned} \frac{\text{rise}}{\text{run}} &= \frac{-4}{+6} \text{ right} \\ &= -\frac{2}{3} \end{aligned}$$



slope is just a *ratio*; comparison of two numbers

11. Determine the slope of lines C and D.

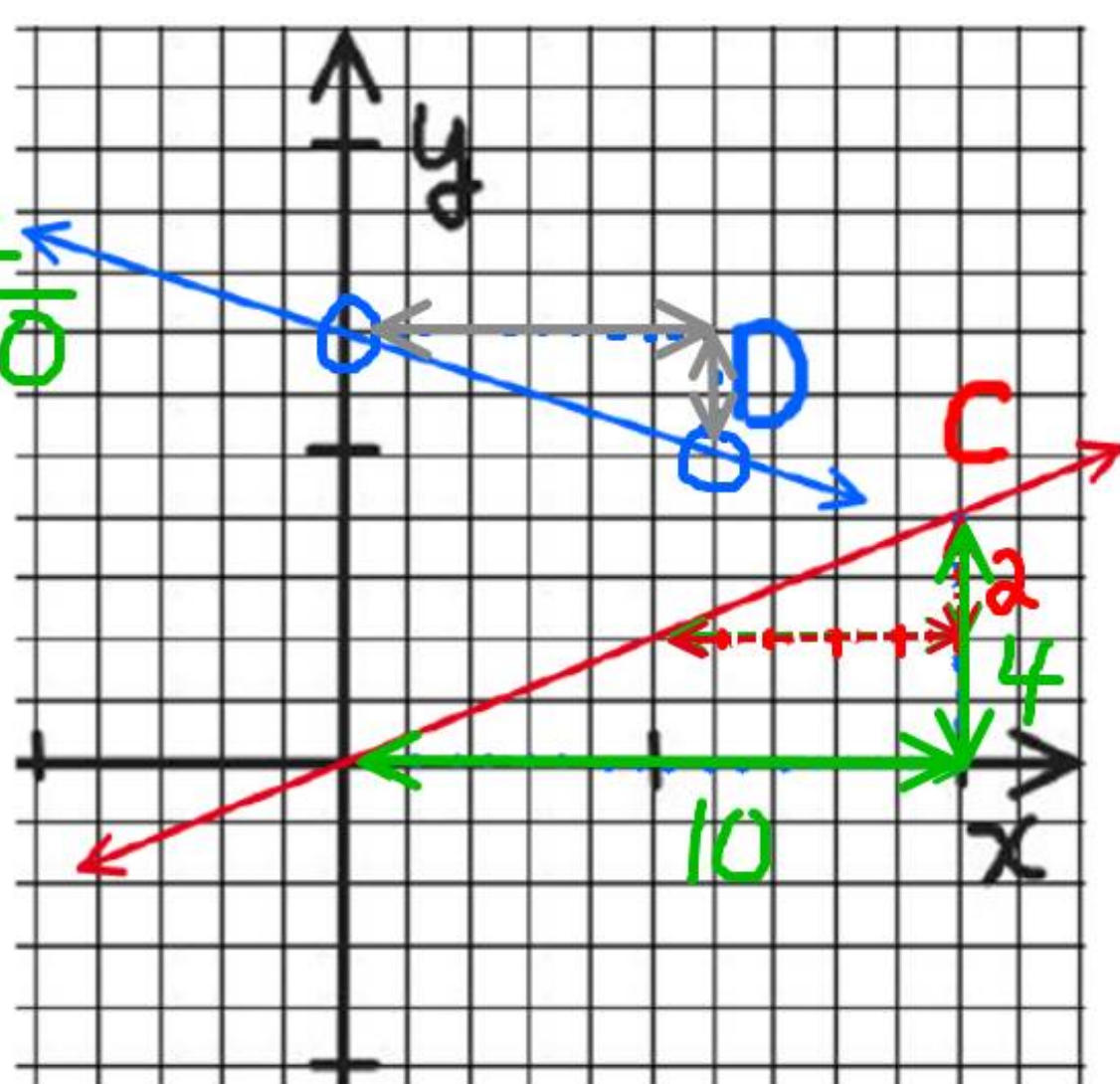
0.4

$$\text{Slope}_C = \frac{\text{rise}}{\text{run right}} = \frac{2}{5} = \frac{4}{10}$$

$$\text{Slope}_D = \frac{\text{rise}}{\text{run right}} = \frac{-2}{+6} = -\frac{1}{3}$$

a negative rise is
a drop

Notice how a fraction is a
perfect way to express a
slope!



fractions and ratios should
always be simplified!

