Variables and Expressions (for Holt Algebra 1, Lesson 1-1)

- A **variable** is a letter that represents a value that can change.
- A **constant** is a value that does not change.
- A **numerical expression** contains only constants and operations.
- An **algebraic expression** also contains variables.
- To **evaluate** an expression is to find its value.
- To translate between algebraic expressions and words, look for key words or phrases.
- To evaluate algebraic expressions, substitute the given values for the variables and simplify.

Key Words and Phrases for Translating Between Expressions and Words			
Addition +	**Subtraction –**	**Multiplication ×**	**Division ÷**
plus, sum, increased by, put together, combined	minus, difference, less than, find how much more or less	times, product, equal groups of, put together equal groups	divided by, quotient, separate into equal groups

Examples

TRANSLATING BETWEEN WORDS AND ALGEBRA

1. Give two ways to write each algebraic expression in words.

A $x + 3$

the sum of x and 3
x increased by 3

B $m - 7$

the difference of m and 7
7 less than m

C $2 \bullet y$

2 times y
the product of 2 and y

D $k \div 5$

k divided by 5
the quotient of k and 5

2. Eva reads 25 pages per hour. Write an expression for the number of pages she reads in *h* hours.

$25 \bullet h$ or $25h$ *Multiply to put together h equal groups of 25.*

EVALUATING ALGEBRAIC EXPRESSIONS

3. Evaluate each expression for $x = 8$, $y = 5$, and $z = 4$.

A $x + y$

$$x + y = 8 + 5$$
$$= 13$$

B $\frac{x}{z}$

$$\frac{x}{z} = \frac{8}{4}$$
$$= 2$$

C yz

$$y \bullet z = 5 \bullet 4$$
$$= 20$$

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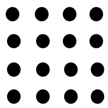
Powers and Exponents (for Holt Algebra 1, Lesson 1-4)

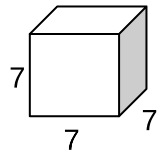
- A **power** is an expression written with an *exponent* and a *base*.
- The **base** is the number used as a factor.
- The **exponent** tells how many times the base is used as a factor.

base → 5^3 ← exponent $\qquad 5^3 = 5 \bullet 5 \bullet 5 = 125$

- Powers of 2 and 3 can be represented by geometric models.



4^2



7^3

- Powers can be written and evaluated using repeated multiplication.

Reading Exponents			
Words	**Multiplication**	**Power**	**Value**
3 to the first power	3	3^1	3
3 to the second power, or 3 squared	$3 \bullet 3$	3^2	9
3 to the third power, or 3 cubed	$3 \bullet 3 \bullet 3$	3^3	27
3 to the fourth power	$3 \bullet 3 \bullet 3 \bullet 3$	3^4	81
3 to the fifth power	$3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$	3^5	243

Examples

EVALUATING POWERS

1. Evaluate each expression.

A $(-2)^3$

Use –2 as a factor 3 times.

$(-2)(-2)(-2) = -8$

B -5^2

Think of a negative in front of a power as –1.

$-1 \bullet 5 \bullet 5 = -25$

C $\left(\frac{2}{3}\right)^2$

Use $\frac{2}{3}$ as a factor 2 times.

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

WRITING POWERS

2. Write each number as the power of a given base.

A 8; base 2

The product of three 2's is 8.

$2 \bullet 2 \bullet 2 = 2^3$

B –125; base –5

The product of three –5's is –125.

$(-5)(-5)(-5) = (-5)^3$

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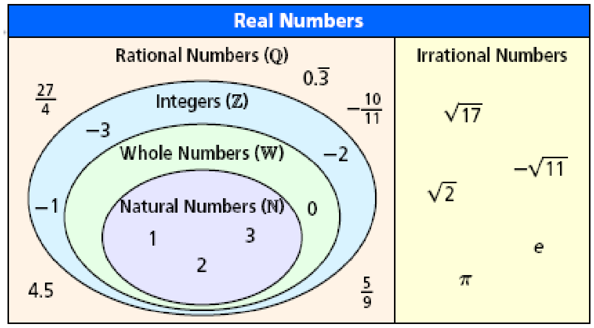
Square Roots and Real Numbers (for Holt Algebra 1, Lesson 1-5)

- A number that is multiplied by itself to form a product is called a **square root** of that product.
- A **perfect square** is a number whose positive square root is a *whole number*. Some perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.
- **Real numbers** can be represented on the number line. They can be classified according to their characteristics, as follows:
- The **natural numbers** are the counting numbers (1, 2, 3...); the **whole numbers** are the natural numbers and 0.
- **Integers** are whole numbers and their opposites (...–3, –2, –1, 0, 1, 2, 3...).
- **Rational numbers** can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not zero. They include both **terminating decimals** and **repeating decimals**.



- **Irrational numbers** cannot be expressed in the form $\frac{a}{b}$.

Examples

FINDING SQUARE ROOTS OF PERFECT SQUARES

1. Find each square root.

A $\sqrt{49}$

$7^2 = 49$ *Think: 7 squared is 49.*

$\sqrt{49} = 7$ *Positive square root → positive 7*

B $-\sqrt{36}$

$6^2 = 36$ *Think: 6 squared is 36.*

$-\sqrt{36} = -6$ *Negative square root → negative 6.*

CLASSIFYING REAL NUMBERS

2. Write all classifications that apply to each real number.

A $\frac{8}{9}$

$8 \div 9 = 0.888... = 0.\overline{8}$

$\frac{8}{9}$ can be written as a repeating decimal.

rational number, repeating decimal

B 18

$18 = \frac{18}{1} = 18.0$

18 can be written as a fraction and a decimal.

rational number, terminating decimal, integer, whole number, natural number

C $\sqrt{20}$

$\sqrt{20} = 4.472135...$

The digits of $\sqrt{20}$ continue with no pattern.

irrational number

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Order of Operations (for Holt Algebra 1, Lesson 1-6)

- The **order of operations** is a set of rules that tells what sequence to use when simplifying expressions that contain more than one operation.

Order of Operations	
First:	Perform operations inside grouping symbols.
Second:	Simplify powers.
Third:	Perform multiplication and division from left to right.
Fourth:	Perform addition and subtraction from left to right.

- Grouping symbols include parentheses ( ), brackets [ ], braces { }, fraction bars, radical symbols, and absolute-value symbols.
- If an expression contains more than one grouping symbol, simplify the innermost set first. Within each set, follow the order of operations.
- Grouping symbols may be used when translating from words to math. The product of 6 and the sum of 9 and 8 is written 6(9 + 8).

Examples

SIMPLIFYING NUMERICAL EXPRESSIONS

Simplify each expression.

1. $-4^2 + 24 \div 3 \bullet 2$

$-4^2 + 24 \div 3 \bullet 2$	*There are no grouping symbols.*
$-16 + 24 \div 3 \bullet 2$	*Simplify the power. The exponent belongs only to the 4.*
$-16 + 8 \bullet 2$	*Divide.*
$-16 + 16$	*Multiply.*
0	*Add.*

2. $\left|10 - 5^2\right| \div 5$

$\left|10 - 5^2\right| \div 5$	*The absolute-value symbols act as grouping symbols.*
$\left|10 - 25\right| \div 5$	*Simplify the power.*
$\left|-15\right| \div 5$	*Subtract within the absolute-value symbols.*
$15 \div 5$	*The absolute value of –15 is 15.*
3	*Divide.*

EVALUATING ALGEBRAIC EXPRESSIONS

3. Evaluate 21 – x + 2 • 5 for x = 7.

$21 - x + 2 \bullet 5$	
$21 - 7 + 2 \bullet 5$	*Substitute 7 for x. Then follow the order of operations.*
$21 - 7 + 10$	*Multiply.*
$14 + 10$	*Subtract.*
24	*Add.*

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Simplifying Expressions (for Holt Algebra 1, Lesson 1-7)

- The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange and simplify an expression.
- The Distributive Property can be used with addition or subtraction. It is often used as a mental math strategy.

Properties of Addition and Multiplication

PROPERTY	WORDS	NUMBERS	ALGEBRA
Commutative	You can add numbers in any order and multiply numbers in any order.	$2 + 7 = 7 + 2$ $3 \bullet 9 = 9 \bullet 3$	$a + b = b + a$ $ab = ba$
Associative	When you are only adding or multiplying, you can group any of the numbers together.	$6 + 8 + 2$ $= (6 + 8) + 2$ $= 6 + (8 + 2)$ $7 \bullet 4 \bullet 5$ $= (7 \bullet 4) \bullet 5$ $= 7 \bullet (4 \bullet 5)$	$a + b + c$ $= (a + b) + c$ $= a + (b + c)$ abc $= (ab)c$ $= a(bc)$
Distributive	You can multiply a number by a sum or multiply by each number in the sum and then add. The result is the same.	$3(4 + 8) = 3(4) + 3(8)$	$a(b + c) = a(b) + a(c)$

- The **like terms** of an expression contain the same variables raised to the same powers. You can combine like terms by adding or subtracting the **coefficients** (the numbers multiplying the variables) and keeping the variables and exponents the same.

Examples

USING THE COMMUTATIVE, ASSOCIATIVE, AND DISTRIBUTIVE PROPERTIES

1. Simplify each expression.

A $4 \bullet 9 \bullet 25$

$9 \bullet 4 \bullet 25$ *Commutative Property*

$9 \bullet (4 \bullet 25)$ *Associative Property*

$9 \bullet 100$ *Simplify parentheses.*

900

B $15(103)$

$15(100 + 3)$ *Rewrite 103 as 100 + 3.*

$15(100) + 15(3)$ *Distributive Property*

$1500 + 45$ *Multiply.*

1545 *Add.*

COMBINING LIKE TERMS

2. Simplify each expression by combining like terms.

A $12x + 30x$

These are like terms. Add the coefficients.

$12x + 30x = 42x$

B $8y^2 - y^2$

These are like terms. y^2 has a coefficient of 1.

