GRADE 11 ESSENTIAL
UNIT F - RELATIONS AND PATTERNS
WORK BOOK

Name:____________
Date: ____________

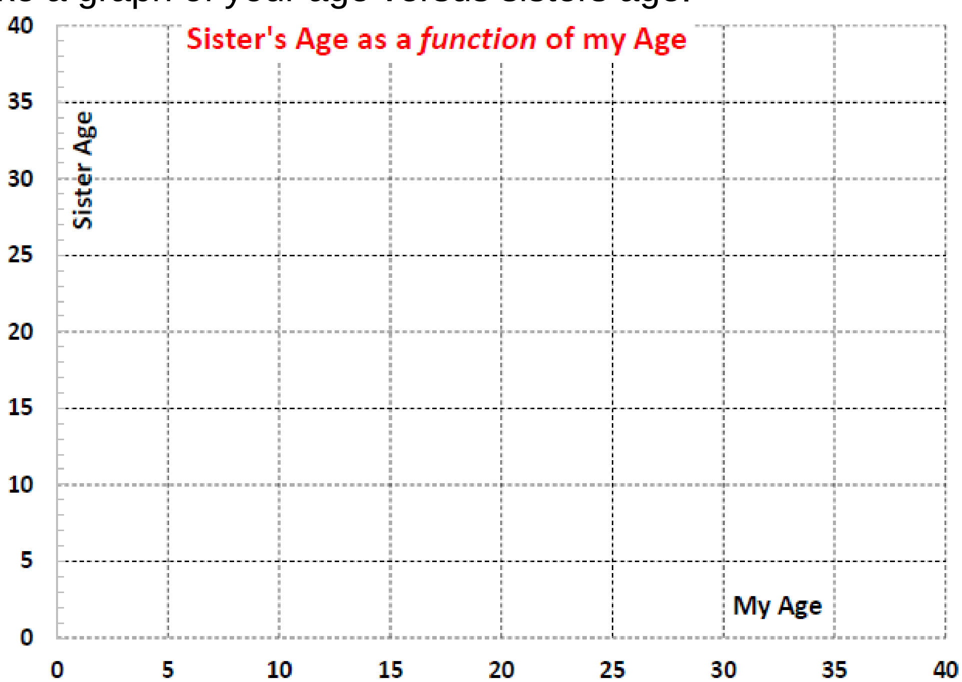
1. A relationship between two different variables can readily be graphed to show any pattern. A pattern allows us to make predictions. Where we have been, where we are going. Finding patterns is what Math is all about.

Relations and patterns occur every day of your life, multiple times! Hopefully you notice them! Lots of books in your backpack, it is heavier. The bus is slow, it will take longer to get to school. Study better→ better mark. What goes up, must come down.

2. Here is a simple pattern. Your sister is three years younger than you. Make a table of the relationship of the ages, what her age is depending on what age you select for yourself.

You	5	8	15	22	23	35
Sister	2	5	12			

Now make a graph of your age versus sisters age.



An interesting pattern! A line. A linear relationship.

Making Lines Using a ‘T’-Table

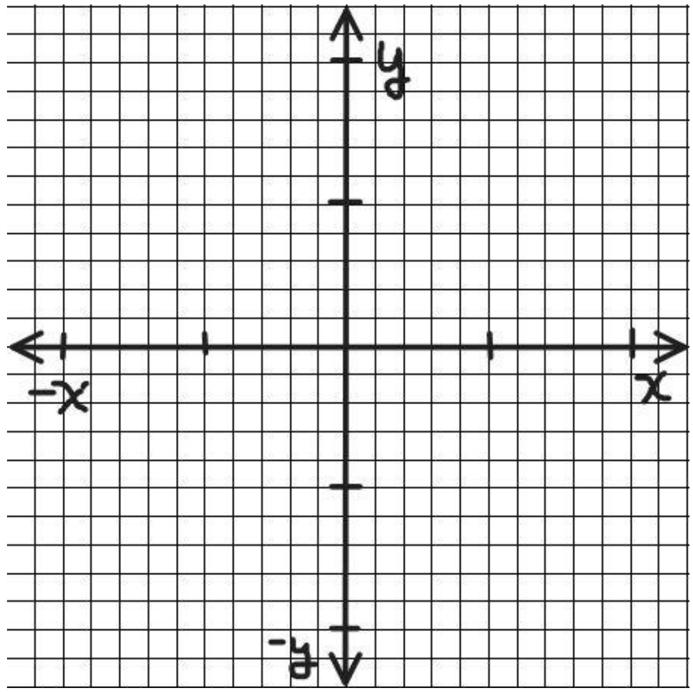
3. Complete the table for the given equation

Equation:

$$y = 3x + 2$$

x	3(x) + 2	y
0	*3(0)+ 2*	2
1	*3(1) + 2*	
2		

Plot the points and neatly connect them.



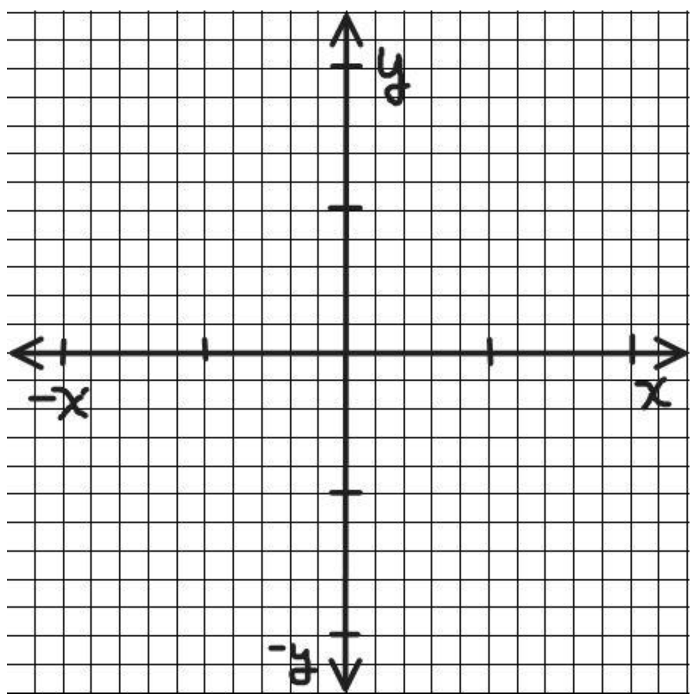
4. Complete the table for the given equation

Equation:

$$y = 2x - 5$$

x	2(x) – 5	y
0	*2(0) – 5*	–5
1		
2		
3	*2(3) – 5*	

Plot the points and neatly connect them.
(technically you only need two points to make a line, but an extra one makes sure you did not mess it up)

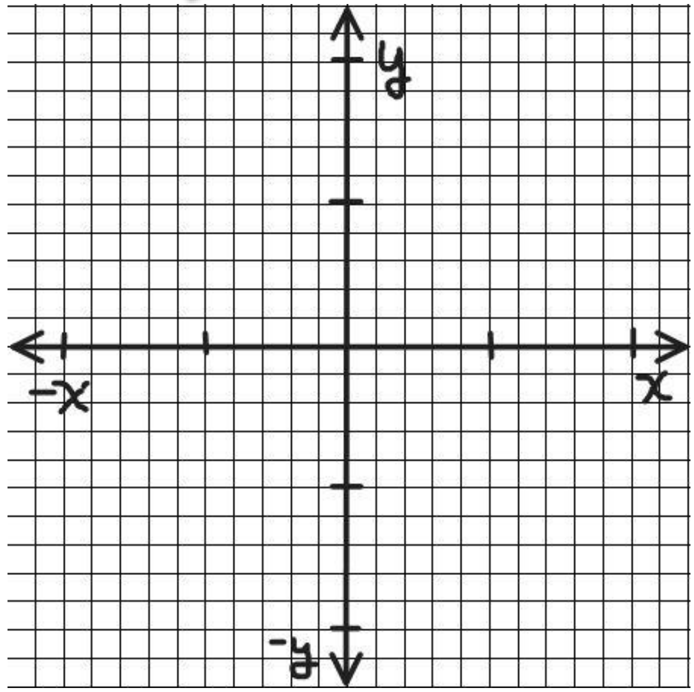


5. Complete the table for the given equation

Equation:

$$y = \frac{1}{2}x + 6$$

x	$y = \frac{1}{2}x + 6$	y
0	$y = \frac{1}{2}(0) + 6$	*6*
1	$y = \frac{1}{2}(1) + 6$	*6.5*
2		
4		



Plot the points and neatly connect them.