~~4/10~~ Frac

~~-2/6~~ Frac

2/5

-1/3

15. You graph and label a line (any line) that has a slope of:

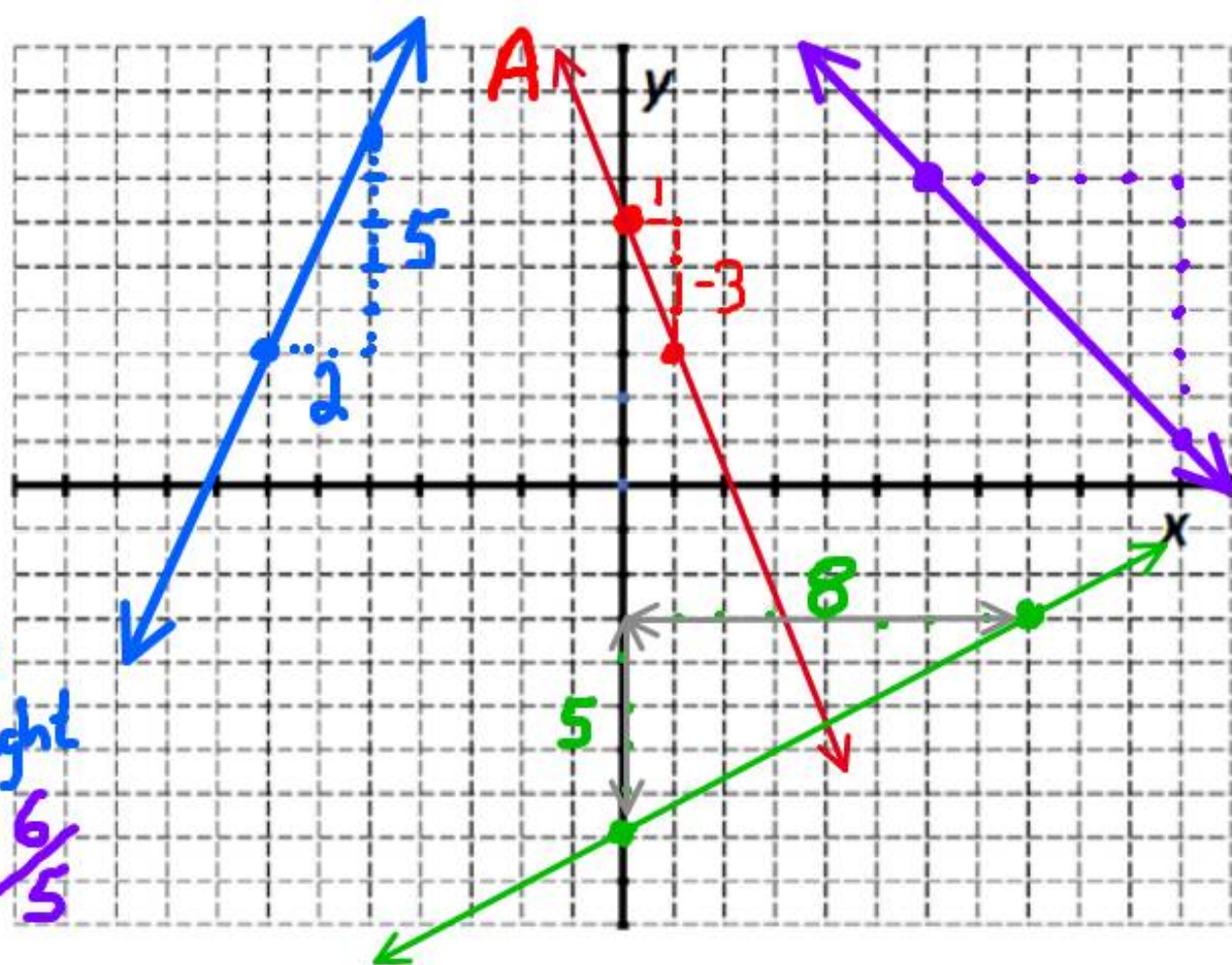
A. $m = -3 = \frac{-3}{1}$

B. $m = +5/8 = \frac{\text{up } 5}{\text{right } 8}$

C. $m = 2.5 = \frac{2.5}{1} = \frac{5 \text{ up}}{2 \text{ right}}$

D. $m = -1.2 = \frac{-1.2}{1} = \frac{-12}{10} = \frac{-6}{5}$

An infinite number of correct answers



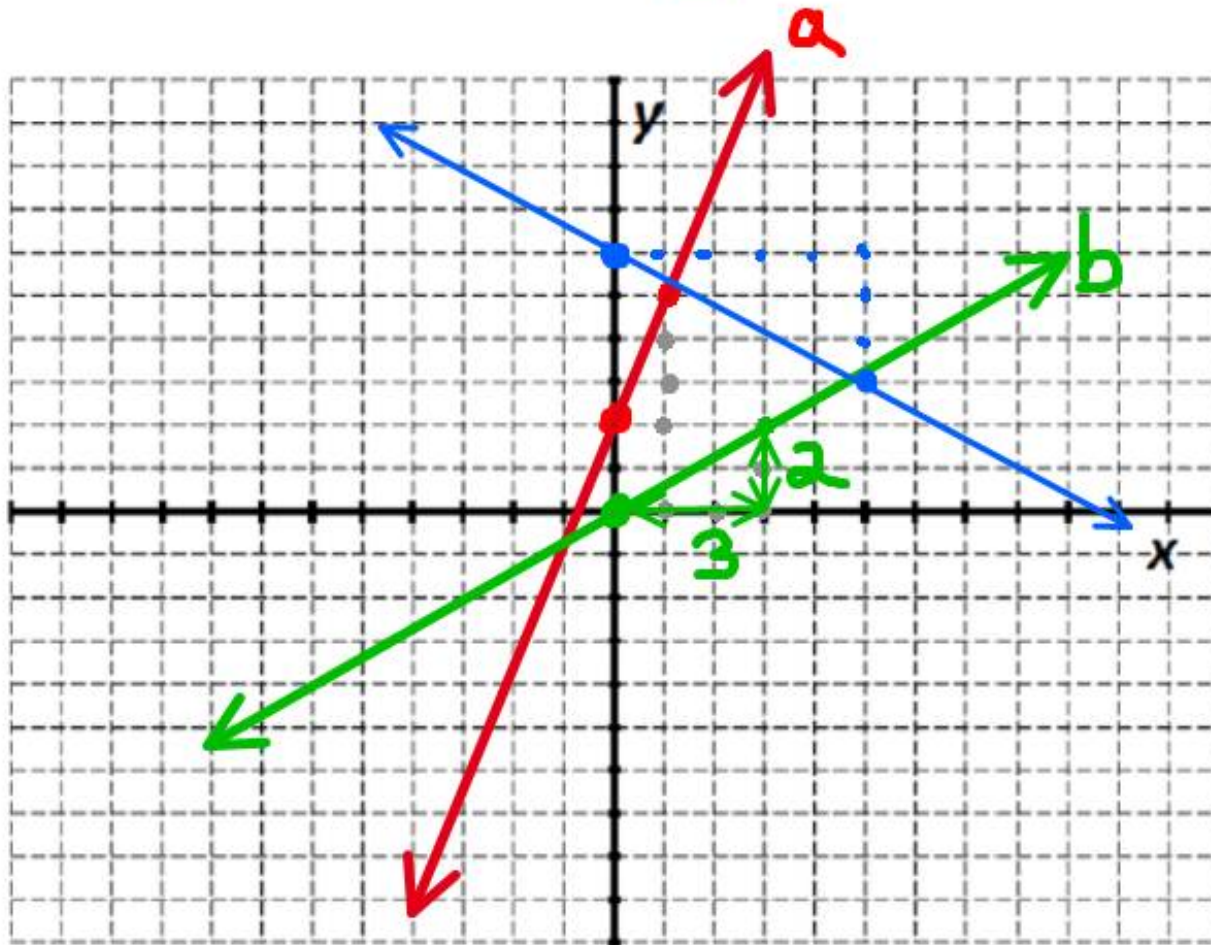
start anywhere

lots of triangles!

So much easier to count whole lines, go to intersections!

16. Line through a given point

- a. You graph a line that has a slope of 3 and goes through the point $(0, 2)$ ✓
- b. You graph a line that has a slope of $\frac{2}{3}$ and goes through the point $(0, 0)$ ✓
- c. You graph a line that has a slope of $-\frac{3}{5}$ and goes through the point $(0, 6)$



$$\text{slope} = 3 = \frac{3}{1}$$