$8y^2 - 1y^2 = 7y^2$

C $4n + 11n^2$

These are not like terms. Do not combine them.

$4n + 11n^2$

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Solving Equations by Adding or Subtracting

(for Holt Algebra 1, Lesson 2-1)

- An **equation** is a statement that two expressions are equal.
- A **solution of an equation** is a value of a variable that makes the equation true.
- To solve an equation, *isolate the variable* by getting it by itself on one side of the equal sign.
- To isolate a variable, use *inverse operations* to "undo" operations on the variable.

Inverse Operations	
Operation	**Inverse Operation**
Addition	Subtraction
Subtraction	Addition

Examples

SOLVING EQUATIONS USING ADDITION AND SUBTRACTION

Solve each equation.

1. $x - 10 = 4$

$$\begin{aligned} x - 10 &= 4 \\ \underline{+10} &\ \underline{+10} \\ x + 0 &= 14 \\ x &= 14 \end{aligned}$$

10 is subtracted from x.
So, add 10 to both sides to undo the subtraction.

Check $x - 10 = 4$

$$\begin{array}{r|l} 14 - 10 & 4 \\ 4 & 4\ \checkmark \end{array}$$

To check, substitute 14 for x in the original equation.

2. $x + 7 = 9$

$$\begin{aligned} x + 7 &= 9 \\ \underline{-7} &\ \underline{-7} \\ x + 0 &= 2 \\ x &= 2 \end{aligned}$$

7 is added to x.
So, subtract 7 from both sides to undo the addition.

Check $x + 7 = 9$

$$\begin{array}{r|l} 2 + 7 & 9 \\ 9 & 9\ \checkmark \end{array}$$

To check, substitute 2 for x in the original equation.

Properties of equality justify the use of inverse operations. They say that you can add or subtract the same number from both sides of an equation.

Addition and Subtraction Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$

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Solving Equations by Multiplying or Dividing

(for Holt Algebra 1, Lesson 2-2)

- Solving an equation by multiplying or dividing is similar to solving an equation by adding or subtracting. The goal is to isolate the variable on one side of the equal sign.
- Use *inverse operations* to "undo" operations on the variable.
- Whatever you do on one side of the equal sign, you must do on the other.

Inverse Operations	
Operation	**Inverse Operation**
Multiplication	Division
Division	Multiplication

Examples

SOLVING EQUATIONS USING MULTIPLICATION AND DIVISION

Solve each equation.

1. $-4 = \frac{k}{-5}$

$(-5)(-4) = (-5)\left(\frac{k}{-5}\right)$ *Since k is divided by –5, multiply both sides by –5 to undo the division.*

$20 = k$

2. $7x = 56$

$\frac{7x}{7} = \frac{56}{7}$ *Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.*

$x = 8$

3. $\frac{5}{9}v = 35$ *Dividing is the same as multiplying by the reciprocal.*

$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$ *Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.*

$v = 63$

The properties of equality justify the use of inverse operations.

Multiplication and Division Properties of Equality		
WORDS	**NUMBERS**	**ALGEBRA**
Multiplication Property of Equality You can multiply both sides of an equation by the same number, and the statement will still be true.	$6 = 6$ $6(3) = 6(3)$ $18 = 18$	$a = b$ $ac = bc$
Division Property of Equality You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$8 = 8$ $\frac{8}{4} = \frac{8}{4}$ $2 = 2$	$a = b$ $(c \neq 0)$ $\frac{a}{c} = \frac{b}{c}$

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Solving Two-Step and Multi-Step Equations
(for Holt Algebra 1, Lesson 2-3)

- To solve equations with more than one operation:
  - First identify the operations in the equation and the order in which they are applied to the variable.
  - Then use inverse operations and work backward to undo them one at a time.
- Some equations may have to be simplified before using inverse operations.
- Check answers by substituting the solution into the original equation.

Examples

SOLVING TWO-STEP AND MULTI-STEP EQUATIONS

Solve each equation.

1. $2x + 6 = 10$

$2x + 6 = 10$ — *First x is multiplied by 2. Then 6 is added.*

$\underline{\quad -6 \quad -6}$ — *Work backward: First subtract 6 from both sides.*

$2x = 4$

$\dfrac{2x}{2} = \dfrac{4}{2}$ — *Then divide both sides by 2 to undo the multiplication.*

$1x = 2$

$x = 2$

2. $\dfrac{q}{15} - \dfrac{1}{5} = \dfrac{3}{5}$ — *To eliminate fractions, multiply by the least common denominator.*

$15\left(\dfrac{q}{15} - \dfrac{1}{5}\right) = 15\left(\dfrac{3}{5}\right)$ — *Multiply both sides by 15.*

$15\left(\dfrac{q}{15}\right) - 15\left(\dfrac{1}{5}\right) = 15\left(\dfrac{3}{5}\right)$ — *Distribute 15 on the left side.*

$q - 3 = 9$ — *Simplify.*

$\underline{\quad +3 \quad +3}$ — *Add 3 to both sides.*

$q = 12$

3. $2(3x + 1) - 8x = 12$

$2(3x + 1) - 8x = 12$ — *Distribute 2 on the left side.*

$2(3x) + 2(1) - 8x = 12$

$6x + 2 - 8x = 12$ — *Simplify.*

$6x - 8x + 2 = 12$ — *Use the Commutative Property of Addition.*

$-2x + 2 = 12$ — *Combine like terms.*

$\underline{\quad -2 \quad -2}$ — *Subtract 2 from both sides.*

$-2x = 10$

$\dfrac{-2x}{-2} = \dfrac{10}{-2}$ — *Divide by –2 on both sides.*

$x = -5$

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Rates, Ratios, and Proportions (for Holt Algebra 1, Lesson 2-6)

- A **ratio** is a comparison of two quantities by division. For example, the ratio of men to women at a movie theater can be written as 13:18 or $\frac{13}{18}$.
- A **proportion** is a statement that two ratios are equivalent, like $\frac{1}{12} = \frac{2}{24}$.
- In the proportion $\frac{a}{b} = \frac{c}{d}$, the **cross products** are $a \bullet d$ and $b \bullet c$. In a proportion, cross products are equal: $ad = bc$ (if $b \neq 0$ and $d \neq 0$).

$$\frac{a}{b} \times \frac{c}{d}$$

Example

USING RATIOS AND SOLVING PROPORTIONS

1. The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?

$$\frac{\text{faculty}}{\text{students}} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{1}{15}$$

Write a ratio comparing faculty to students.

$$\frac{1}{15} = \frac{x}{675}$$

Write a proportion. Let x be the number of faculty members.

Method A - Solve as an equation.

$$\frac{1}{15} = \frac{x}{675}$$

$$675\left(\frac{1}{15}\right) = 675\left(\frac{x}{675}\right)$$

$$45 = x$$

Method B - Solve using cross products.

$$\frac{1}{15} \times \frac{x}{675}$$

$$1(675) = 15(x)$$

$$\frac{675}{15} = \frac{15x}{15}$$

$$45 = x$$

There are 45 faculty members.

- A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$.

  A **unit rate** has a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal.

- If the two quantities are equal, but use different units, such as $\frac{12 \text{ in.}}{1 \text{ ft}}$, the rate is a **conversion factor**.

Example

CONVERTING RATES

2. A runner ran at a rate of 6 mi/h. What is this speed in mi/min?

$$\frac{6 \text{ mi}}{1 \text{ h}} \bullet \frac{1 \text{ h}}{60 \text{ min}}$$

The second quantity is being converted. So, multiply by a conversion factor with that unit in the first quantity.

$$\frac{1 \text{ mi}}{10 \text{ min}}$$

Multiply and simplify.

The speed is 0.1 mi/min.

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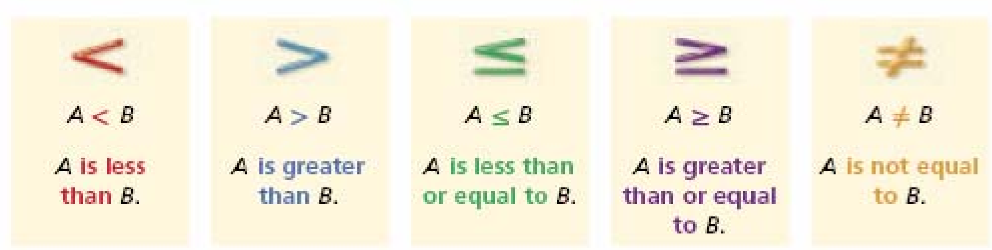
Graphing and Writing Inequalities (for Holt Algebra 1, Lesson 3-1)

- An **inequality** is a statement that two quantities are not equal.
- The quantities are compared using one of the following signs.



- A **solution of an inequality** is any value that makes the inequality true.
- Because most inequalities have too many solutions to list, they are typically shown on a number line. The solutions are shaded and an arrow shows that solutions continue past those that are shown on the graph.

  A *solution circle* indicates that the endpoint *is* a solution. It corresponds to ≤ and ≥ signs.

  An *empty circle* indicates that the endpoint *is not* a solution. It corresponds to < and > signs.

Examples

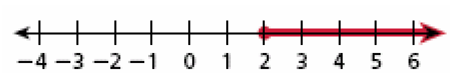
GRAPHING INEQUALITIES

Graph each inequality.

1. $r \geq 2$

The inequality states that r is all real numbers greater than or equal to 2.

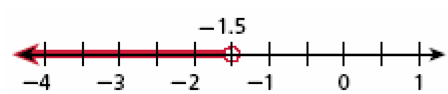
Draw a solid circle at 2.
Shade all the numbers greater than 2.
Draw an arrow pointing to the right.



2. $b < -1.5$

The inequality states that b is all real numbers less than –1.5.

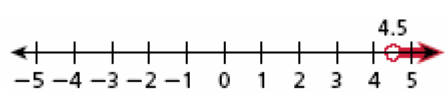
Draw an empty circle at –1.5.
Shade all the numbers less than –1.5.
Draw an arrow pointing to the left.



WRITING AN INEQUALITY FROM A GRAPH

Write the inequality shown by each graph.