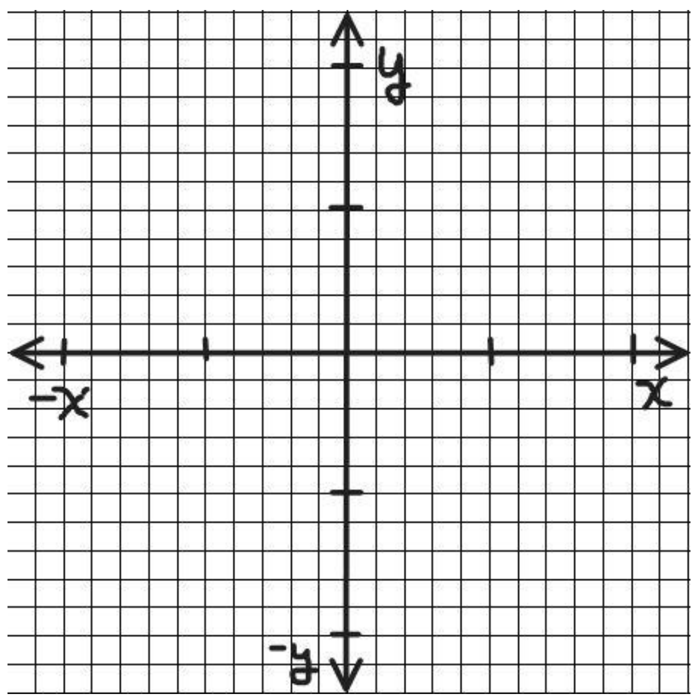
You can pick whatever x you want, but zero is pretty easy!

6. Complete the table for the given equation

Equation:

$$y = -\frac{2}{3}x + 6$$

x	$y = \frac{-2}{3}x + 6$	y
0	$y = \frac{-2}{3}(0) + 6$	*6*
3	$y = -\frac{2}{3}(3) + 6$	*4*
9		



Plot the points and neatly connect them.

You can pick whatever x you want, but zero is pretty easy! But notice that picking multiples of 3 here made nice easy values to plot!

7. Complete the table for the given equation

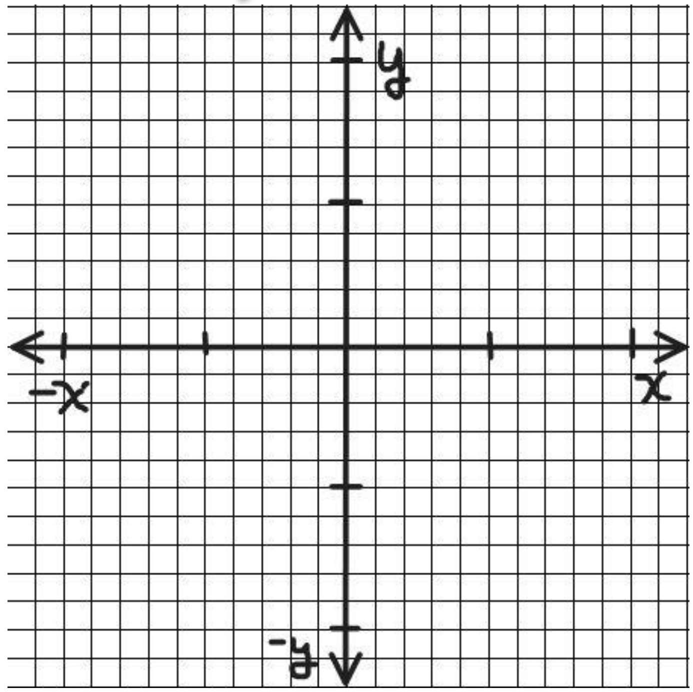
Equation:

$$y = -\frac{3}{5}x$$

x	$y = \frac{-3}{5}x$	y
0	$y = -\frac{3}{5}(0)$	*0*
3	$y = -\frac{3}{5}(3)$	*-1.8*
5		
–5		

Plot the points and neatly connect them.

You can pick whatever x you want, but zero is pretty easy! But notice that picking multiples of 5 here made nice easy values to plot!



8. Complete the table for the given equation

Equation:

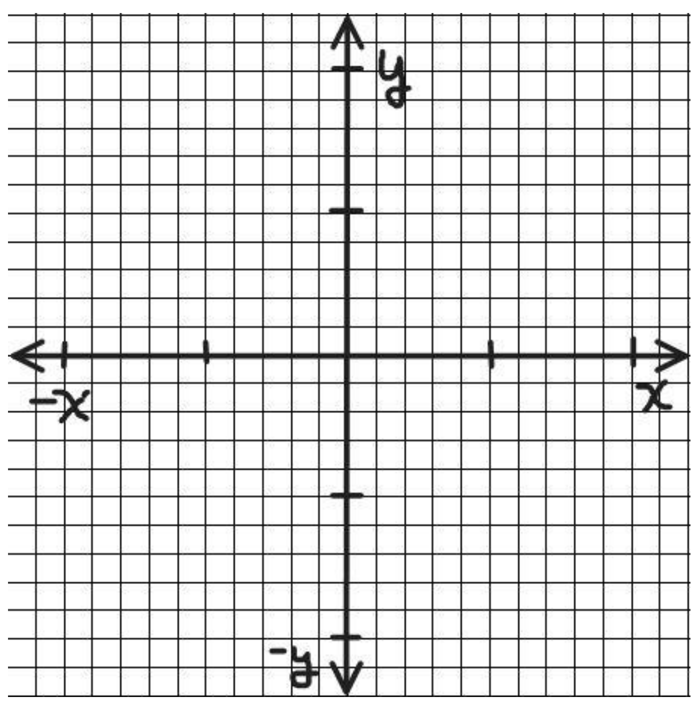
$$y = 4$$

x	$y = 4$	y
0	$y = 4$	*4*
3	$y = 4$	*4*
9.66		
-135		

Plot the points and neatly connect them.

A horizontal line! Does not matter what the x is!

The y IS 4

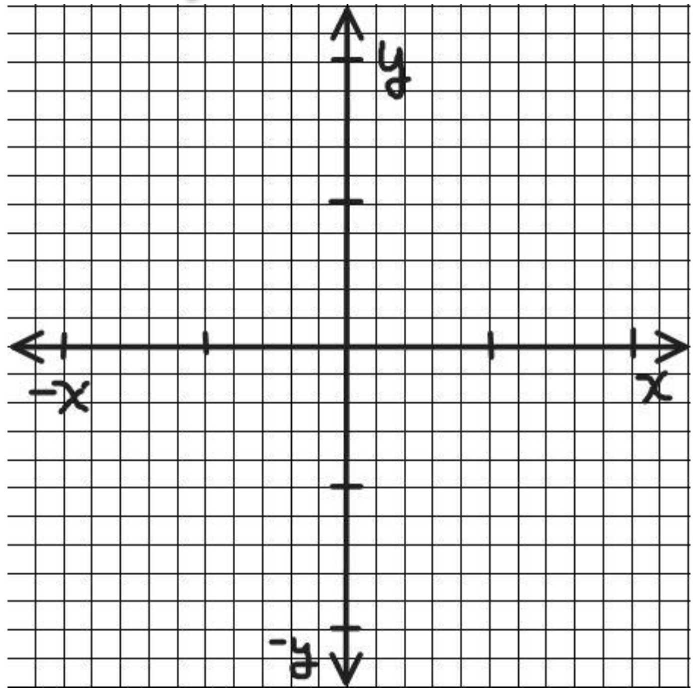


9. Complete the table for the given equation

Equation: $y = \frac{4-x}{2}$

x	$\frac{(4-x)}{2}$	y
0		
2	(4 – 2) / 2	*1*
4		
−8		

Plot the points and neatly connect them.



Calculating Slope of a Line

The line has a slope: steep, shallow, flat, …
Formulae for calculating slope :

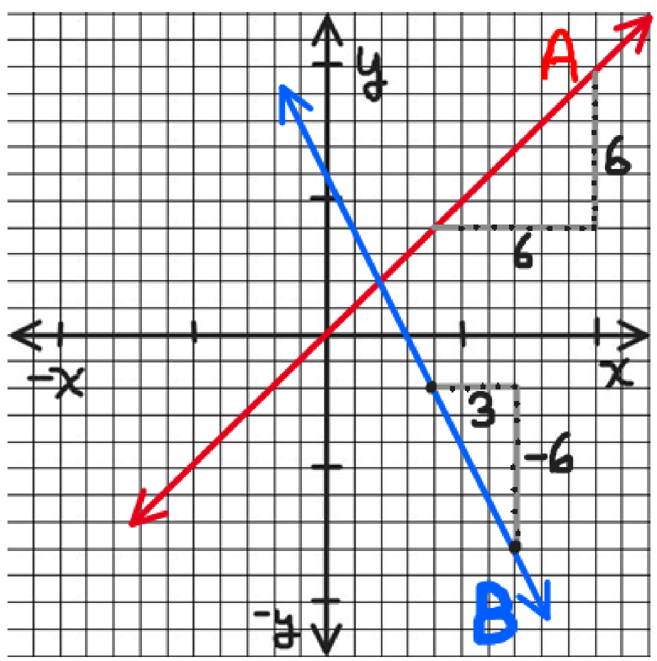
$$slope \equiv m \equiv \frac{change\ in\ y}{change\ in\ x} = \frac{rise\ up}{run\ right}$$

10. Determine the slope of lines A and B.

slope$_A$ = $\frac{rise}{run\ right}$ =

slope$_B$ = $\frac{rise}{run\ right}$ =

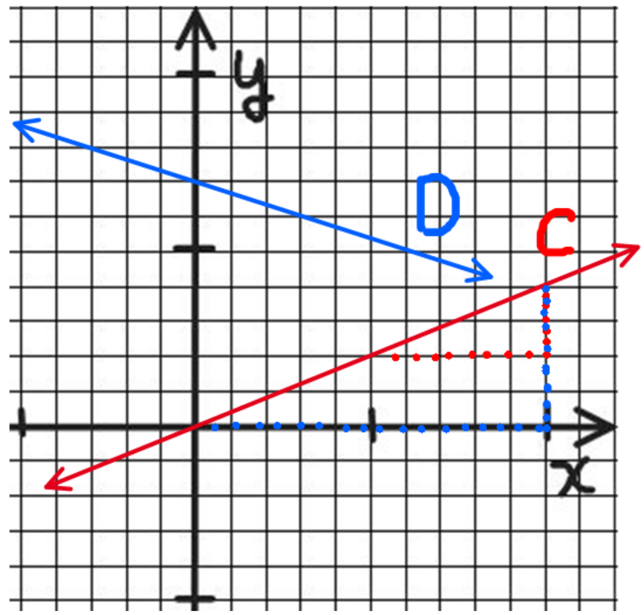
You make one and calculate it!