$$\text{slope}_b = \frac{2 \text{ rise}}{3 \text{ right}}$$

$$\begin{aligned} \text{slope}_c &= -\frac{3}{5} \\ &= \frac{\text{drop } 3}{5 \text{ right}} \end{aligned}$$

Calculating Slope without a graph; using just two points

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

17. Δ is the Greek letter 'delta'. We use it to mean 'difference'. Δx means the change in x , so if x starts at 4 and goes to 9, we say the change of x , Δx , is $9 - 4$. So the Δx is 5. It went from the first x , x_1 , to the second x , x_2 , so the difference is, $\Delta x = x_2 - x_1 = 9 - 4 = 5$.

$$\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

the second y; where you finish

the first y; where you start

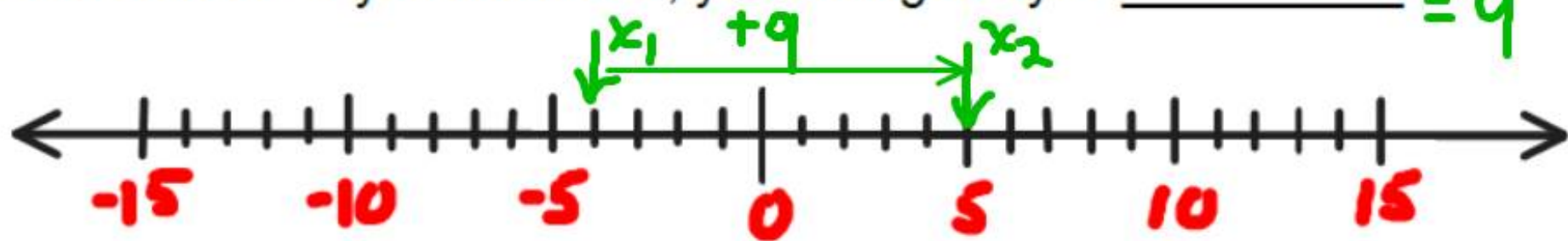
the second x; where you finish

the first x; where you start

duh!
if you want to know how you changed, you find the difference from where you are to where you started