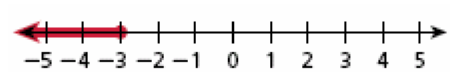
3.



Use any variable. The arrow points to the right and the circle is empty, so use >.

$h > 4.5$

4.



Use any variable. The arrow points to the left and the circle is solid, so use ≤.

$m \leq -3$

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Solving Inequalities by Adding or Subtracting

(for Holt Algebra 1, Lesson 3-2)

- Solving one-step inequalities is much like solving one-step equations.
- To solve an inequality you need to isolate the variable using the properties of inequality and inverse operations.

Addition and Subtraction Properties of Inequality		
WORDS	**NUMBERS**	**ALGEBRA**
Addition Property of Inequality You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction Property of Inequality You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$
These properties are also true for inequalities that use the symbols $>$, $\geq$, **and** $\leq$.		

Examples

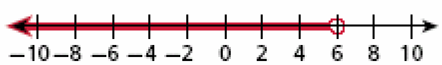
USING ADDITION AND SUBTRACTION TO SOLVE INEQUALITIES

Solve each inequality and graph the solutions.

1. $x + 9 < 15$

$$\begin{aligned} x + 9 &< 15 \\ \underline{\quad -9} &\;\; \underline{-9} \\ x + 0 &< 6 \\ x &< 6 \end{aligned}$$

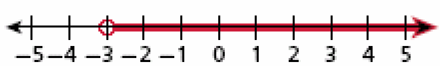
Since 9 is added to x, subtract 9 from both sides to undo the addition.



2. $d - 3 > -6$

$$\begin{aligned} d - 3 &> -6 \\ \underline{\quad +3} &\;\; \underline{+3} \\ d + 0 &> -3 \\ d &> -3 \end{aligned}$$

Since 3 is subtracted from d, add 3 to both sides to undo the subtraction.



- It is not possible to check all the solutions to an inequality. You can check the endpoint and the direction of the inequality symbol.

The solutions of $d - 3 > -6$ are given by $d > -3$.

Step 1: Check the endpoint.

Substitute –3 for d in d – 3 = –6.

$$\begin{array}{r|l} \multicolumn{2}{c}{d - 3 = -6} \\ \hline -3 - 3 & -6 \\ -6 & -6 \; \checkmark \end{array}$$

Step 2: Check the inequality symbol.

Substitute a number greater than –3 into d – 3 > –6.

$$\begin{array}{r|c|l} \multicolumn{3}{c}{d - 3 > -6} \\ \hline 5 - 3 & > & -6 \\ 2 & > & -6 \; \checkmark \end{array}$$

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Solving Inequalities by Multiplying or Dividing
(for Holt Algebra 1, Lesson 3-3)

- Solving inequalities is much like solving equations. But the properties of inequality are different when multiplying or dividing by a *positive* number, than when multiplying or dividing by a *negative* number.

Properties of Inequality - Multiplication and Division by Positive Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication You can multiply both sides of an inequality by the same *positive* number, and the statement will still be true.	$7 < 12$ $7(3) < 12(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division You can divide both sides of an inequality by the same *positive* number, and the statement will still be true.	$15 < 35$ $\frac{15}{5} < \frac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.
These properties are also true for inequalities that use the symbols $>$, $\geq$, **and** $\leq$.		

Properties of Inequality - Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication If you multiply both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$8 > 4$ $8(-2) \quad 4(-2)$ $-16 \quad -8$ $-16 < -8$	If $a > b$ and $c < 0$, then $ac < bc$.
Division If you divide both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$12 > 4$ $\frac{12}{-4} \quad \frac{4}{-4}$ $-3 \quad -1$ $-3 < -1$	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.
These properties are also true for inequalities that use the symbols $>$, $\geq$, **and** $\leq$.		

Examples

MULTIPLYING AND DIVIDING BY POSITIVE AND NEGATIVE NUMBERS

Solve each inequality and graph the solutions.

1. $3x > -27$

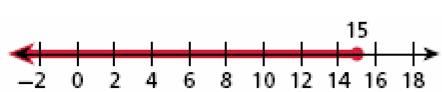
$$\frac{3x}{3} > \frac{-27}{3}$$
$$1x > -9$$
$$x > -9$$

Since x is multiplied by 3, divide by 3 on both sides to undo the multiplication.

2. $\frac{x}{-5} \geq -3$

$$(-5)\frac{x}{-5} \geq (-5)-3$$
$$x \leq 15$$

Since x is divided by –5, multiply both sides by –5. Change ≥ to ≤.



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Solving Two-Step and Multi-Step Inequalities

(for Holt Algebra 1, Lesson 3-4)

- Inequalities with more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.
- Some inequalities may need to be simplified first, as shown in Example 3.

Examples

SOLVING MULTI-STEP INEQUALITIES

Solve each inequality and graph the solutions.

1. $160 + 4f \le 500$

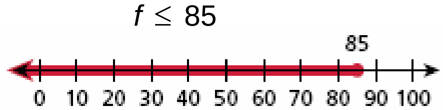
$$\begin{aligned} 160 + 4f &\le 500 \\ \underline{-160\qquad} &\quad \underline{-160} \\ 4f &\le 340 \end{aligned}$$

Since 160 is added to 4f, subtract 160 from both sides to undo the addition.

$$\frac{4f}{4} \le \frac{340}{4}$$

$$f \le 85$$

Since f is multiplied by 4, divide by 4 on both sides to undo the multiplication.



2. $7 - 2t \le 21$

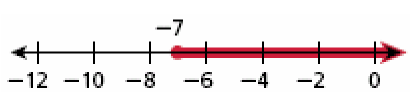
$$\begin{aligned} 7 - 2t &\le 21 \\ \underline{-7\qquad} &\quad \underline{-7} \\ -2t &\le 14 \end{aligned}$$

Since 7 is added to –2t, subtract 7 from both sides to undo the addition.

$$\frac{-2t}{-2} \le \frac{14}{-2}$$

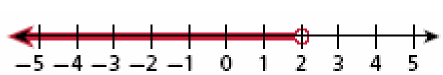
$$t \ge -7$$

Since t is multiplied by –2, divide both sides by –2 to undo the multiplication.
Change $\le$ to $\ge$ (dividing by a negative).



3. $-3^2 + 4 < -5(c - 1)$

$$\begin{aligned} -3^2 + 4 &< -5(c - 1) && \\ -9 + 4 &< -5(c - 1) && \textit{Square 3 on the left side.} \\ -5 &< -5(c - 1) && \textit{Simplify the left side.} \\ -5 &< -5c + 5 && \textit{Distribute –5 on the right side.} \\ \underline{-5} &\quad \underline{\qquad -5} && \textit{Subtract 5 from both sides.} \\ -10 &< -5c && \\ \frac{-10}{-5} &< \frac{-5c}{-5} && \textit{Divide both sides by –5. Change < to > (dividing by a negative).} \\ 2 &> c \text{ (or } c < 2) && \end{aligned}$$



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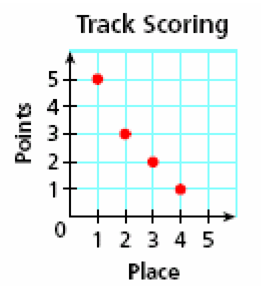
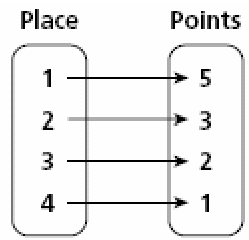
- A **relation** is a set of ordered pairs that represents a relationship. For example, the relation {(1, 5), (2, 3), (3, 2), (4, 1)} represents the points earned for each place at a track meet. A relation can also be shown as a table, a graph, or a mapping diagram.

Track Scoring	
Place	Points
1	5
2	3
3	2
4	1





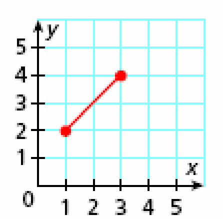
- The **domain** of a relation is the set of first coordinates (or *x*-values) of the ordered pairs. In the example above, the domain is D: {1, 2, 3, 4}.
- The **range** of a relation is the set of second coordinates (or *y*-values) of the ordered pairs. In the example above, the range is R: {5, 3, 2, 1}.
- A relation is a **function** if it pairs each domain value with exactly one range value. The relation shown above is a function.

Examples

FINDING DOMAIN AND RANGE AND IDENTIFYING FUNCTIONS

Give the domain and range of the relation. Then tell whether the relation is a function. Explain.

1.



The domain is all x-values from 1 through 3, inclusive.

The range is all y-values from 2 through 4, inclusive.

D: $1 \le x \le 3$ and R: $2 \le x \le 4$

This relation is a function. Each domain value is paired with exactly one range value.

2.

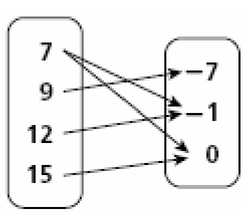
x	4	0	0	–4
y	0	4	–4	0

D: {–4, 0, 4} *The domain is all x-values in the table.*

R: {–4, 0, 4} *The range is all y-values in the table.*

This relation is not a function. The domain value 0 is paired with both 4 and –4.

3.



D: {7, 9, 12, 15} and R: {–7, –1, 0}

Use the arrows to determine which domain values correspond to each range value.

This relation is not a function. The domain value 7 is paired with both –1 and 0.

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Writing Functions (for Holt Algebra 1, Lesson 4-3)

- The input of a function is also called the **independent variable**.
  The output of a function is also called a **dependent variable**.
  A way to remember these names is that the value of the dependent variable *depends* on, or is a function of, the value of the independent variable.
- Sometimes functions are written in **function notation**. If x is the independent variable and y is the dependent variable, then function notation for y is $f(x)$, which is read "f of x". For example, the relation $y = 5x$ can be written in function notation as $f(x) = 5x$.
- The algebraic expression that defines a function is called a **function rule**.
  In $f(x) = 5x$, the function rule is $5x$.
- You can *evaluate a function* by using the function rule.