11. Determine the slope of lines C and D.

Slope$_C$ = $\frac{rise}{run\ right}$ =

Slope$_D$ = $\frac{rise}{run\ right}$ =



Notice how a fraction is a perfect way to express a slope!

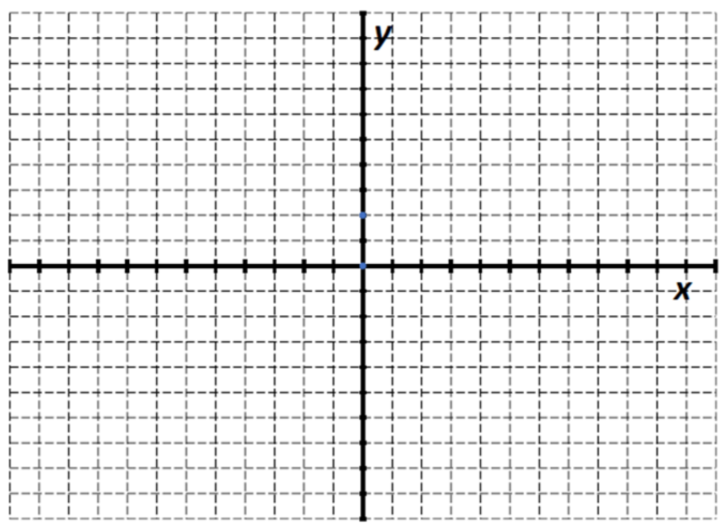
15. **You** graph and label a line (any line) that has a slope of:

A. m = –3

B. m = +5/8

C. m = 2.5

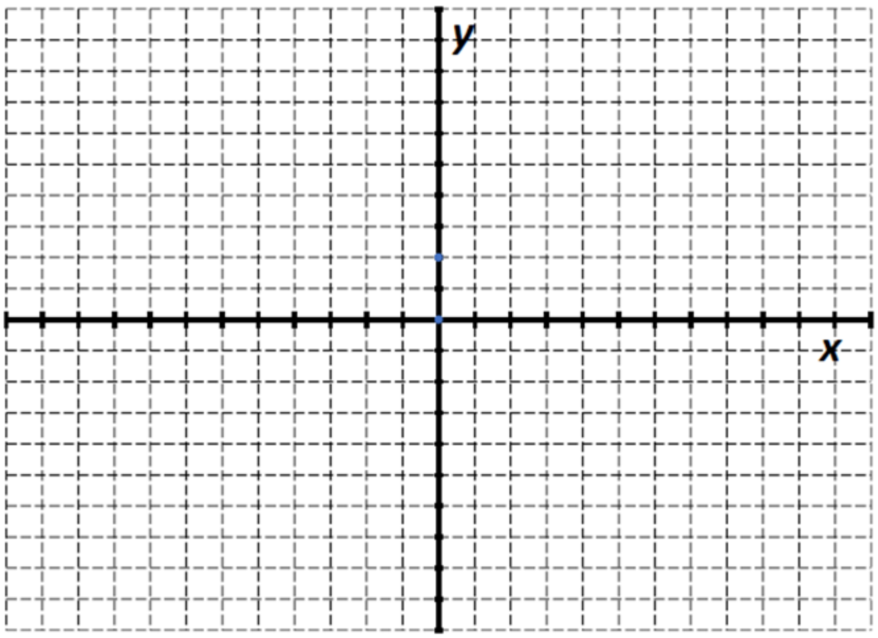
D. m = –1.2



An infinite number of correct answers

16. Line through a given point

a. You graph a line that has a slope of 3 and goes through the point (0, 2)
b. You graph a line that has a slope of 2/3 and goes through the point (0, 0)
b. You graph a line that has a slope of –3/5 and goes through the point (0, 6)



Calculating Slope without a graph; using just two points

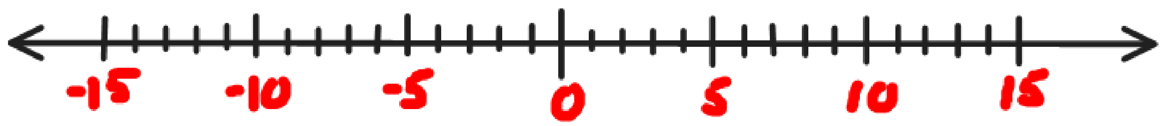
$$slope = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

17. **Δ** is the Greek letter '**d**elta'. We use it to mean '**d**ifference'. **Δx** means the **change in x**, so if x starts at 4 and goes to 9, we say the change of x, Δx, is 9 **–** 4. So the Δx is 5. It went from the first x; x_1, to the second x; x_2, so the difference is, $\boldsymbol{\Delta x} = x_2 - x_1$ which is = 9 – 4 which = **5**.

18. Finding a change, Δ, in a value.

a. You start at 7 you finish at 12, you changed by? __________

b. You start at –4 you finish at 5; you changed by? : __________

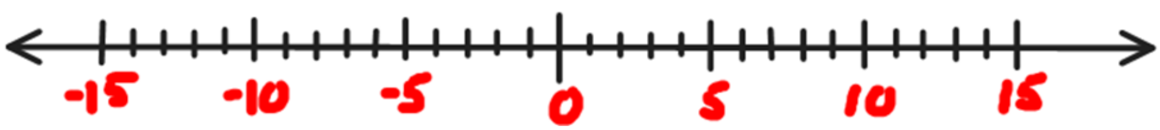


c. You start at 0 you finish at 10; what is your change in value, Δ: _______

d. Your x changes from to $x_1 = 2$ to an $x_2 = 7.5$, $\Delta x =$ __________

e. Your x changes from to $x_1 = -3.5$ to an $x_2 = 8.5$, $\Delta x =$ __________

f. Your x changes from to $x_1 = -3.5$ to an $x_2 = -4.5$, $\Delta x =$ __________



g. Your y changes from to $y_1 = -3.5$ to a $y_2 = 0$, $\Delta y =$ __________

h. Your y changes from to $y_1 = 0$ to a $y_2 = -8.375$ 0, $\Delta y =$ __________

Did you know that subtracting a negative is the same as adding a positive? Taking away two bad things is the same as getting two good things! (from my experience)

Eg: 3 – (–5) = 3 + (+5) = 8

And adding negative things to your life is same as subtracting good things!

3 **+** (–5) = 3 **–** (+5)

20. Calculate more *delta* differences, Δ. Calculate the Δ by finding the difference from the **x_2** subtract the **x_1**. ie: **$\Delta x = x_2 - x_1$.**

$x_2 = 5$, $x_1 = 7$ $\Delta x =$ _______	$x_2 = 5$, $x_1 = 0$ $\Delta x =$ _______	$x_2 = 5$, $x_1 = -7$ $\Delta x =$ _______
$x_2 = -5$, $x_1 = -8.3$ $\Delta x =$ _______	$x_2 = -9.2$, $x_1 = -7$ $\Delta x =$ _______	$x_2 = 8$, $x_1 = -8$ $\Delta x =$ _______
$x_2 = 56.3$, $x_1 = 56.3$ $\Delta x =$ _______	$x_2 = 0$, $x_1 = 0$ $\Delta x =$ _______	$x_2 = 0$, $x_1 = -7$ $\Delta x =$ _______

Calculator. If you are unfamiliar with how to add or subtract negative amounts, make sure you know how to do it on your calculator.

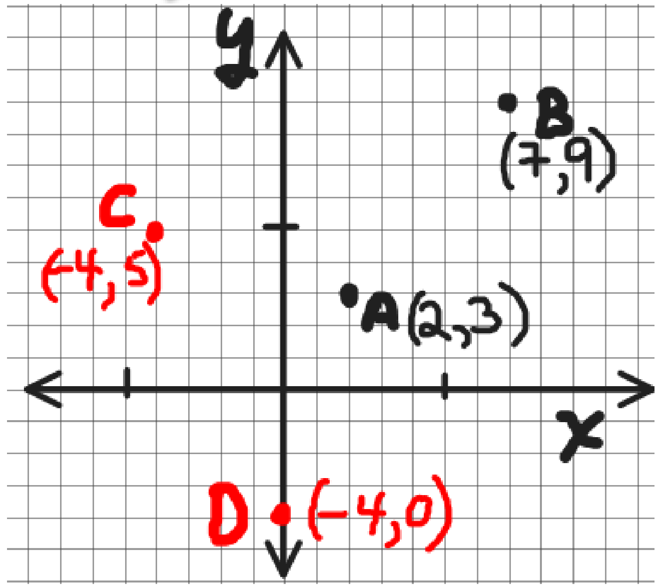
There is a button on your calculator looks like this: **+ / –** or this **(–)**
On a decent calculator you select the button before the number (just as you would write it); on a Dollarama calculator (or equivalent) you select the button after you insert the number.