18. Finding a change, Δ , in a value.

- a. You start at $x_1 = 7$ you finish at $x_2 = 12$, you changed by? $\frac{x_2 - x_1 = 5}{+5}$
- b. You start at $x_1 = -4$ you finish at $x_2 = 5$; you changed by? : $\frac{x_2 - x_1 = 5 - (-4)}{= 9}$



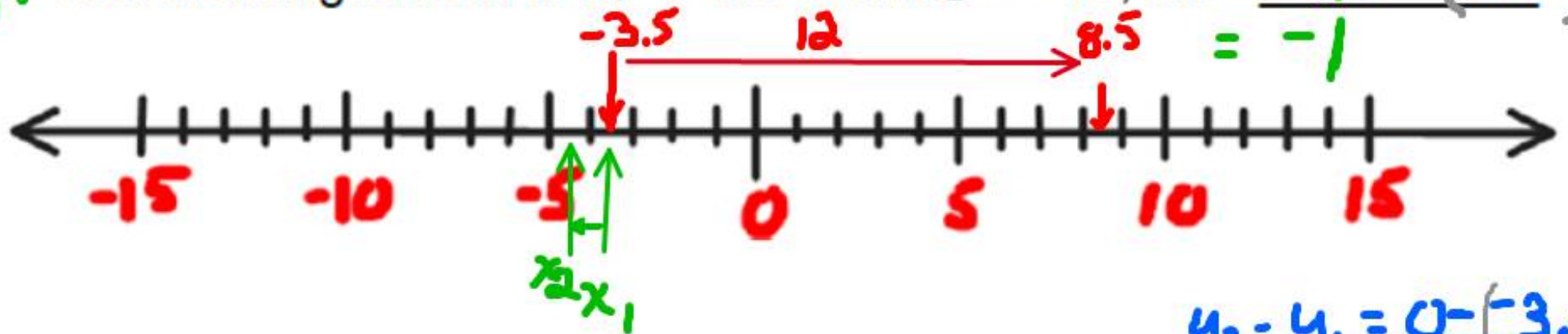
- c. You start at 0 you finish at 10; what is your change in value, Δ : $\frac{10 - 0}{= 10}$
- d. Your x changes from to $x_1 = 2$ to an $x_2 = 7.5$, $\Delta x = \frac{x_2 - x_1 = 7.5 - 2}{= +5.5}$

$5 - -4 = 5 + +4$
subtracting a negative
is adding a positive!

" Δ " is the secret
symbol for change

e. Your x changes from to $x_1 = -3.5$ to an $x_2 = 8.5$, $\Delta x = \frac{\textcircled{12}}{x_2 - x_1 = 8.5 - (-3.5) = 12}$

f. Your x changes from to $x_1 = -3.5$ to an $x_2 = -4.5$, $\Delta x = \frac{-4.5 - (-3.5)}{= -1}$



g. Your y changes from to $y_1 = -3.5$ to a $y_2 = 0$, $\Delta y = \frac{y_2 - y_1 = 0 - (-3.5)}{\textcircled{+3.5}}$

h. Your y changes from to $y_1 = 0$ to a $y_2 = -8.375$, $\Delta y = \frac{y_2 - y_1}{= -8.375 - 0}{= -8.375}$

Curious how subtracting a negative thing is the same as getting a positive thing

a six week old puppy knows that!

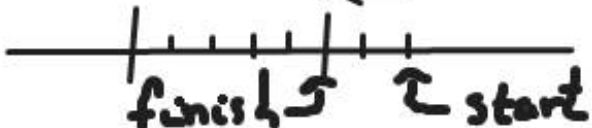
Did you know that subtracting a negative is the same as adding a positive? Taking away two bad things is the same as getting two good things! (from my experience)

$$\text{Eg: } 3 - (-5) = 3 + (+5) = 8$$

And adding negative things to your life is same as subtracting good things!

$$3 + (-5) = 3 - (+5)$$

20. Calculate more *delta* differences, Δ . Calculate the Δ by finding the difference from the x_2 subtract the x_1 . ie: $\Delta x = x_2 - x_1$.