Examples

WRITING FUNCTIONS

1. Identify the independent and dependent variables. Write a rule in function notation for each situation.

A A lawyer's fee is $200 per hour for her services.

The fee for the lawyer depends on how many hours she works.
Dependent: fee   Independent: hours

Let h represent the number of hours the lawyer works.
The function for the lawyer's fee is $f(h) = 200h$.

B The admission fee to a local carnival is $8. Each ride costs $2.

The total cost depends on the number of rides ridden.
Dependent: total cost   Independent: number of rides

Let r represent the number of rides ridden.
The function for the total cost is $f(r) = 2r + 8$.

EVALUATING FUNCTIONS

2. Evaluate each function for the given input value.

A For $f(x) = 2x + 10$, find $f(x)$ when $x = 6$.

$f(x) = 2x + 10$
$f(6) = 2(6) + 10$ *Substitute 6 for x.*
$= 12 + 10$ *Simplify.*
$= 22$

B For $g(t) = \frac{1}{2}t - 3$, find $g(t)$ when $t = 12$.

$g(t) = \frac{1}{2}t - 3$
$g(12) = \frac{1}{2}(12) - 3$ *Substitute 12 for t.*
$= 6 - 3$ *Simplify.*
$= 3$

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Arithmetic Sequences (for Holt Algebra 1, Lesson 4-6)

- A **sequence** is a list of numbers that often forms a pattern.
- Each number in a sequence is called a **term**.

6, 8, 10, 12, ... In this sequence, you can add 2 to each term to get the next term.

- When the terms differ by the same non-zero number *d*, the sequence is an **arithmetic sequence** and *d* is the **common difference**.

Example

IDENTIFYING ARITHMETIC SEQUENCES

1. Determine whether 12, 8, 4, 0, ... appears to be an arithmetic sequence. If so, find the common difference and the next 3 terms.

Step 1: Find the difference between successive terms.

$$12, \underbrace{\quad 8,}_{-4} \underbrace{\quad 4,}_{-4} \underbrace{\quad 0,}_{-4} \ldots$$

You add –4 to each term to find the next term. The common difference is –4.

Step 2: Use the common difference to find the next three terms.

$$12, 8, 4, 0, \underbrace{\quad -4,}_{-4} \underbrace{\quad -8,}_{-4} \underbrace{\quad -12}_{-4}$$

The sequence appears to be an arithmetic sequence with a common difference of –4. The next three terms are –4, –8, and –12.

- The variable *a* is often used to represent the terms in a sequence. The variable a_n represents any term, or the *n*th term in a sequence. So the 9th term would be designated a_9, read "*a* sub 9."
- The *n*th term of an arithmetic sequence with common difference *d* and first term a_1 is $a_n = a_1 + (n - 1)d$.

Example

FINDING THE NTH TERM OF AN ARITHMETIC SEQUENCE

2. Find the 15th term of the arithmetic sequence 7, 10, 13, 16, ...

Step 1: Find the common difference.

$$7, \underbrace{\quad 10,}_{+3} \underbrace{\quad 13,}_{+3} \underbrace{\quad 16,}_{+3} \ldots$$

The common difference is 3.

Step 2: Write a rule to find the 15th term.

$a_n = a_1 + (n - 1)d$	*Write the rule to find the nth term.*
$a_{15} = 7 + (15 - 1)3$	*Substitute 7 for a_1, 15 for n, and 3 for d.*
$= 7 + (14)3$	*Simplify the expression in parentheses.*
$= 7 + 42$	*Multiply.*
$= 49$	*Add.*

The 15th term is 49.

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Rate of Change and Slope (for Holt Algebra 1, Lesson 5-3)

- A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

The table shows the cost of mailing a 1-ounce letter in different years.

Year	1985	1988	1990	1991
Cost (cents)	22	25	25	29

The independent variable is the year.
The dependent variable is the cost.

The rates of change for each interval are found as follows.

1985 to 1988: $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{25-22}{1988-1985} = \dfrac{3}{3} = 1 \Rightarrow \dfrac{1 \text{ cent}}{\text{year}}$

1988 to 1990: $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{25-25}{1990-1988} = \dfrac{0}{2} = 0 \Rightarrow \dfrac{0 \text{ cents}}{\text{year}}$

1990 to 1991: $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{29-25}{1991-1990} = \dfrac{4}{1} = 4 \Rightarrow \dfrac{4 \text{ cents}}{\text{year}}$

When graphed, each line segment would have a different steepness, corresponding to the rate of change for that interval.

- If all the segments of a graph have the same steepness, they form a straight line. This constant rate of change is called the **slope** of a line.
- The **rise** is the difference in the *y*-values of two points on a line. The **run** is the difference in the *x*-values of two points on a line.
- The slope of a line is the ratio of rise to run for any two points on the line.

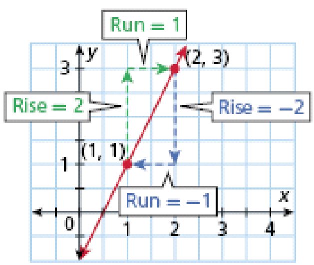
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

Examples

FINDING SLOPE

Find the slope of each line.

1.

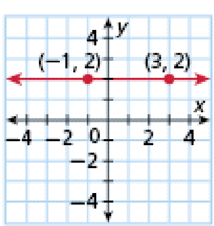


Begin at one point and count vertically to find the rise.
Then count horizontally to the second point to find the run.
It does not matter which point you start with. The slope is the same.

$\text{slope} = \dfrac{2}{1} = 2$

$\text{slope} = \dfrac{-2}{-1} = 2$

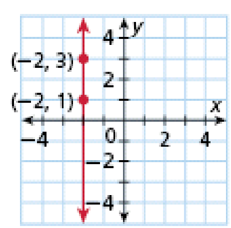
2.



$\dfrac{\text{rise}}{\text{run}} = \dfrac{0}{4} = 0$

The slope of all horizontal lines is 0.

3.



$\dfrac{\text{rise}}{\text{run}} = \dfrac{2}{0}$ *You cannot divide by 0.*

The slope of all vertical lines is undefined.

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The Slope Formula (for Holt Algebra 1, Lesson 5-4)

- The slope m of a line can be found using a formula. The formula can be used whether you are given a graph, a table, or an equation. All you need is the coordinates of two different points on the line.

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in y-values to the difference in x-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If (6, 2) and (8, 4) are two points on a line, the slope of the line is $m = \frac{4-2}{8-6} = \frac{2}{2} = 1$.

Example

FINDING SLOPE BY USING THE SLOPE FORMULA

1. Find the slope of the line that contains (4, –2) and (–1, 2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

$$m = \frac{2-(-2)}{-1-4}$$ *Substitute (4, –2) for (x_1, y_1) and (–1, 2) for (x_2, y_2).*

$$m = \frac{4}{-5}$$ *Simplify.*

The slope of the line is $-\frac{4}{5}$.

- If you are given a table or graph that describes a line, you can find the slope by substituting any two different points into the slope formula.
- If you are given an equation that describes a line, you can find the slope from any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

Example

FINDING SLOPE FROM AN EQUATION

2. Find the slope of the line described by $6x - 5y = 30$.

Step 1: Find the x-intercept.

$$6x - 5y = 30$$
$$6x - 5(0) = 30 \quad \textit{Let } y = 0.$$
$$6x = 30 \quad \textit{Simplify.}$$
$$x = 5$$

Step 2: Find the y-intercept.

$$6x - 5y = 30$$
$$6(0) - 5y = 30 \quad \textit{Let } x = 0.$$
$$-5y = 30 \quad \textit{Simplify.}$$
$$y = -6$$

The x-intercept is 5. The y-intercept is –6.

Step 3: The line contains (5, 0) and (0, –6). Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-5} = \frac{-6}{-5} = \frac{6}{5}$$

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Slope-Intercept Form (for Holt Algebra 1, Lesson 5-6)

- If you know the slope and y-intercept of a line, you can write an equation that describes the line.

Slope-Intercept Form of a Linear Equation
If a line has slope m and y-intercept b, then the line is described by the equation $y = mx + b$.

- Any linear equation can be written in *slope-intercept form* by solving for y and simplifying. In this form, you can immediately identify the slope and y-intercept and quickly graph the line.

Examples

WRITING LINEAR EQUATIONS IN SLOPE-INTERCEPT FORM

1. Write the equation that describes each line in slope-intercept form.

A slope = $\frac{1}{3}$, y-intercept = 6

$y = mx + b$

$y = \frac{1}{3}x + 6$ *Substitute the given values for m and b.*

B slope = 0, y-intercept = –5

$y = mx + b$

$y = 0x + (-5)$ *Substitute.*

$y = -5$ *Simplify.*

USING SLOPE-INTERCEPT FORM TO GRAPH

2. Write the equation $3x - 4y = 8$ in slope-intercept form. Then graph the line described by the equation.

Step 1: Solve the equation for y.

$$\begin{aligned} 3x - 4y &= 8 \\ -3x \qquad &\ \ -3x \end{aligned}$$ *Subtract 3x from both sides.*

$$-4y = -3x + 8$$

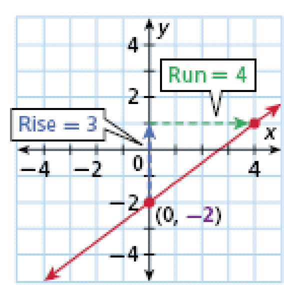
$$\frac{-4y}{-4} = \frac{-3x+8}{-4}$$ *Since y is multiplied by –4, divide both sides by –4.*

$$y = \frac{3}{4}x - 2$$

Step 2: The y-intercept is –2, so the line contains (0, –2). Plot the point (0, –2).

Step 3: Slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{3}{4}$. Count 3 units up and 4 units right from (0, –2) and plot another point.

Step 4: Draw the line through the two points.