Very modern calculators are smart enough to know that if there is a **'–'** without a number in front of it from which to subtract it must be a 'negative' sign.

$-6-(-8) \qquad = 2$

$-3+(-2) \qquad = -5$

21. Calculate the slope:



A to B:

$$Slope_{AB} = \frac{(y_B - y_{A)}}{(x_B - x_A)} = \frac{9-3}{7-2} =$$

B to A

$$Slope_{BA} = \frac{(y_A - y_{B)}}{(x_A - x_B)} = \frac{3-9}{2-7} =$$

C to D

$$Slope_{CD} = \frac{(y_D - y_{C)}}{(x_D - x_C)} = \frac{\quad}{\quad} =$$

D to A

$$Slope_{DA} = \frac{(y_A - y_{D)}}{(x_A - x_D)} = \frac{\quad}{\quad} =$$

Slope expressed as a reduced fraction is preferred! A decimal equivalent or approximation is acceptable sometimes.

22. If you are unfamiliar with the idea of negative numbers, and that a negative divided by a negative is a positive for example, see teacher now!

If you are unfamiliar with how a fraction is reduced or how a fraction and a decimal are pretty much the same thing , see the teacher now!

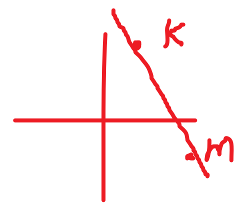
25. Calculate the slope, m, between the given pairs of points:
A rough tiny sketch is always useful too! If you aren't doodling, or sketching, then you are not truly thinking !

a. A(0, 0) & B (4, 4);
m = _________

a. G(0, 0) & H (–4, –4);
m = _________

b. C (2, 8) & D (7, 13);
m = _________

b. K (2, 7) & M (7, –3);
m_{KM} = _________



c. D(7,13) & E (22, 13);
m = _________

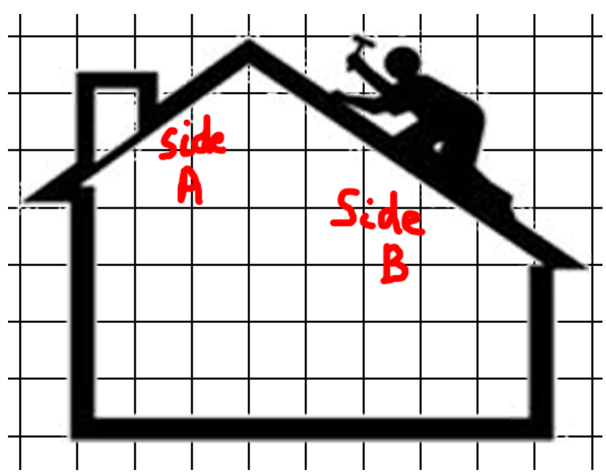
c. P (7,0) & Q (7, 14);
m_{PQ} = _________

26. **Advanced thinking**. *Curious*! How do you divide by zero? Try it on a calculator. Think about it. How many zeros can you subtract from 3 ie: **3 ÷ 0**? If you have 12 candies and want to share them with zero kids, how many candies does eack kid get?

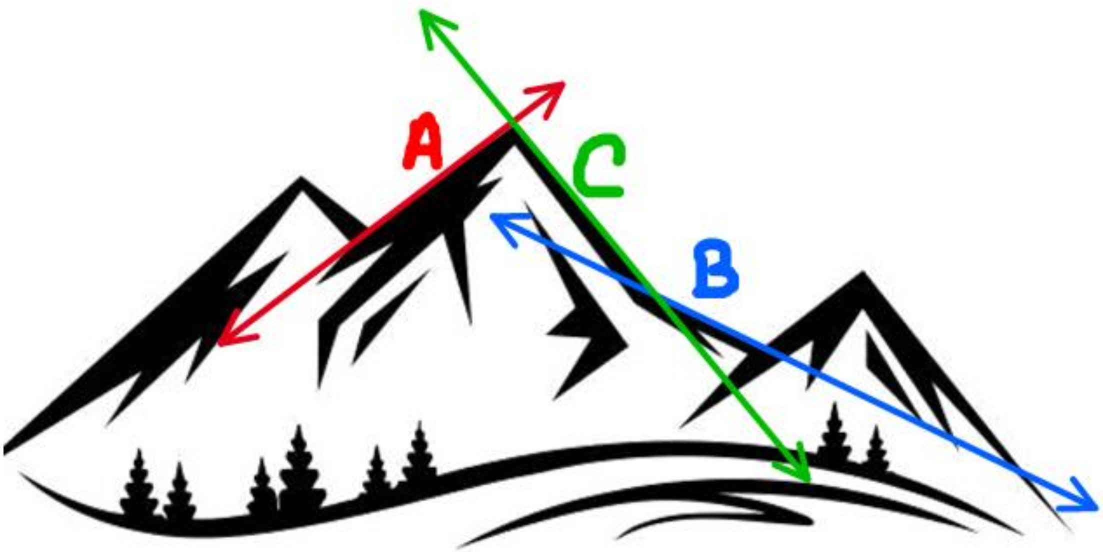
30. What is the slope of this roof? (*approx*)

a. Side A slope:

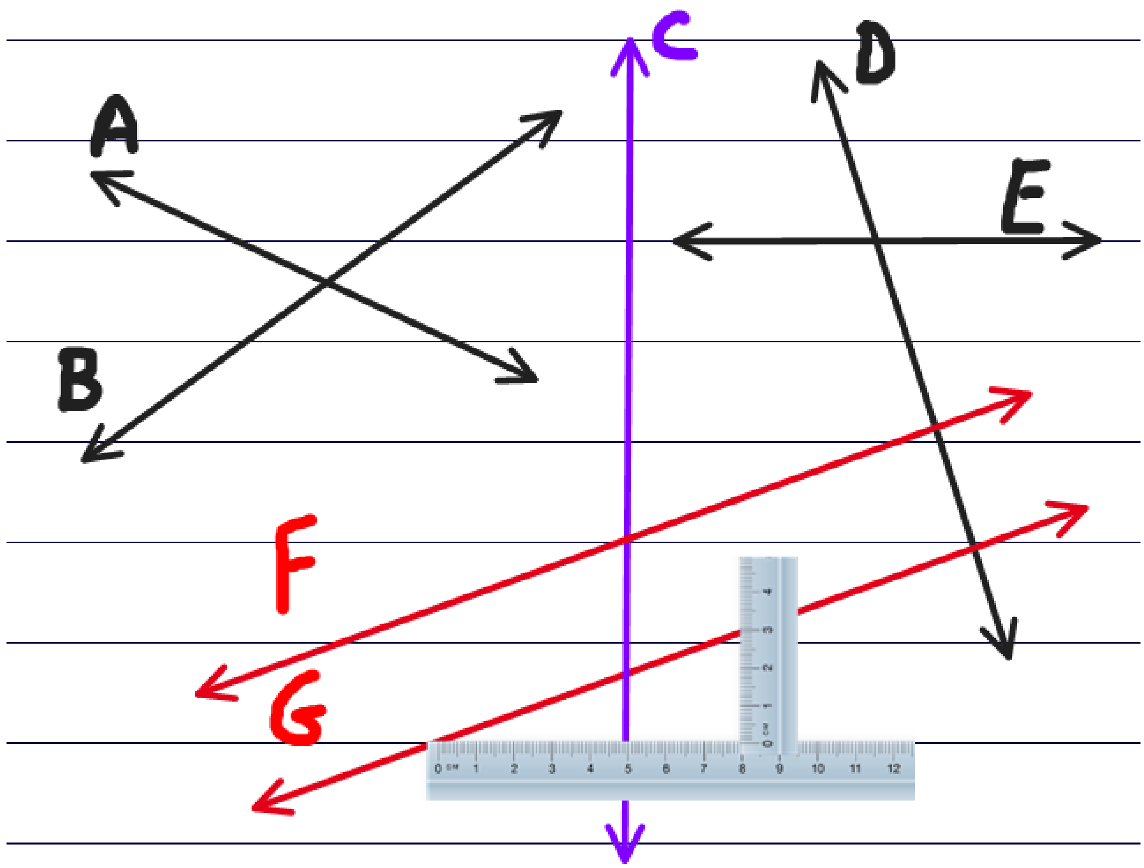
b. Side B slope:



31 Determine the slope of the mountain(s) along the indicated slopes, A, B, and C. You will need a ruler to make your own measurements of changes, a metric ruler of course is better for this.



32. Determine the slopes of the labelled lines. Write the value adjacent to the label of the line. You will need a ruler of some type.



GRAPHING LINES USING THE SLOPE AND INTERCEPT METHOD

No need to make a table.