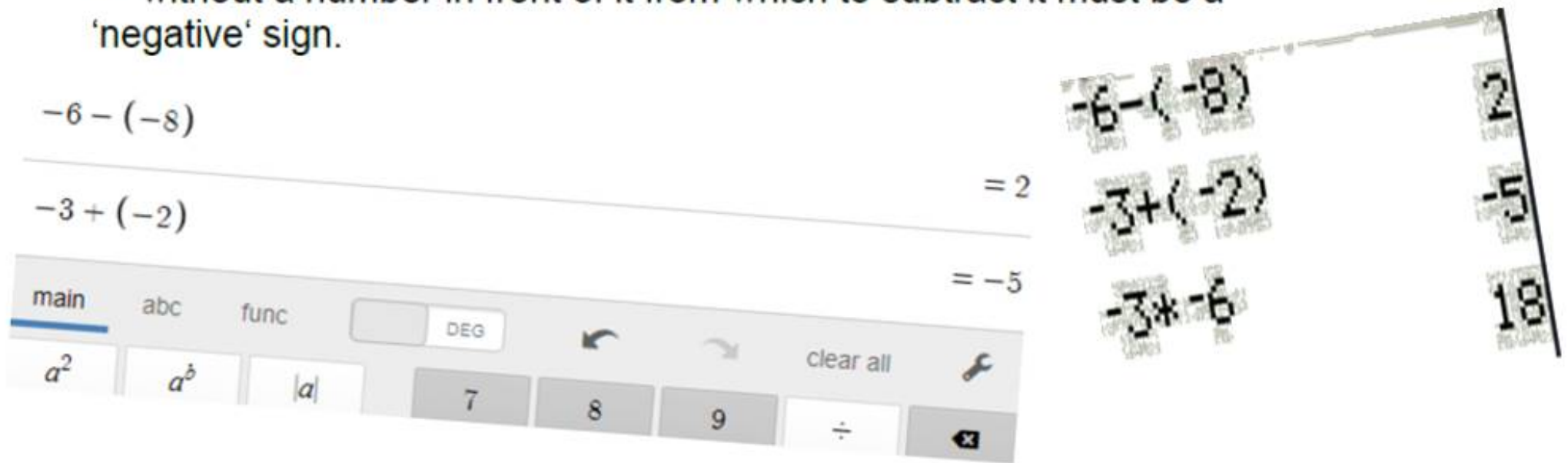
| | | |
|---|--|--|
| $x_2 = 5, x_1 = 7$ $\Delta x = \underline{5-7} = -2$  | $x_2 = 5, x_1 = 0$ $\Delta x = \underline{x_2 - x_1} = 5 - 0$ $= (5)$ | $x_2 = 5, x_1 = -7$ $\Delta x = \underline{5 - (-7)} = (12)$ |
| $x_2 = -5, x_1 = -8.3$ $\Delta x = \underline{-5 - (-8.3)}$ $= -5 + 8.3$ $= +3.3$ | $x_2 = -9.2, x_1 = -7$ $\Delta x = \underline{-9.2 - (-7)}$ $= (-2.2)$ | $x_2 = 8, x_1 = -8$ $\Delta x = \underline{8 - (-8)}$ $= (16)$ |
| $x_2 = 56.3, x_1 = 56.3$ $\Delta x = \underline{56.3 - 56.3}$ $= 0$ | $x_2 = 0, x_1 = 0$ $\Delta x = \underline{0 - 0}$ $= 0$ | $x_2 = 0, x_1 = -7$ $\Delta x = \underline{0 - (-7)}$ $= (+7)$ |

Calculator. If you are unfamiliar with how to add or subtract negative amounts, make sure you know how to do it on your calculator.

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There is a button on your calculator looks like this: $+/-$ or this $(-)$
On a decent calculator you select the button before the number (just as you would write it); on a Dollarama calculator (or equivalent) you select the button after you insert the number.

Very modern calculators are smart enough to know that if there is a '-' without a number in front of it from which to subtract it must be a 'negative' sign.



21. Calculate the slope:

A to B:

$$\begin{aligned} \text{Slope}_{AB} &= \frac{(y_B - y_A)}{(x_B - x_A)} = \frac{9 - 3}{7 - 2} \\ &= \frac{6}{5} \checkmark \end{aligned}$$