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Point-Slope Form (for Holt Algebra 1, Lesson 5-7)

- You can graph a line if you know its slope and a point on the line.
- If you know the slope and a point on a line, you can write the equation of the line in *point-slope form*.

Point-Slope Form of a Linear Equation

The line with slope m and point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

- An equation written in point-slope form can be written in slope-intercept form and then quickly graphed.

Examples

USING SLOPE AND A POINT TO GRAPH

1. Graph the line that has a slope of $-\frac{1}{2}$ and contains the point (3, –2).

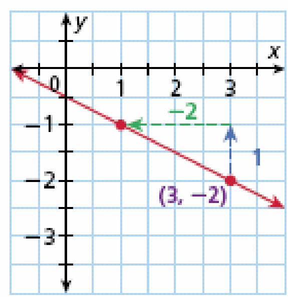
Step 1: Plot (3, –2).

Step 2: Use the slope to move from (3, –2) to another point.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = -\frac{1}{2} = \frac{1}{-2}$$

Move 1 unit up and 2 units left and plot another point.

Step 3: Draw the line connecting the two points.



WRITING LINEAR EQUATIONS IN POINT-SLOPE FORM

2. Write an equation in point-slope form for the line that has a slope of 7 and contains the point (4, 2).

$y - y_1 = m(x - x_1)$ *Write the general point-slope form of an equation.*

$y - 2 = 7(x - 4)$ *Substitute 7 for m, 4 for x_1, and 2 for y_1.*

USING TWO POINTS TO WRITE AN EQUATION

3. Write an equation in slope-intercept form for the line through the points (1, –4) and (3, 2).

Step 1: Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{3-1} = \frac{6}{2} = 3$$

Step 2: Substitute the slope and one of the points into the point-slope form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$ *Choose (3, 2).*

Step 3: Write the equation in slope-intercept form.

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$ *Distribute.*

$\underline{+2} \quad \underline{+2}$ *Add 2.*

$y = 3x - 7$

An equation for the line is $y = 3x - 7$.

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Solving Systems by Graphing (for Holt Algebra 1, Lesson 6-1)

- A **system of linear equations** is a set of two or more linear equations containing two or more variables.
- A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. This ordered pair is a point of intersection of the lines.

Examples

IDENTIFYING SOLUTIONS OF SYSTEMS

1. Tell whether the ordered pair is a solution of the given system.

A (4, 1); $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

$x + 2y$	$= 6$
$4 + 2(1)$	6
$4 + 2$	6
6	6 ✓

$x - y$	$= 3$
$4 - 1$	3
3	3 ✓

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair (4, 1) makes both equations true.

(4, 1) is a solution of the system.

B (–1, 2); $\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

$2x + 5y$	$= 8$
$2(-1) + 5(2)$	8
$-2 + 10$	8
8	8 ✓

$3x - 2y$	$= 5$
$3(-1) - 2(2)$	5
$-3 - 4$	5
-7	5 ✗

Substitute –1 for x and 2 for y in each equation in the system.

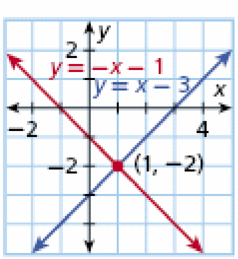
The ordered pair (–1, 2) makes one equation true but not the other.

(–1, 2) is not a solution of the system.

SOLVING A SYSTEM OF LINEAR EQUATIONS BY GRAPHING

2. Solve the system by graphing.

$\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$



Graph the system. Find the point of intersection.

The solution appears to be (1, –2).

Check by substituting (1, –2) into the system.

y	$= x - 3$
(-2)	$(1) - 3$
-2	-2 ✓

y	$= -x - 1$
(-2)	$-(1) - 1$
-2	-2 ✓

(1, –2) is a solution of the system.

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Solving Systems by Substitution (for Holt Algebra 1, Lesson 6-2)

- Sometimes it is difficult to identify the exact solution of a system by graphing. In this case you can use a method called *substitution*.

Solving Systems of Equations by Substitution
Step 1 Solve for one variable in at least one equation, if necessary.
Step 2 Substitute the resulting expression into the other equation.
Step 3 Solve that equation to get the value of the first variable.
Step 4 Substitute that value into one of the original equations and solve.
Step 5 Write the values from Steps 3 and 4 as an ordered pair, (x, y), and check.

Example

SOLVING A SYSTEM OF LINEAR EQUATIONS BY SUBSTITUTION

Solve $\begin{cases} x + 4y = 6 \\ x + y = 3 \end{cases}$ **by substitution.**

Step 1:
$$\begin{aligned} x + 4y &= 6 \\ -4y \quad & \;\; -4y \\ x &= 6 - 4y \end{aligned}$$

Solve the first equation for x.

Step 2:
$$\begin{aligned} x + y &= 3 \\ (6 - 4y) + y &= 3 \end{aligned}$$

Substitute 6 – 4y into the second equation for x.

Step 3:
$$\begin{aligned} 6 - 3y &= 3 \\ -6 \qquad & \;\; -6 \\ \frac{-3y}{-3} &= \frac{-3}{-3} \\ y &= 1 \end{aligned}$$

Simplify by combining like terms.
Solve for y. Subtract 6 from both sides.

Divide both sides by –3.

Step 4:
$$\begin{aligned} x + y &= 3 \\ x + 1 &= 3 \\ -1 \; & \;\; -1 \\ x &= 2 \end{aligned}$$

Write one of the original equations.
Substitute 1 for y.
Subtract 1 from both sides.

Step 5: (2, 1)

Write the solution as an ordered pair.

Check Substitute (2, 1) into both equations in the system.

$x + 4y$	$= 6$
$2 + 4(1)$	6
$2 + 4$	6
6	6 ✓

$x + y$	$= 3$
$2 + 1$	3
3	3 ✓

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Solving Systems by Elimination (for Holt Algebra 1, Lesson 6-3)

- Systems can also be solved by a process called *elimination*. This involves adding equations to eliminate opposite terms, such as $2y$ and $-2y$.

Solving Systems of Equations by Elimination
Step 1 Write the system so that like terms are aligned.
Step 2 Eliminate one of the variables and solve for the other variable.
Step 3 Substitute that value into one of the original equations and solve for the other variable.
Step 4 Write the answers from Steps 2 and 3 as an ordered pair, (x, y), and check.

- In some cases, you will first need to multiply one or both of the equations by a number to create opposite terms.

Examples

USING ELIMINATION

1. Solve $\begin{cases} \mathbf{2y - x = 11} \\ \mathbf{-2y + 5x = 1} \end{cases}$ **by elimination.**

Step 1: $\begin{aligned} 2y - x &= 11 \\ -2y + 5x &= 1 \end{aligned}$ *Write the system so that like terms are aligned.*

Step 2: $\begin{aligned} 0 + 4x &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned}$ *Add the equations to eliminate the y-terms.* *Simplify and solve for x.*

Step 3: $\begin{aligned} 2y - x &= 11 \\ 2y - 3 &= 11 \\ 2y &= 14 \\ y &= 7 \end{aligned}$ *Write one of the original equations.* *Substitute 3 for x.* *Simplify and solve for y.*

Step 4: (3, 7) *Write the solution as an ordered pair.*

2. Solve $\begin{cases} \mathbf{2x + y = 3} \\ \mathbf{-x + 3y = -12} \end{cases}$ **by elimination.**

Step 1: $\begin{aligned} 2x + y &= 3 \\ +\ 2(-x + 3y &= -12) \\ 2x + y &= 3 \\ +\ (-2x + 6y &= -24) \end{aligned}$ *Multiply each term in the second equation by 2 to get opposite terms.* *Add the new equation to the first equation, eliminating the x-terms.*

Step 2: $\begin{aligned} 7y &= -21 \\ y &= -3 \end{aligned}$ *Simplify and solve for y.*

Step 3: $\begin{aligned} 2x + y &= 3 \\ 2x - 3 &= 3 \\ 2x &= 6 \\ x &= 3 \end{aligned}$ *Write one of the original equations.* *Substitute –3 for y.* *Simplify and solve for x.*

Step 4: (3, –3) *Write the solution as an ordered pair.*

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Solving Linear Inequalities (for Holt Algebra 1, Lesson 6-5)

- A **linear inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol.
- A **solution of a linear inequality** is any ordered pair that makes the inequality true.
- A linear inequality describes a region of a coordinate plane called a *half-plane*. All points in the region are solutions of the linear inequality.
- The boundary line is the graph of the related equation.

Graphing Linear Inequalities
Step 1 Solve the inequality for y (slope-intercept form).
Step 2 Graph the boundary line. Use a solid line for $\le$ or $\ge$. Use a dashed line for $<$ or $>$.
Step 3 Shade the half-plane above the line for $>$ or $\ge$. Shade the half-plane below the line for $<$ or $\le$. Check your answer.

Examples

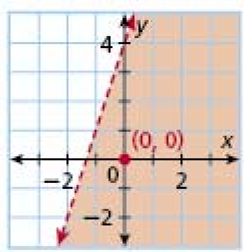
GRAPHING LINEAR INEQUALITIES IN TWO VARIABLES

1. Graph the solutions of the linear inequality $y < 3x + 4$.

Step 1 The inequality is already solved for y.

Step 2 Graph the boundary line $y = 3x + 4$.
Use a dashed line for <.

Step 3 Shade below the line for <.



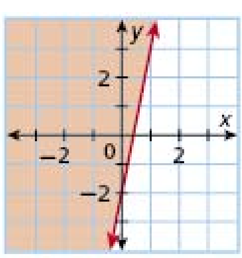
Check

y	$<$	$3x + 4$
0		3(0) + 4
0		0 + 4
0	$<$	4 ✓

Substitute (0, 0) for (x, y) because it is not on the boundary line. The point (0, 0) satisfies the inequality so the graph is shaded correctly.

WRITING AN INEQUALITY FROM A GRAPH

2. Write an inequality to represent the graph.



Identify the y-intercept and slope.
y-intercept: –2; slope: 5

Write an equation in slope-intercept form.
$y = mx + b \rightarrow y = 5x - 2$

The graph is shaded above a solid boundary line so replace = with $\ge$ to write the inequality.