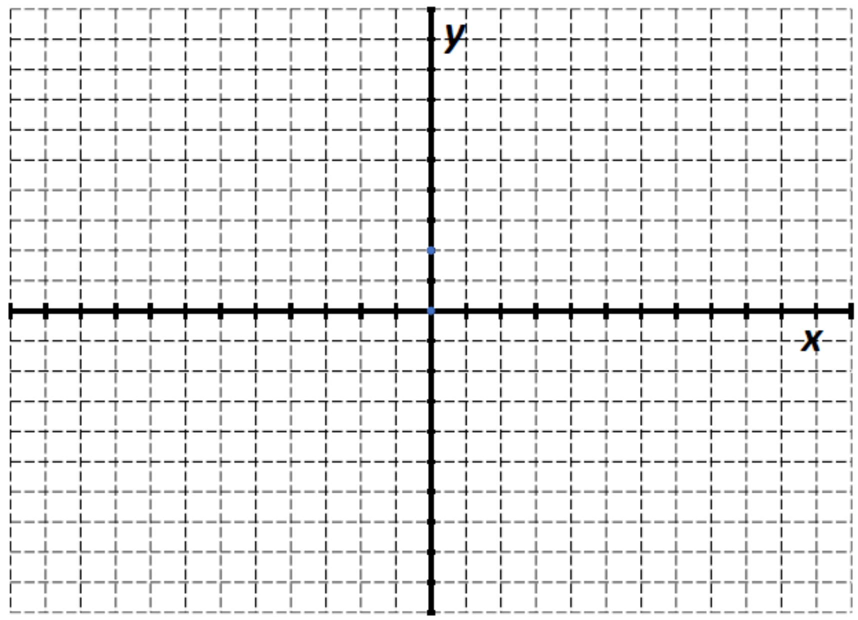
33. Graph the line(s):

a. $y = 2x + 4$

y-intercept point:
(0 , __)
Slope: _______
x- intercept : (____,0)

b. $y = -\frac{1}{2}x + 3$

y-intercept: (0, __)
Slope: _______
x-intercept : (____, 0)



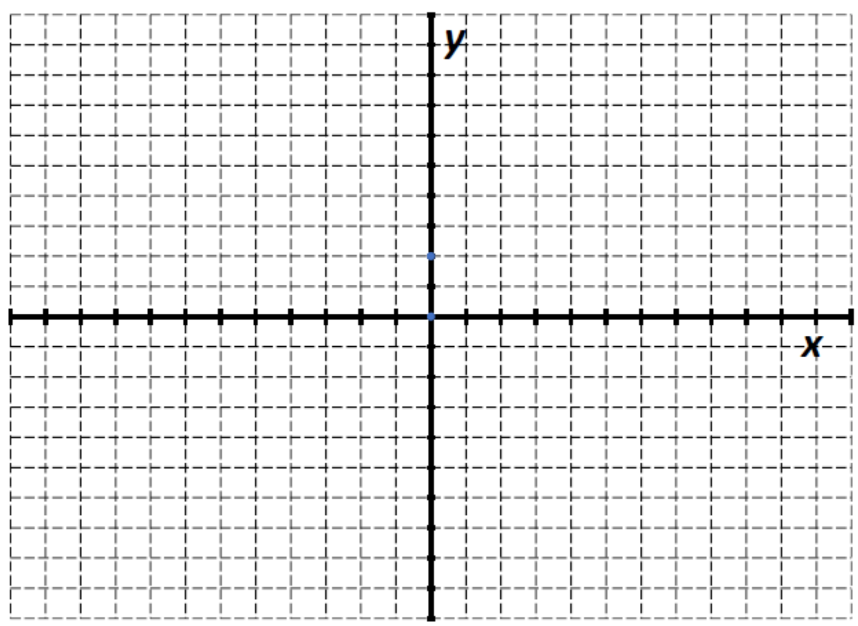
34. Graph the line(s):

a. $y = x - 4$

y-intercept point:
(0 , __)
Slope: _______

b. $y = -4 + x$

y-intercept point:
(0 , __)
x- intercept:
(_____, 0)
Slope: _______



Notice that x - 4 and -4 + x are the same thing!

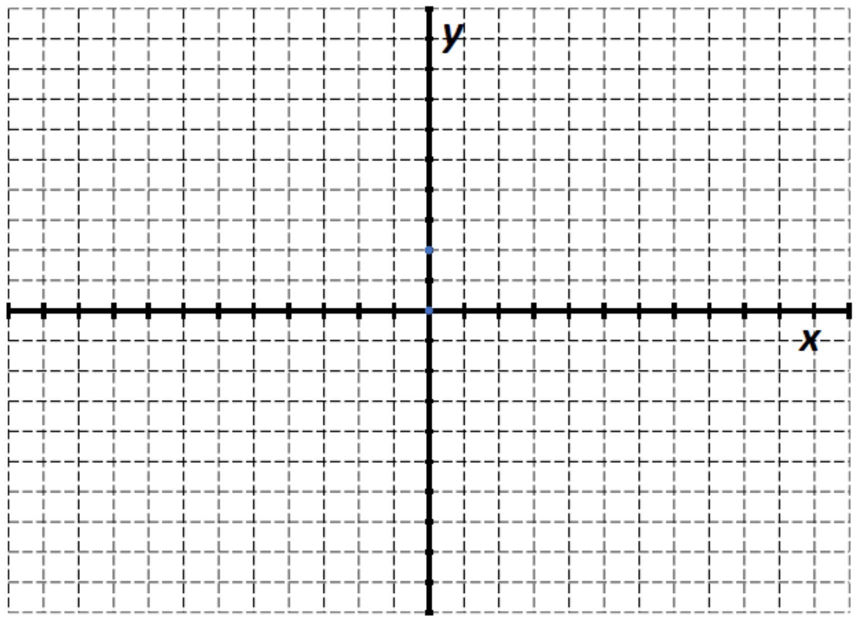
35. Graph the line(s):

$$y = \frac{3}{5}x + 2$$

y-intercept point: (0 , __)
Slope: _______

$$y = -0.4x - 5$$

y-intercept point: (0 , __)
Slope: _______
x-intercept : (___, 0)



Did you know that 0.4 means 4 tenths, ie: $\frac{4}{10}$; which is really 2/5^{ths}.
Earthlings break whole things into 10 pieces for some reason!

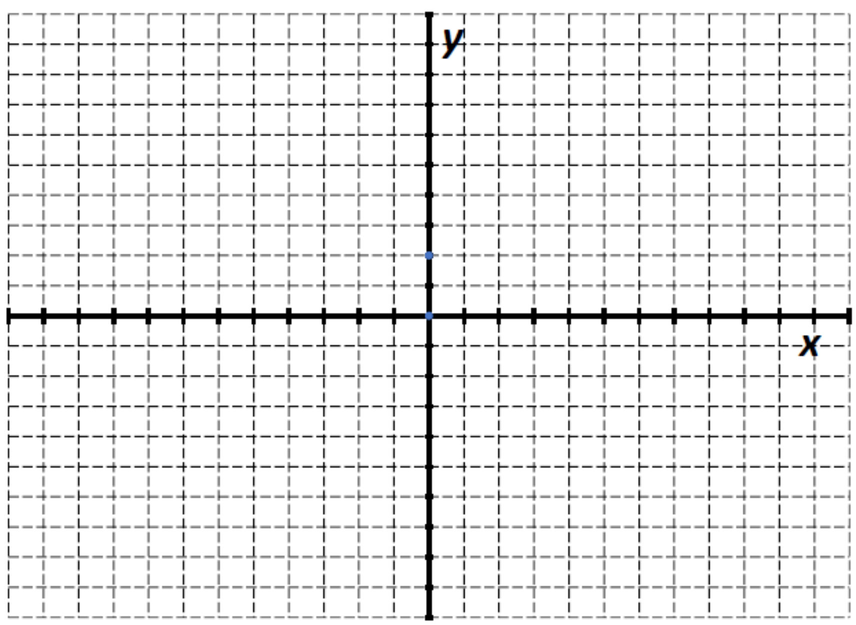
36. Graph and label the line(s):

A. $y = 5$

B. $y = -0.75x$

C. $y = -7.3$

D. **x = 8**



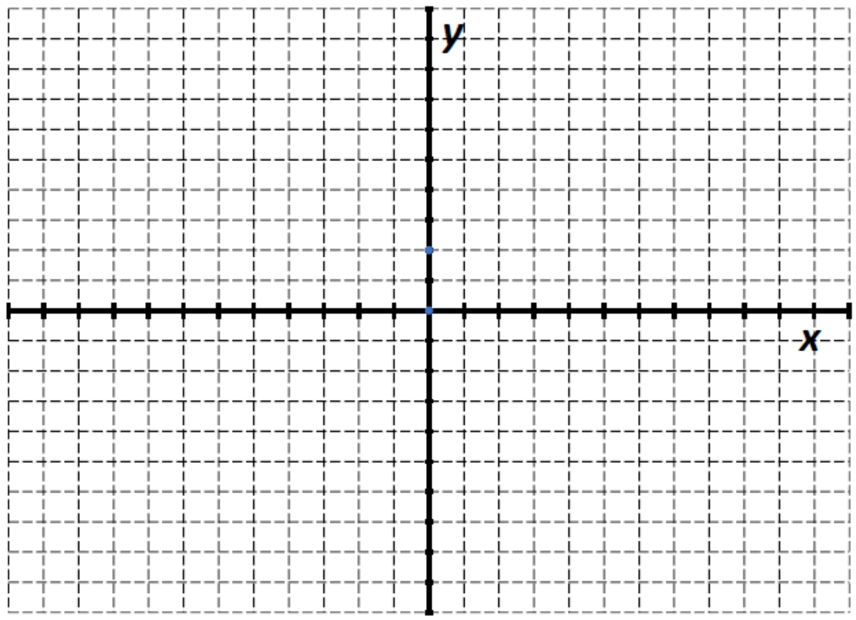
37. Graph the line(s):

E. $y = -\frac{2}{3}x + 2$

F. $y = -0.1x - 3$

G. **y = 0**

H. **x = 0**



38. Make up your own. Of course you can make up your own linear equations and graph them on grid paper and check them with a graphing tool on your device.