B to A

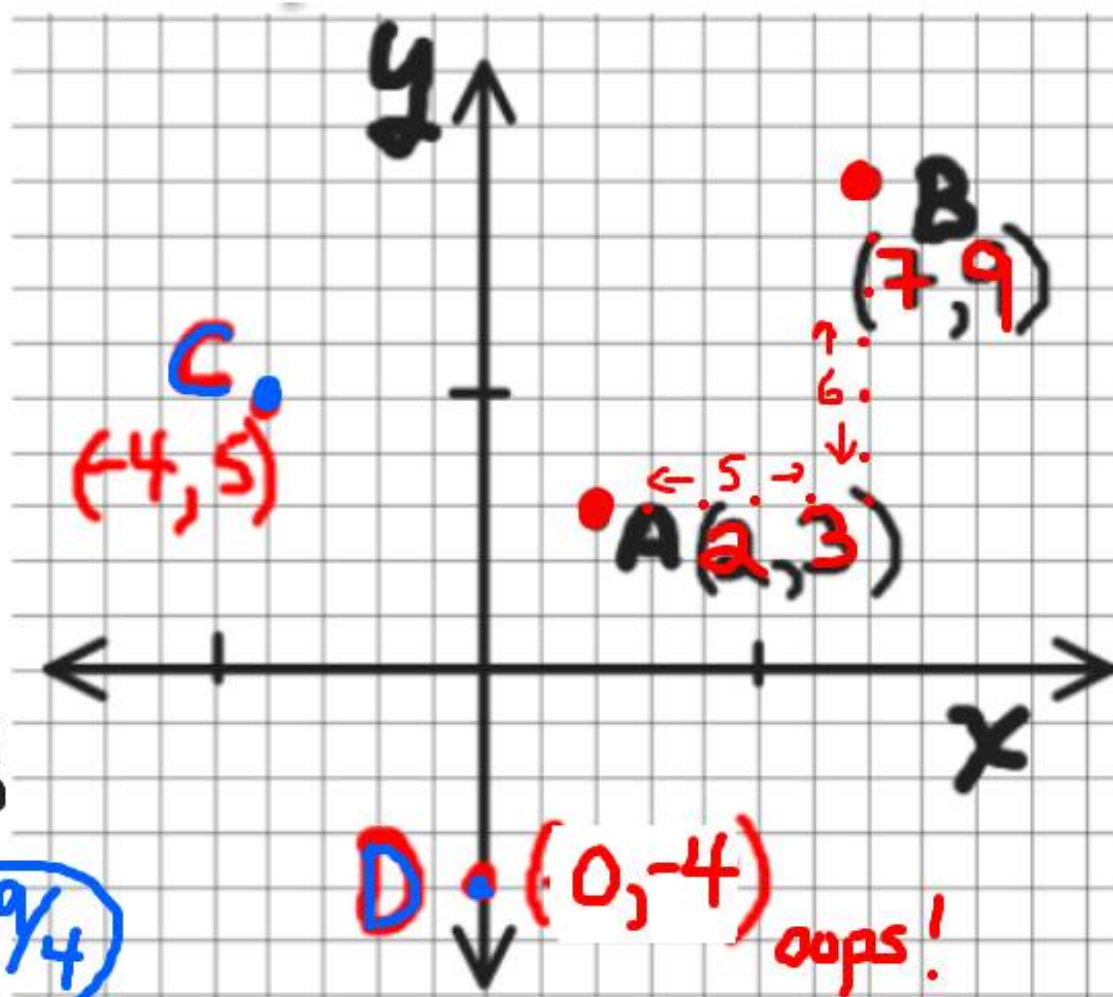
$$\begin{aligned} \text{Slope}_{BA} &= \frac{(y_A - y_B)}{(x_A - x_B)} = \frac{3 - 9}{2 - 7} \\ &= \frac{-6}{-5} = +\frac{6}{5} \end{aligned}$$

C to D

$$\text{Slope}_{CD} = \frac{(y_D - y_C)}{(x_D - x_C)} = \frac{-4 - 5}{0 - (-4)} = \frac{-9}{4} = -\frac{9}{4}$$

D to A

$$\text{Slope}_{DA} = \frac{(y_A - y_D)}{(x_A - x_D)} = \frac{3 - (-4)}{2 - 0} = \frac{7}{2} = \frac{7}{2} \text{ or } 3.5$$



notice then that slope A to B
is same as slope B to A (of course)
It is still the same slope whether
going up it or down it!

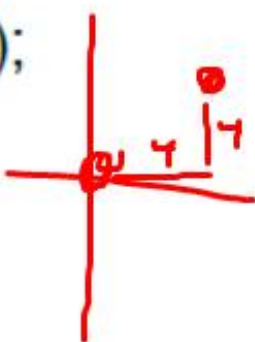
Slope expressed as a reduced fraction is preferred! A decimal equivalent or approximation is acceptable sometimes.

22. If you are unfamiliar with the idea of negative numbers, and that a negative divided by a negative is a positive for example, see teacher now!

If you are unfamiliar with how a fraction is reduced or how a fraction and a decimal are pretty much the same thing , see the teacher now!

25. Calculate the slope, m , between the given pairs of points:
 A rough tiny sketch is always useful too! If you aren't doodling, or sketching, then you are not truly thinking!

a. $A(0, 0)$ & $B(4, 4)$;
 $m = \frac{4}{4} = 1$



e. $G(0, 0)$ & $H(-4, -4)$;
 $m = 1$

$$\frac{-4-0}{-4-0} = 1$$

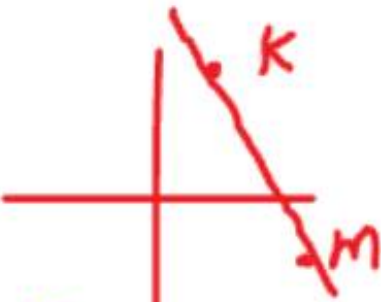
b. $C(2, 8)$ & $D(7, 13)$;
 $m = 1$

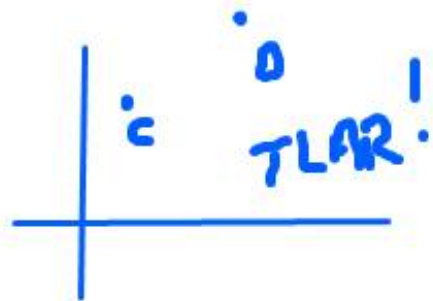
$$\text{slope}_{CD} = \frac{13-8}{7-2} = \frac{5}{5} = 1$$