$y \ge 5x - 2$

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Solving Systems of Linear Inequalities

(for Holt Algebra 1, Lesson 6-6)

- A **system of linear inequalities** is a set of two or more linear inequalities containing two or more variables.
- The **solution of a system of linear inequalities** consists of all the ordered pairs that satisfy all the linear inequalities in the system.
- To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions are represented by the overlapping shaded regions.
- If the boundary lines are parallel, solutions may or may not exist.

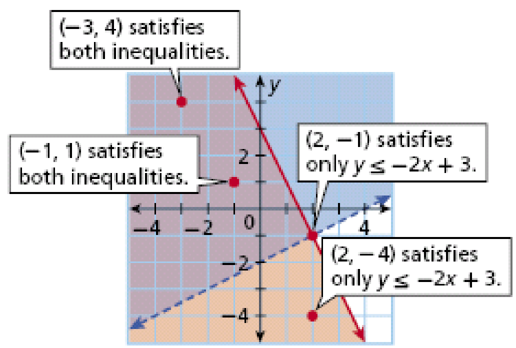
Examples

SOLVING A SYSTEM OF LINEAR INEQUALITIES BY GRAPHING

Graph the systems of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

1. $\begin{cases} \boldsymbol{y \le -2x + 3} \\ \boldsymbol{y > \frac{1}{2}x - 2} \end{cases}$ *Graph each inequality on the same grid.*

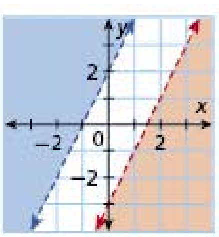
$y \le -2x + 3$	$y > \frac{1}{2}x - 2$
y-intercept: 3	*y*-intercept: –2
slope: –2	slope: $\frac{1}{2}$
solid line shaded below	dashed line shaded above



The points (–1, 1) and (–3, 4) are solutions.
The points (2, –1) and (2, –4) are not solutions.

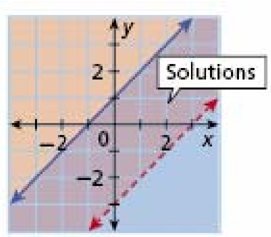
Graph each system of linear inequalities.

2. $\begin{cases} \boldsymbol{y < 2x - 3} \\ \boldsymbol{y > 2x + 2} \end{cases}$



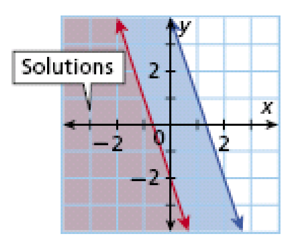
This system has no solutions.

3. $\begin{cases} \boldsymbol{y > x - 3} \\ \boldsymbol{y \le x + 1} \end{cases}$



The solutions are all points between the parallel lines and on the solid line.

4. $\begin{cases} \boldsymbol{y \le -3x - 2} \\ \boldsymbol{y \le -3x + 4} \end{cases}$



The solutions are the same as the solutions of $y \le -3x - 2$.

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Multiplication Properties of Exponents

(for Holt Algebra 1, Lesson 7-3)

- An exponential expression is simplified if…
  - there are no negative exponents.
  - the same base does not appear more than once in a product or quotient.
  - no powers, products, or quotients are raised to powers.
  - numeral coefficients in a quotient have no common factor other than 1.
- The properties below are used to simplify exponential expressions containing multiplication.

Product of Powers Property

WORDS	NUMBERS	ALGEBRA
The product of two powers with the same base equals that base raised to the sum of the exponents.	$6^7 \bullet 6^4 = 6^{7+4} = 6^{11}$	If a is any nonzero real number and m and n are integers, then $a^m \bullet a^n = a^{m+n}$.

Power of a Power Property

WORDS	NUMBERS	ALGEBRA
A power raised to another power equals that base raised to the product of the exponents.	$(6^7)^4 = 6^{7 \bullet 4} = 6^{28}$	If a is any nonzero real number and m and n are integers, then $(a^m)^n = a^{mn}$.

Power of a Product Property

WORDS	NUMBERS	ALGEBRA
A product raised to a power equals the product of each factor raised to that power.	$(2 \bullet 4)^3 = 2^3 \bullet 4^3$	If a and b are any nonzero real numbers and n is any integer, then $(ab)^n = a^n b^n$.

Examples

SIMPLIFYING EXPONENTIAL EXPRESSIONS WITH MULTIPLICATION

Simplify.

1. $2^5 \bullet 2^6$

2^{5+6} *Product of Powers*

2^{11} *Simplify.*

2. $a^4 \bullet b^5 \bullet a^2$

$(a^4 \bullet a^2) \bullet b^5$ *Group powers with the same base. Add the exponents of powers with same base.*

$a^6 \bullet b^5$

a^6b^5

3. $(7^4)^3$

$7^{4 \bullet 3}$ *Power of a Power*

7^{12} *Simplify.*

4. $(3^6)^0$

$3^{6 \bullet 0}$ *Power of a Power*

3^0 *Simplify.*

1 *A number to the 0 power is 1.*

5. $(3x)^2$

$3^2 \bullet x^2$ *Power of a Product*

$9x^2$ *Simplify.*

6. $(4x^3y^5)^2$

$4^2 \bullet (x^3)^2 \bullet (y^5)^2$ *Power of a Product*

$4^2 \bullet x^6 \bullet y^{10}$ *Simplify.*

$16x^6y^{10}$

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Division Properties of Exponents (for Holt Algebra 1, Lesson 7-4)

- The properties below are used to simplify exponential expressions containing division.

Quotient of Powers Property

WORDS	NUMBERS	ALGEBRA
The quotient of two non-zero powers with the same base equals the base raised to the difference of the exponents.	$\frac{6^7}{6^4} = 6^{7-4} = 6^3$	If a is a nonzero real number and m and n are integers, then $\frac{a^m}{a^n} = a^{m-n}$.

Positive Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a positive power equals the quotient of each base raised to that power.	$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Negative Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a negative power equals the reciprocal of the quotient raised to the opposite (positive) power.	$\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$.

Examples

SIMPLIFYING EXPONENTIAL EXPRESSIONS WITH DIVISION

Simplify.

1. $\frac{3^6}{3^2}$

$\frac{3^6}{3^2} = 3^{6-2}$ *Quotient of Powers*

$= 3^4$ *Simplify.*

$= 81$

2. $\frac{a^5b^9}{a^4b^4}$

$\frac{a^5b^9}{a^4b^4} = a^{5-4} \bullet b^{9-4}$ *Quotient of Powers*

$= a^1 \bullet b^5$ *Simplify.*

$= ab^5$

3. $\left(\frac{3}{4}\right)^3$

$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3}$ *Power of a Quotient*

$= \frac{27}{64}$ *Simplify.*

4. $\left(\frac{3x}{y^2}\right)^{-3}$

$\left(\frac{3x}{y^2}\right)^{-3} = \left(\frac{y^2}{3x}\right)^3$ *Write with a positive exponent.*

$= \frac{(y^2)^3}{(3x)^3}$ *Power of a Quotient*

$= \frac{y^6}{3^3x^3}$ *Power of a Power and Power of a Product*

$= \frac{y^6}{27x^3}$ *Simplify.*

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Adding and Subtracting Polynomials

(for Holt Algebra 1, Lesson 7-6)

- To add polynomials, combine like terms. Like terms are constants or terms with the same variable(s) raised to the same power(s).
- To subtract polynomials, add the opposite of each term.

Examples

ADDING AND SUBTRACTING MONOMIALS

1. Add or subtract.

A $15m^3 + 6m^2 + 2m^3$

$15m^3 + 6m^2 + 2m^3$	*Identify like terms.*
$15m^3 + 2m^3 + 6m^2$	*Rearrange terms so that like terms are together.*
$17m^3 + 6m^2$	*Combine like terms.*

B $2x^2y - x^2y + 5xy - x^2y$

$2x^2y - x^2y + 5xy - x^2y$	*Identify like terms.*
$2x^2y - x^2y - x^2y + 5xy$	*Rearrange terms so that like terms are together.*
$0x^2y + 5xy$	*Combine like terms.*
$5xy$	*Simplify.*

ADDING POLYNOMIALS

2. Add.

$(2x^2 - x) + (x^2 + 3x - 1)$

$(2x^2 - x) + (x^2 + 3x - 1)$	*Identify like terms.*
$(2x^2 + x^2) + (-x + 3x) + (-1)$	*Group like terms together.*
$3x^2 + 2x - 1$	*Combine like terms.*

SUBTRACTING POLYNOMIALS

3. Subtract.

$(a^4 - 2a) - (3a^4 - 3a - 1)$

$(a^4 - 2a) + (-3a^4 + 3a + 1)$	*Rewrite subtraction as addition of the opposite.*
$(a^4 - 2a) + (-3a^4 + 3a + 1)$	*Identify like terms.*
$(a^4 - 3a^4) + (-2a + 3a) + 1$	*Rearrange terms so that like terms are together.*
$-2a^4 + a + 1$	*Combine like terms.*

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Multiplying Polynomials (for Holt Algebra 1, Lesson 7-7)

- To multiply monomials, use properties of exponents.
- To multiply a polynomial by a monomial, use the Distributive Property.
- To multiply a binomial by a binomial, you can use the Distributive Property more than once, or you can use the FOIL method:
  - Multiply the First terms.
  - Multiply the Outer terms.
  - Multiply the Inner terms.
  - Multiply the Last terms.

$$(x + 1)(2x + 4)$$

- To multiply polynomials with more than two terms, use the Distributive property several times.