There should be tons of grid paper available from your teacher.

Graphing Lines in the General Form

39. Some plain simple x's and some plain simple y's in an equation will make a linear pattern. $\sqrt{x}$, **x^2**, 3^x, etc would NOT be lines.

Example: **y = x + 4** makes a line! You, **y**, are 4 years older than your brother, **x.**

40. But that also means that the difference of your age subtract his age is 4.
So: **y – x = 4,** your age subtract brother's age is 4. It is the same relationship.

A little algebra?

y = x + 4

Subtract x from both sides they are still equal! ↓

y – x = x + 4 – x

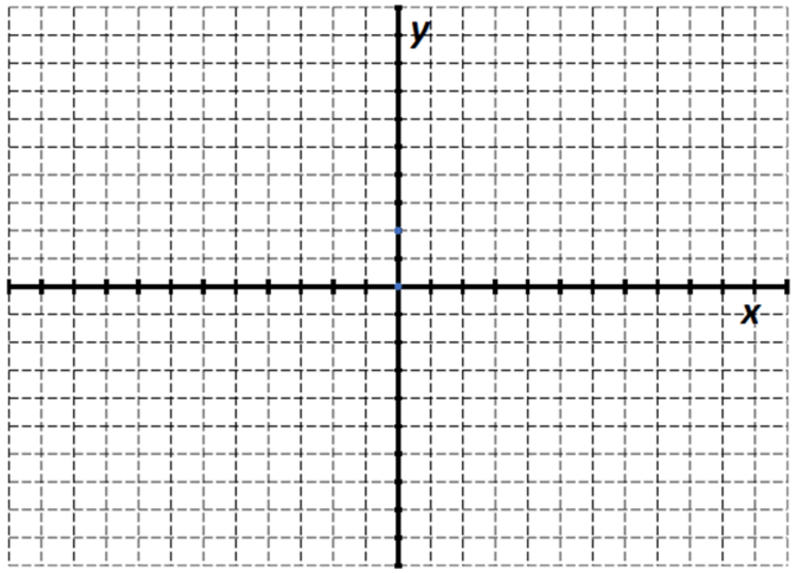
so: ………….. y – x = 4

is exactly the same relationship.

General Form Equation. If you do not like the algebra just make a simple t-table. Find the "zeros".

41. Graph **y + x = 10**

x	y
If x = 0→	Then y = 10
x = 10	←If y = 0



42. Graph **y + 2x = 8**

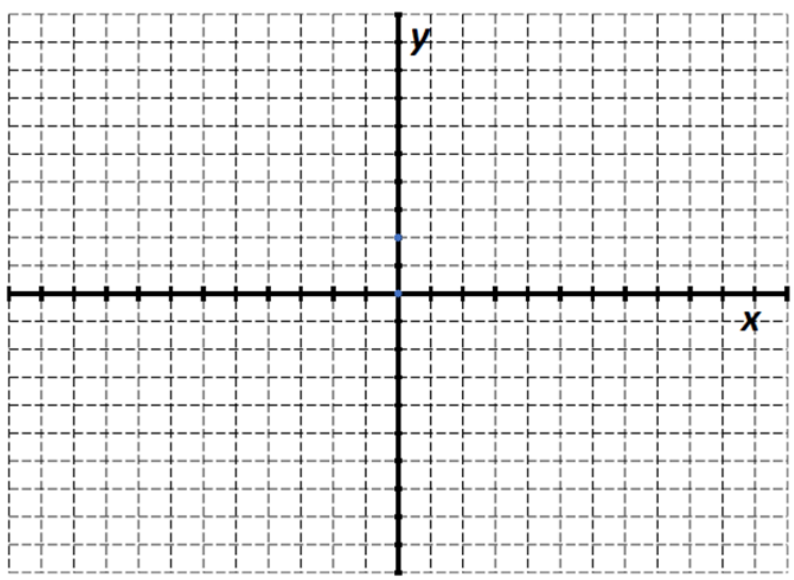
x	y
0→	8
4	←0

43. Graphing in the general form is so easy! Just find where the x is zero (ie: the y-intercept) and where the y is zero (ie: x-intercept)

If you are traveling in a straight line and we know where you cross Main street and we know where you cross Portage Ave., we can see a pattern that predicts where you have been, and where you are going!