d. $K(2, 7)$ & $M(7, -3)$;
 $m_{KM} = -2$

$$m_{km} = \frac{-3-7}{7-2} = \frac{-10}{5} = -2$$

TLAR





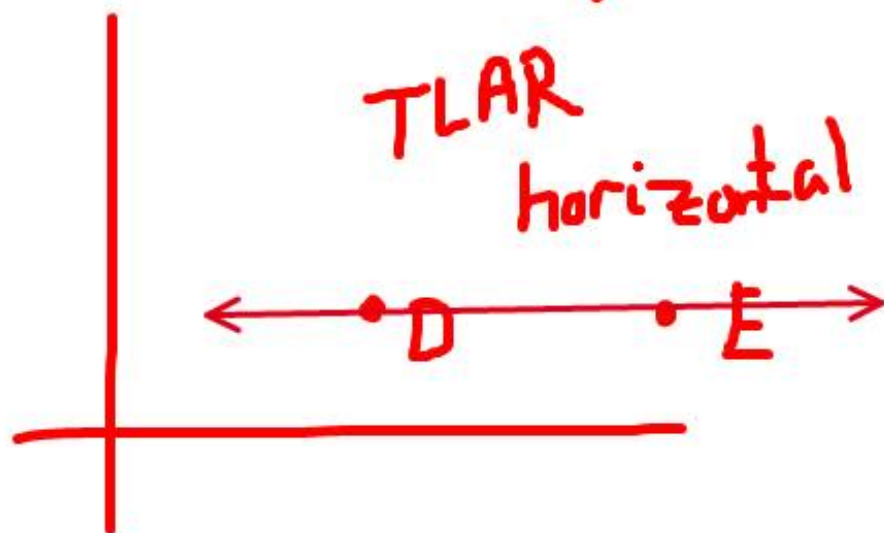
TLAR: That Looks About Right

25. Calculate the slope, m , between the given pairs of points:
 A rough tiny sketch is always useful too! If you aren't doodling, or sketching, then you are not truly thinking!

c. $D(7, 13)$ & $E(22, 13)$;
 $m =$ _____

$$m = \frac{13 - 13}{22 - 7} = \frac{0}{15} = 0$$

check with a sketch?



e. $P(7, 0)$ & $Q(7, 14)$;
 $m_{PQ} =$ _____

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 0}{7 - 7}$$

$$= \frac{14}{0} \quad \text{WTH?}$$

```
14/0 Error
ERR: DIVIDE BY 0
Quit
2: Goto
```

$$\frac{14}{0}$$

= undefined



26. **Advanced thinking.** *Curious!* How do you divide by zero? Try it on a calculator. Think about it. How many zeros can you subtract from 3 ie: $3 \div 0$? If you have 12 candies and want to share them with zero kids, how many candies does each kid get?

First of all, the question does not make any sense

how do you share 12 candies with zero kids!!????

how many times can you subtract nothing from something???

See teacher if you want to learn about:



infinity, limits, ...

30. What is the slope of this roof? (approx)

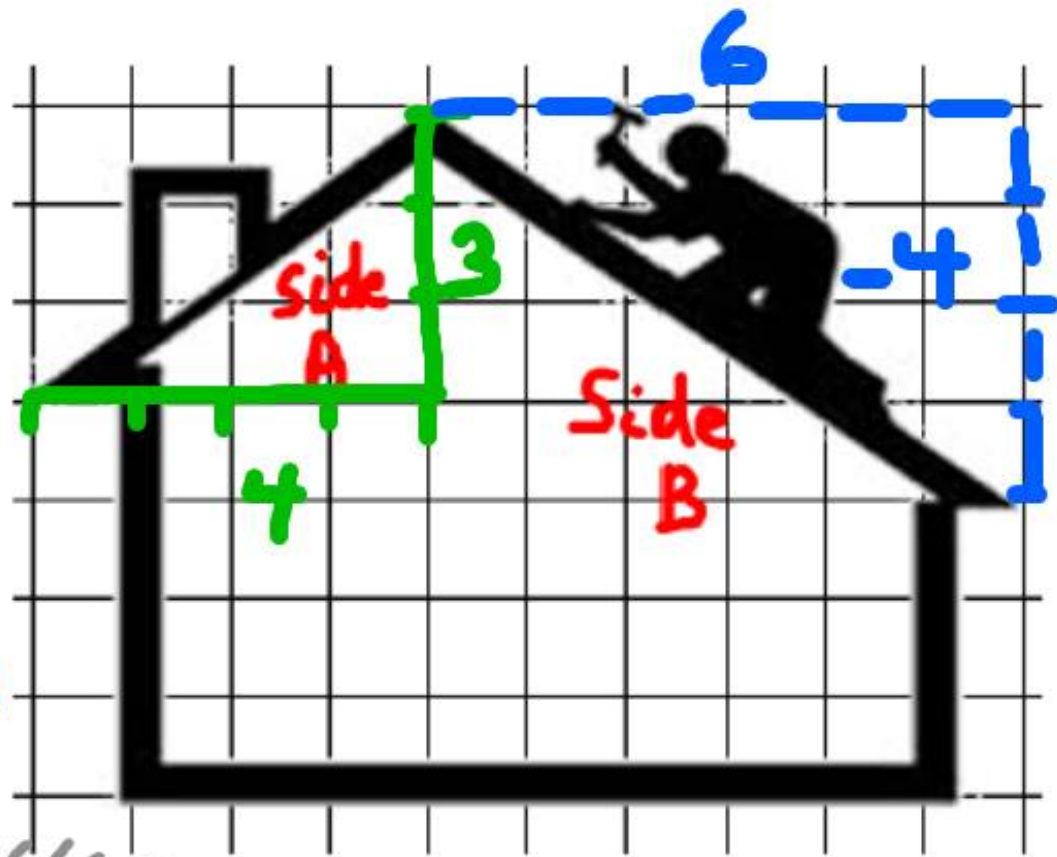
a. Side A slope:

$$\frac{3}{4}$$

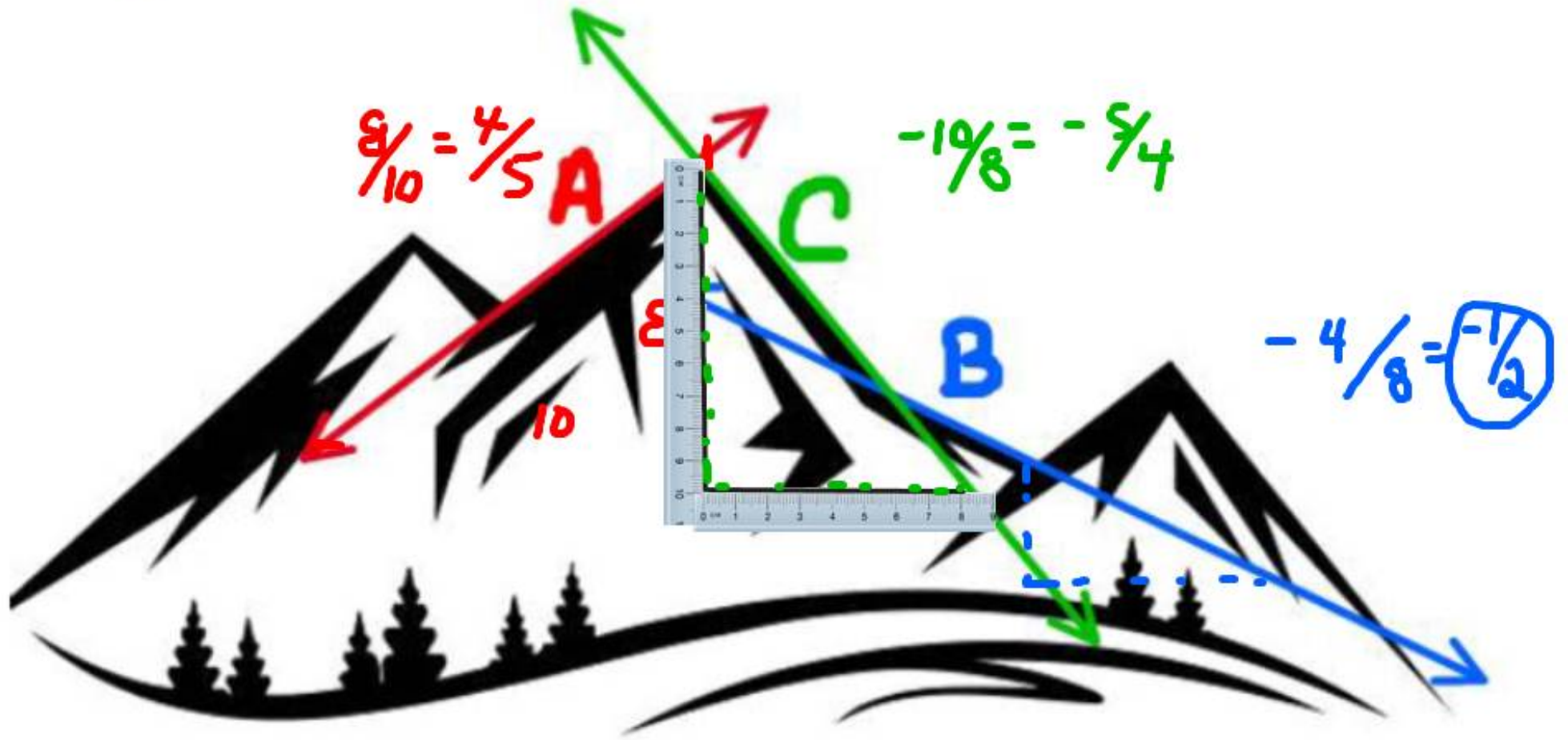
b. Side B slope:

$$-\frac{4}{6} = -\frac{2}{3}$$

$$= -0.6666666666...$$



31 Determine the slope of the mountain(s) along the indicated slopes, A, B, and C. You will need a ruler to make your own measurements of changes.



32. Determine the slopes of the labelled lines. Write the value adjacent to the label of the line. You will need a ruler of some type.