Examples

MULTIPLYING MONOMIALS

1. Multiply $(5x^2) \bullet (4x^3)$.

$(5x^2) \bullet (4x^3)$	
$(5 \bullet 4)(x^2 \bullet x^3)$	*Group terms with like bases together.*
$20x^5$	*Multiply.*

MULTIPLYING A POLYNOMIAL BY A MONOMIAL

2. Multiply $5(2x^2 + x + 4)$.

$5(2x^2 + x + 4)$	
$5(2x^2) + 5(x) + 5(4)$	*Use the Distributive Property.*
$10x^2 + 5x + 20$	*Multiply.*

MULTIPLYING BINOMIALS

3. Multiply.

Method 1: Apply the Distributive Property.

A $(x + 2)(x – 5)$

$x(x – 5) + 2(x – 5)$	*Distribute x and 2.*
$x(x) + x(–5) + 2(x) + 2(–5)$	*Distribute x and 2 again.*
$x^2 – 5x + 2x – 10$	*Multiply.*
$x^2 – 3x – 10$	*Combine like terms.*

Method 2: Use the FOIL method.

B $(x + 3)(x + 2)$

$(x \bullet x) + (x + 2) + (3 \bullet x) + (3 \bullet 2)$	*Use the FOIL Method.*
$x^2 + 2x + 3x + 6$	*Multiply.*
$x^2 + 5x + 6$	*Combine like terms.*

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Special Products of Binomials (for Holt Algebra 1, Lesson 7-8)

- A **perfect-square trinomial** is the result of squaring a binomial. It is one of two *special products*. The general forms are shown below with the FOIL method.

$$\begin{aligned}\mathbf{(a+b)^2} &= (a+b)(a+b)\\ &= a^2+ab+ab+b^2\\ &= \mathbf{a^2+2ab+b^2}\end{aligned}$$

$$\begin{aligned}\mathbf{(a-b)^2} &= (a-b)(a-b)\\ &= a^2-ab-ab+b^2\\ &= \mathbf{a^2-2ab+b^2}\end{aligned}$$

- Another special product, called a **difference of two squares**, is the result of multiplying binomials of the form $(a + b)(a - b)$. The general form is shown below with the FOIL method.

$$\begin{aligned}\mathbf{(a+b)(a-b)} &= a^2+ab-ab-b^2\\ &= \mathbf{a^2-b^2}\end{aligned}$$

Examples

FINDING PRODUCTS IN THE FORM $(A + B)^2$ AND $(A - B)^2$

Multiply.

1. $\mathbf{(x+4)^2}$

$(a+b)^2 = a^2 + 2ab + b^2$	*Use the rule for $(a + b)^2$.*
$(x+4)^2 = x^2 + 2(x)(4) + 4^2$	*Identify a and b: a = x and b = 4.*
$= x^2 + 8x + 16$	*Simplify.*

2. $\mathbf{(x-5)^2}$

$(a-b)^2 = a^2 - 2ab + b^2$	*Use the rule for $(a - b)^2$.*
$(x-5)^2 = x^2 - 2(x)(5) + 5^2$	*Identify a and b: a = x and b = 5.*
$= x^2 - 10x + 25$	*Simplify.*

FINDING PRODUCTS IN THE FORM $(A + B)(A - B)$

Multiply.

3. $\mathbf{(x+6)(x-6)}$

$(a+b)(a-b) = a^2 - b^2$	*Use the rule for $(a + b)(a - b)$.*
$(x+6)(x-6) = x^2 - 6^2$	*Identify a and b. a = x and b = 6.*
$= x^2 - 36$	*Simplify.*

4. $\mathbf{(7+n)(7-n)}$

$(a+b)(a-b) = a^2 - b^2$	*Use the rule for $(a + b)(a - b)$.*
$(7+n)(7-n) = 7^2 - n^2$	*Identify a and b. a = 7 and b = n.*
$= 49 - n^2$	*Simplify.*

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Special Products of Binomials (for Holt Algebra 1, Lesson 7-8)

- A **perfect-square trinomial** is the result of squaring a binomial. It is one of two *special products*. The general forms are shown below with the FOIL method.

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ab+b^2\\ &= \mathbf{a^2+2ab+b^2}\end{aligned}$$

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ab+b^2\\ &= \mathbf{a^2-2ab+b^2}\end{aligned}$$

- Another special product, called a **difference of two squares**, is the result of multiplying binomials of the form $(a+b)(a-b)$. The general form is shown below with the FOIL method.

$$\begin{aligned}(a+b)(a-b) &= a^2+ab-ab-b^2\\ &= \mathbf{a^2-b^2}\end{aligned}$$

Examples

FINDING PRODUCTS IN THE FORM $(A+B)^2$ AND $(A-B)^2$

Multiply.

1. $(x+4)^2$

$(a+b)^2 = a^2 + 2ab + b^2$	*Use the rule for $(a+b)^2$.*
$(x+4)^2 = x^2 + 2(x)(4) + 4^2$	*Identify a and b: a = x and b = 4.*
$= x^2 + 8x + 16$	*Simplify.*

2. $(x-5)^2$

$(a-b)^2 = a^2 - 2ab + b^2$	*Use the rule for $(a-b)^2$.*
$(x-5)^2 = x^2 - 2(x)(5) + 5^2$	*Identify a and b: a = x and b = 5.*
$= x^2 - 10x + 25$	*Simplify.*

FINDING PRODUCTS IN THE FORM $(A+B)(A-B)$

Multiply.

3. $(x+6)(x-6)$

$(a+b)(a-b) = a^2 - b^2$	*Use the rule for $(a+b)(a-b)$.*
$(x+6)(x-6) = x^2 - 6^2$	*Identify a and b. a = x and b = 6.*
$= x^2 - 36$	*Simplify.*

4. $(7+n)(7-n)$

$(a+b)(a-b) = a^2 - b^2$	*Use the rule for $(a+b)(a-b)$.*
$(7+n)(7-n) = 7^2 - n^2$	*Identify a and b. a = 7 and b = n.*
$= 49 - n^2$	*Simplify.*

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Factoring by GCF (for Holt Algebra 1, Lesson 8-2)

- A polynomial is factored when it is written as a product of monomials and polynomials that cannot be factored further.
- The Distributive Property states that $ab + ac = a(b + c)$. It allows you to "factor out" the GCF of the terms in a polynomial to write a factored form of the polynomial.
- Sometimes the GCF is a binomial. Common binomial factors are used in factoring by grouping.

Examples

FACTORING BY USING THE GCF

1. Factor $4x^2 - 3x$. Check your answer.

$4x^2 = 2 \bullet 2 \bullet \boxed{x} \bullet x$ $3x = 3 \bullet \boxed{x}$	*Find the GCF.*
x	*The GCF of $4x^2$ and $3x$ is x.*
$4x(x) - 3(x)$	*Write terms as products using the GCF as a factor.*
$x(4x - 3)$	*Use the Distributive Property to factor out the GCF.*
Check $x(4x - 3)$	*Multiply to check your answer.*
$4x^2 - 3x$ ✓	*The product is the original polynomial.*

2. Factor $10y^3 + 20y^2 - 5y$.

$10y^3 = 2 \bullet \boxed{5} \bullet \boxed{y} \bullet y \bullet y$ $20y^2 = 2 \bullet 2 \bullet \boxed{5} \bullet \boxed{y} \bullet y$ $5y = \boxed{5} \bullet \boxed{y}$	*Find the GCF.*
$5 \bullet y = 5y$	*The GCF of $10y^3$, $20y^2$, and $5y$ is $5y$.*
$2y^2(5y) + 4y(5y) - 1(5y)$	*Write terms as products using the GCF as a factor.*
$5y(2y^2 + 4y - 1)$	*Use the Distributive Property to factor out the GCF.*

FACTORING BY GROUPING

3. Factor $12a^3 - 9a^2 + 20a - 15$ by grouping.

$(12a^3 - 9a^2) + (20a - 15)$	*Group terms that have a common factor.*
$3a^2(4a - 3) + 5(4a - 3)$	*Factor out the GCF of each group.*
$3a^2(4a - 3) + 5(4a - 3)$	*$(4a - 3)$ is another common factor.*
$(4a - 3)(3a^2 + 5)$	*Factor out $(4a - 3)$.*

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Factoring $x^2 + bx + c$ (for Holt Algebra 1, Lesson 8-3)

- A trinomial of the form $x^2 + bx + c$ can sometimes be factored into two binomials.
- Using the FOIL method, $(x + 3)(x + 4) = x^2 + 7x + 12$.
  - Notice that the constant, 12, is the product of 3 and 4.
  - Notice also that the coefficient of the middle term, 7, is the sum of 3 and 4.

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.

Factors of 18	Sum	
1 and 18	19 ✗	
2 and 9	11 ✗	
3 and 6	9 ✓	$x^2 + 9x + 18 = (x + 3)(x + 6)$

- If c is positive, both factors have the same sign as b. If c is negative, its factors have opposite signs, and the factor with the greater absolute value has the same sign as b.

Examples

FACTORING $x^2 + bx + c$

1. Factor $x^2 + 6x + 8$.

$(x + \square)(x + \square)$

$b = 6$ and $c = 8$; look for factors of 8 whose sum is 6.

Both factors will be positive.

Factors of 8	Sum
1 and 8	9 ✗
2 and 4	6 ✓

The factors needed are 2 and 4.

$(x + 2)(x + 4)$

2. Factor $x^2 – 10x + 16$.

$(x + \square)(x + \square)$

$b = –10$ and $c = 16$; look for factors of 16 whose sum is $–10$.

Both factors will be negative.

Factors of 16	Sum
–1 and –16	–17 ✗
–2 and –8	–10 ✓
–4 and –4	–8 ✗

The factors needed are –2 and –8.

$(x – 2)(x – 8)$

3. Factor $x^2 + 7x – 18$.

$(x + \square)(x + \square)$

$b = 7$ and $c = –18$; look for factors of –18 whose sum is 7.

One factor will be positive; one factor will be negative.

Factors of –18	Sum
–1 and 18	17 ✗
–2 and 9	7 ✓
–3 and 6	3 ✗

The factors needed are –2 and 9.