44. Graph **y − x = 4**

x	y
0→	4
−4	← 0

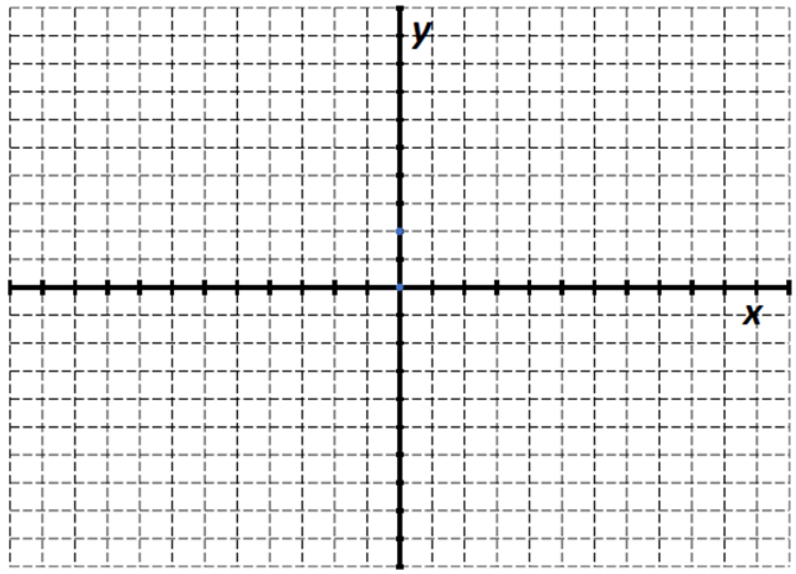


45. Graph
2x + 3y = 12

x	y
0→	4
	←0

50. Graph **3x − y = 9**

x	y
0	
	0

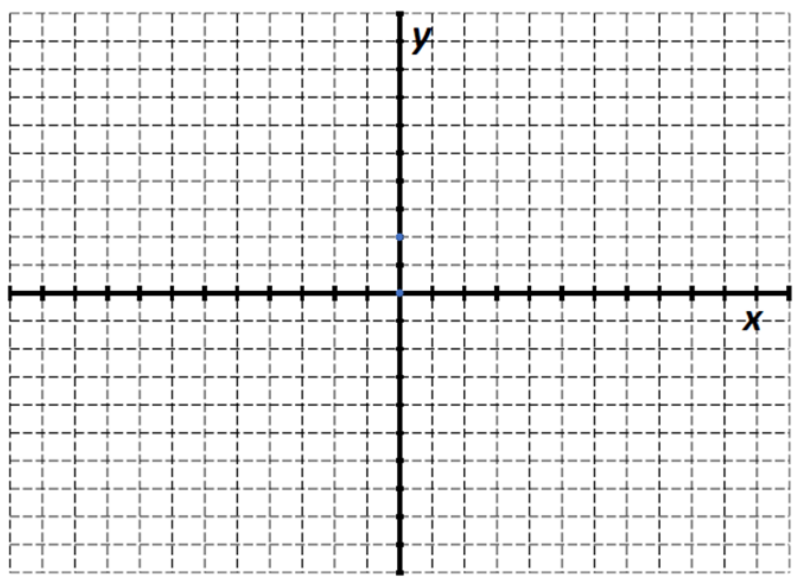


51. Graph:
20x + 15y = 60

x	y
0	
	0

52. Graph **3x – 2y = 6**

x	y
0→	
	← 0

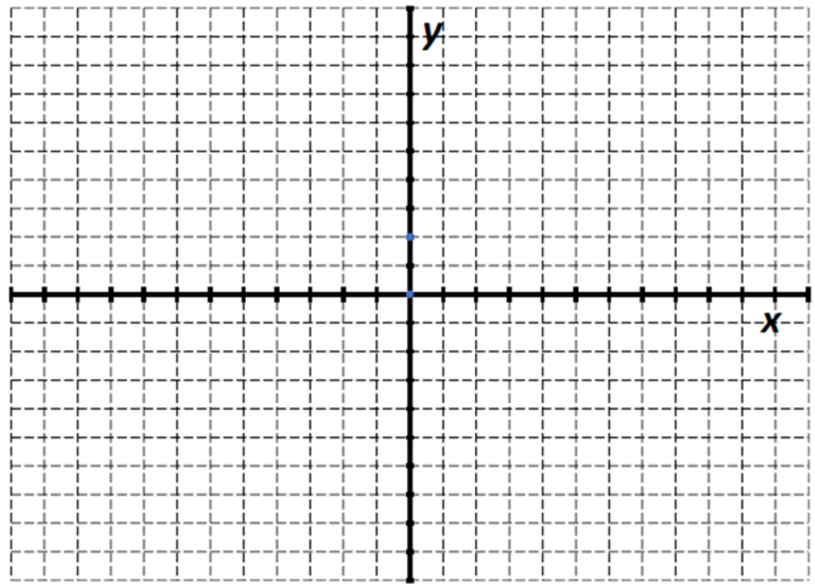


53. Graph
4x + 0.5y = 4

x	y
0	
	0

54. Graph
$\frac{2}{3}x + y = -6$

x	y
0	
	0



55. Graph
$\frac{2}{3}x + \frac{1}{2}y = -9$

x	y
0	
	0

Convert from general equation to y = function

55. The TI-83 Graphing calculator (nor many other graphing tools) does not allow you to enter a general form equation. It must be in the form
y = whatever

So x + 3y = 9 becomes:

3y = 9 –x which becomes

$$y = \frac{(9-x)}{3}$$

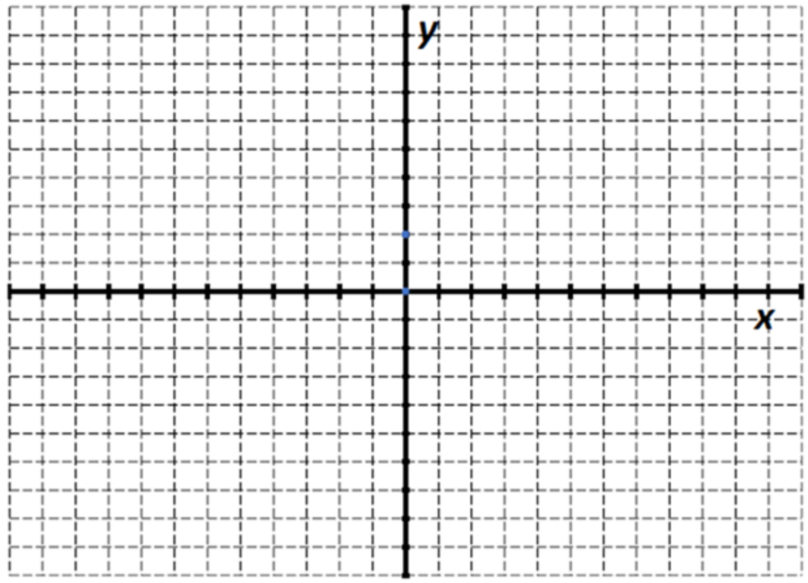
56. Manually graph both equations to see that they are the same:

x + 3y = 9

x	y
0	
	0

$$y = \frac{(9-x)}{3}$$

x	(9 − x) / 3	y
−6		
0		
3		



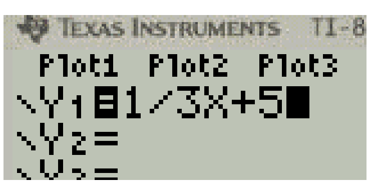
$x + 3y = 9$; $x + 3y - x = 9 - x$;

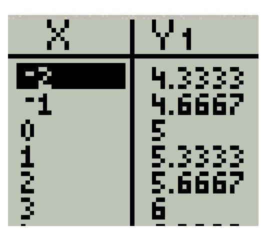
$\frac{3y}{3} = \frac{9 - x}{3}$; $y = \frac{(9-x)}{3}$ ✓

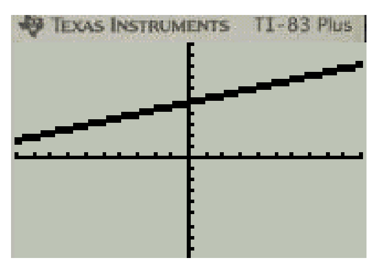
Graphing Tools

60. NOW that you know how to manually graph lines you can more frequently use a graphing tool to make the job quicker, and more accurate, and more reliable.

The TI-83 (or any 20 year old graphing calculator) as you are aware, readily takes an equation in the Slope and Intercept form and evaluates a table and then graphs the line

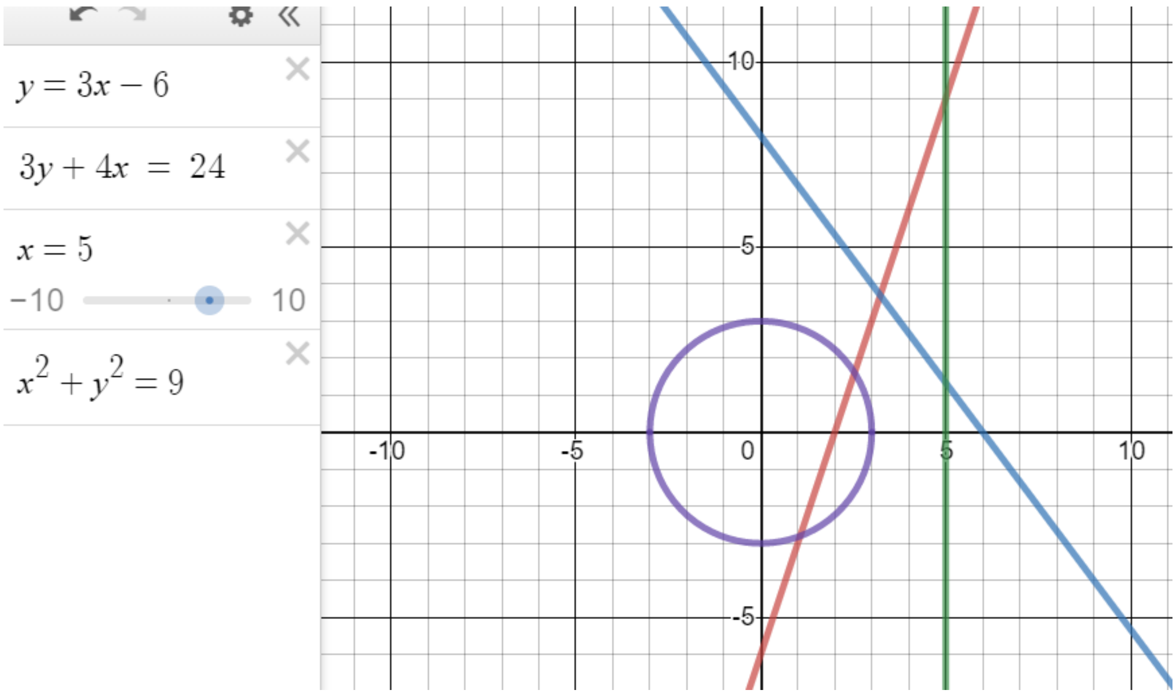






Go back to several of the equations **above** and see if your graphing tool gives the same table(s) and or graph(s).

61. The more modern **DESMOS** Graphing tool has many advantages
It can be done on any device! No App to download.
It can do *General Form* equations. It can graph vertical lines. And animate.



We are now well prepared to start graphing any functions! Very useful if you are going to do Applied Math Grade 12

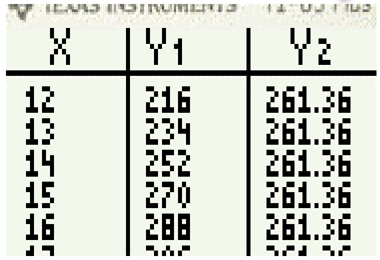
APPLICATIONS OF LINEAR FUNCTION GRAPHING (on a graphing tool)

62. If you are good at graphing you can solve so many problems, without that *nasty algebra*!

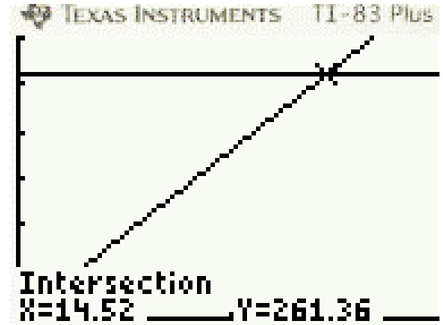
63. **Example**. If you spent $261.36 on t-shirts you got for a fund raiser that you were running and you have 18 t-shirts, how much is one t-shirt?

So 18 times ***what*** is 261.36?
261.36 = 18 * x, where ***x*** is the ***what*** number of t-shirts.

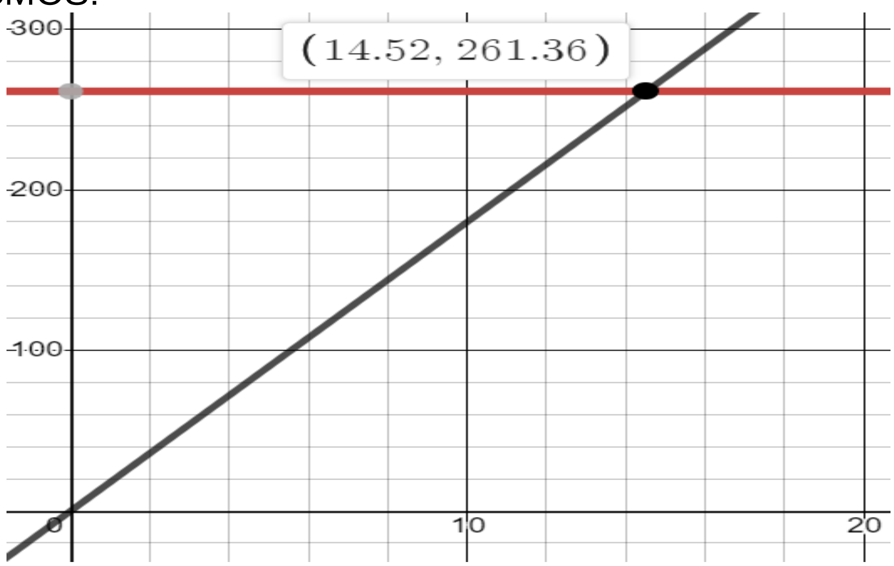
Algebra, guess and check, modelling, etc would solve this eventually
Graphing is better!



X	Y1	Y2
12	216	261.36
13	234	261.36
14	252	261.36
15	270	261.36
16	288	261.36



The solution is that x, the price of a shirt, is $14.52
Or in DESMOS:



65. Solve the following equations by graphing! Answers as decimals or fractions where desirable, either or both, your choice.

a. 2x = 8, solve for x (what is x?); x = ______

b. 2x + 6 = 34; x = _________

c. 4x – 8 = 10; x = _______

d. 37.2x – 22.5 = 4.1; x= _______

e. $4x - \frac{1}{5} = \frac{4}{5}$

f. Here is the type of thing that Grade 11 and 12 Applied does: for what ***x*** does $\mathbf{x^2 - 2x - 4}$ equal **20**? (*There are two answers!*) And if you can do it without graphing you would be doing the Pre-Calculus method.

[Fancy] Systems of Equations – WAY ADVANCED

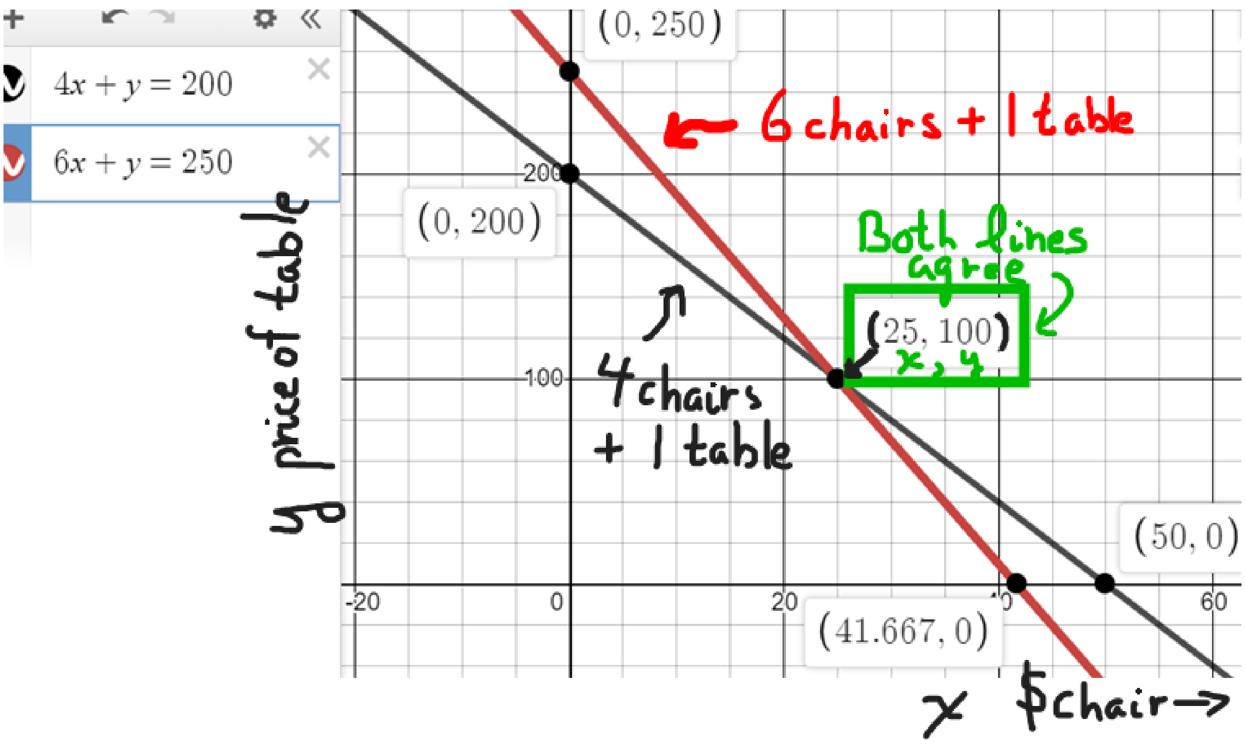
66. If **4 chairs** and **one table** cost $200 but the **6** of the same **chairs** plus the same **table** cost $250, how much does one **chair cost**? How much does one **table cost**?

Let **x** be the price of a chair, let **y** be the price of the table

You have two expressions:

4 x + 1 y = 200 ; and **6x +1 y = 250**

The answer is likely obvious. But graph it



67. **Example 2**. Teacher sends you to the store for donuts.
Quantity. Teacher wants a total quantity of 10 donuts; *some number* of chocolate and *some number* of maple.
Cost. Chocolate donuts cost $2 each and maple donuts cost $1 each. Teacher wants you to spend exactly $15.

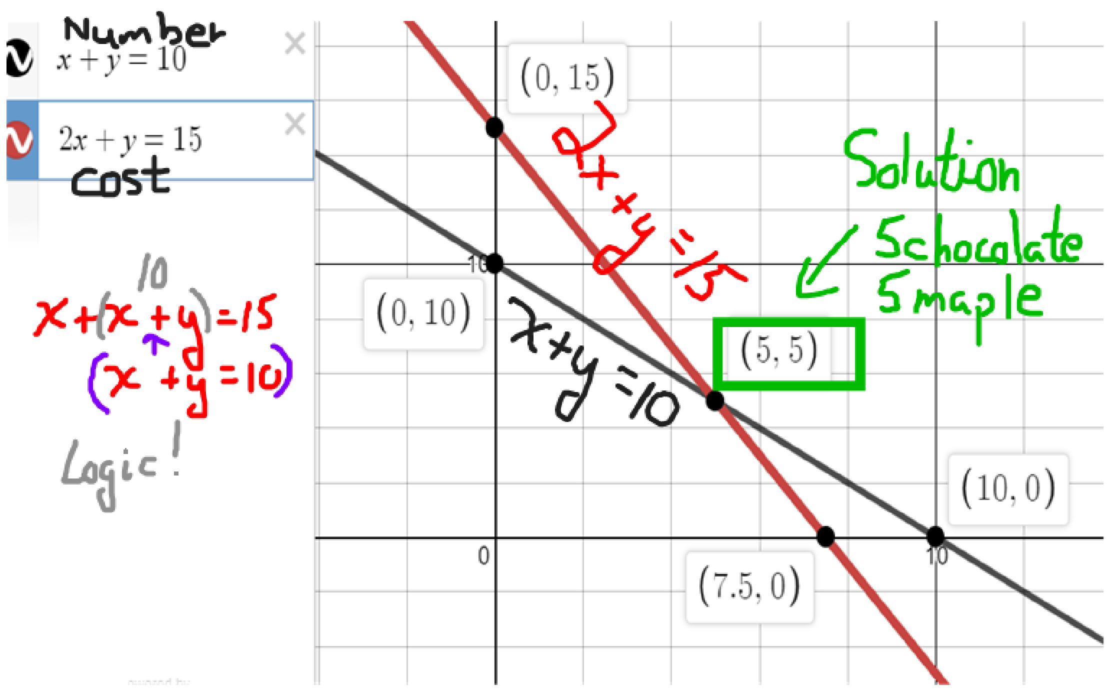
How many of each donut do you buy?

An expression for the number of donuts is x + y = 10; where x is the number of chocolate, y is the number of maple.

An expression for the cost of the purchase is 2*x + 1*y = 15

When are both statements 'satisfied'. How many donuts do you get so that you spend the full $15?

68. Of course Guess and Check would work! Algebra would work too if you know that. Logic would work. Drawing a picture would work! Modelling it with actual donuts would work. But just graph it; too easy.



Solution. 5 Chocolate and 5 Maple is the only solution!

69. This fancy method is called Systems of Linear Equations or Simultaneous Equations. A rather important tool in moderately advanced mathematics. See your teacher if you want to explore more

There is no real need to have a zillion practice exercises, you can invent your own and check them on a graphing tool if you are curious.

70. Enjoy your graphing skills!

There is some way funner stuff ahead if you do the Applied math!