$(x – 2)(x + 9)$

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Factoring $ax^2 + bx + c$ (for Holt Algebra 1, Lesson 8-4)

- To factor $ax^2 + bx + c$, it is helpful to make a table. List all the combinations of the factors of both a and c. Then check the products of the outer and inner terms to see which combination results in the sum of b.
- When a is negative, factor out –1 from each term first. Carry it through the rest of the steps.

Examples

FACTORING $ax^2 + bx + c$

1. Factor $5x^2 - 14x + 8$. Check your answer.

$(\square x + \square)(\square x + \square)$ *$a = 5$ and $c = 8$; Outer + Inner = –14*

Factors of 5	Factors of 8	Outer + Inner
1 and 5	–1 and –8	$1(-8) + 5(-1) = -13$ ✗
1 and 5	–8 and –1	$1(-1) + 5(-8) = -41$ ✗
1 and 5	–2 and –4	$1(-4) + 5(-2) = -14$ ✓

$(x - 2)(5x - 4)$

Check $(x - 2)(5x - 4) = 5x^2 - 4x - 10x + 8$
$= 5x^2 - 14x + 8$ ✓

2. Factor $4x^2 + 19x - 5$.

$(\square x + \square)(\square x + \square)$ *$a = 4$ and $c = -5$; Outer + Inner = 19*

Factors of 4	Factors of –5	Outer + Inner
1 and 4	1 and –5	$1(-5) + 4(1) = -1$ ✗
1 and 4	–1 and 5	$1(5) + 4(-1) = 1$ ✗
1 and 4	5 and –1	$1(-1) + 4(5) = 19$ ✓

$(x + 5)(4x - 1)$

3. Factor $-2x^2 - 15x - 7$.

$-1(2x^2 + 15x + 7)$ *Factor out –1.*
$-1(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 7$; Outer + Inner = 15*

Factors of 2	Factors of 7	Outer + Inner
1 and 2	1 and 7	$1(7) + 2(1) = 9$ ✗
1 and 2	7 and 1	$1(1) + 2(7) = 15$ ✓

$(x + 7)(2x + 1)$
$-1(x + 7)(2x + 1)$

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Factoring Special Products (for Holt Algebra 1, Lesson 8-5)

- There are two special products: perfect-square trinomials and differences of two squares.
- A trinomial is a perfect square if:
  1) The first and last terms are perfect squares.
  2) The middle term is two times one factor from the first term and one factor from the last term.

$$9x^2 + 12x + 4$$
$$3x \bullet 3x \quad 2(3x \bullet 2) \quad 2 \bullet 2$$

- A binomial is a difference of two squares if:
  1) There are two terms, one subtracted from the other.
  2) Both terms are perfect squares.

$$4x^2 - 9$$
$$2x \bullet 2x \quad 3 \bullet 3$$

Factoring Perfect-Square Trinomials	
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 2x + 1 = (x - 1)^2$

Factoring a Difference of Two Squares	
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$

Examples

RECOGNIZING AND FACTORING SPECIAL PRODUCTS

Determine whether each trinomial is a perfect-square. If so, factor. If not, explain.

1. $x^2 + 12x + 36$

$x^2 + 12x + 36$ — *Look for the pattern.*

$x \bullet x \quad 2(x \bullet 6) \quad 6 \bullet 6$ — *This trinomial is a perfect square. Factor.*

$x^2 + 2(x)(6) + 6^2$ — *Write the trinomial as $a^2 + 2ab + b^2$.*

$(x + 6)^2$ — *Write the trinomial as $(a + b)^2$.*

2. $x^2 + 9x + 16$

$x^2 + 9x + 16$

$x \bullet x \quad 2(x \bullet 4) \quad 4 \bullet 4$ — *$2(x \bullet 4) \neq 9x$*

$x^2 + 9x + 16$ is not a perfect-square trinomial because $9x \neq 2(x \bullet 4)$.

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

3. $x^2 - 81$

$x^2 - 81$

$x \bullet x \quad 9 \bullet 9$ — *Difference of two squares*

$x^2 - 9^2$ — *$a = x,\ b = 9$*

$(x + 9)(x - 9)$ — *Write as $(a + b)(a - b)$.*

4. $x^6 - 7y^2$

$x^6 - 7y^2$

$x^3 \bullet x^3$ — *$7y^2$ is not a perfect square.*

$x^6 - 7y^2$ is not a difference of two squares because $7y^2$ is not a perfect square.

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Characteristics of Quadratic Functions

(for Holt Algebra 1, Lesson 9-2)

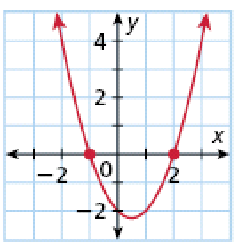
- A **zero of a function** is an x-value that makes the function equal to 0. So, a zero of a function is the same as an x-intercept of a function. A quadratic function may have one, two, or no zeros.
- An **axis of symmetry** is a vertical line that divides a parabola into two symmetrical halves. It always passes through the vertex of the parabola. To find the axis of symmetry, you can find the average of the zeros, or use the formula $x = -\dfrac{b}{2a}$.
- Once you have found the axis of symmetry, you can use it to find the **vertex**. Substitute the x-value into the function, and solve for y. Write the vertex as an ordered pair.

Examples

FINDING ZEROS OF QUADRATIC FUNCTIONS FROM GRAPHS

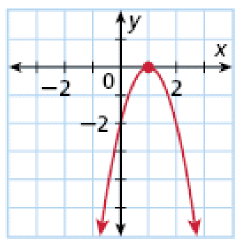
Find the zeros of each quadratic function from its graph.

1.



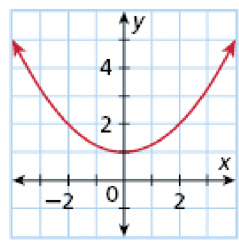
The zeros are –1 and 2.

2.



The only zero is 1.

3.

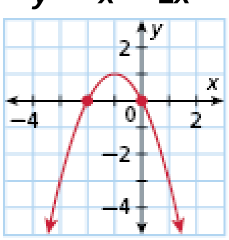


This function has no real zeros.

FINDING THE VERTEX OF A PARABOLA

4. Find the vertex.

$y = -x^2 - 2x$



Step 1: Find the axis of symmetry.

Average the zeros.

$x = \dfrac{-2+0}{2} = \dfrac{-2}{2} = -1$ *The zeros are –2 and 0.*

So, –1 is the x-coordinate of the vertex.

Step 2: Find the corresponding y-coordinate.

$y = -x^2 - 2x$ *Use the function rule.*
$y = -(-1)^2 - 2(-1)$ *Substitute –1 for x.*
$y = -1 + 2$
$y = 1$

Step 3: Write the ordered pair.

(–1, 1)

The vertex is (–1, 1).

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Graphing Quadratic Functions (for Holt Algebra 1, Lesson 9-3)

- For a quadratic function of the form $y = ax^2 + bx + c$, the y-intercept is c.
- You can use the y-intercept, axis of symmetry, and vertex of a parabola to graph a quadratic function.

Example

GRAPHING A QUADRATIC FUNCTION

Graph $y = x^2 - 4x - 5$.

Step 1 Find the axis of symmetry.

$x = -\frac{-4}{2(1)}$ *Use $x = -\frac{b}{2a}$. Substitute 1 for a and –4 for b.*

$= 2$ *Simplify.*

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = x^2 - 4x - 5$ *The x-coordinate of the vertex is 2.*

$= (2)^2 - 4(2) - 5$ *Substitute 2 for x.*

$= -9$ *Simplify.*

The vertex is $(2, -9)$.

Step 3 Find the y-intercept.

$y = x^2 - 4x - 5$ *Identify c.*

The y-intercept is –5; the graph passes through (0, –5).

Step 4 Find two more points on the same side of the axis of symmetry as the point containing the y-intercept.

Let $x = 1$.

$y = 1^2 - 4(1) - 5$

$= -8$

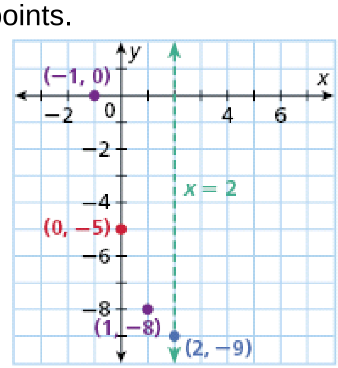
Let $x = -1$.

$y = (-1)^2 - 4(-1) - 5$

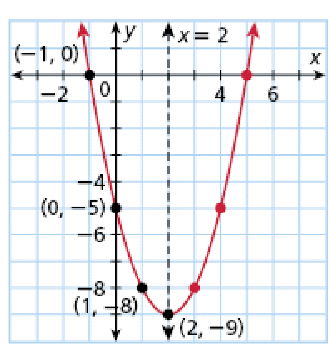
$= 0$

Two other points are (1, –8) and (–1, 0).

Step 5 Graph the axis of symmetry, the vertex, the point containing the y-intercept, and two other points.



Step 6 Reflect the points across the axis of symmetry. Connect the points with a smooth curve.



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Solving Quadratic Equations by Factoring
(for Holt Algebra 1, Lesson 9-6)

- You can solve quadratic equations by factoring and using the Zero Product Property.

Zero Product Property		
WORDS	**NUMBERS**	**ALGEBRA**
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

Examples

USING THE ZERO PRODUCT PROPERTY

1. Use the Zero Product Property to solve $(x - 3)(x + 7) = 0$. Check your answer.

$x - 3 = 0$ or $x + 7 = 0$ *Use the Zero Product Property.*

$x = 3$ or $x = -7$ *Solve each equation.*

The solutions are 3 and –7.

Check

$$\begin{array}{r|l} (x-3)(x+7) & 0 \\ \hline (3-3)(3+7) & 0 \\ (0)(10) & 0 \\ 0 & 0 \checkmark \end{array} \qquad \begin{array}{r|l} (x-3)(x+7) & 0 \\ \hline (-7-3)(-7+7) & 0 \\ (-10)(0) & 0 \\ 0 & 0 \checkmark \end{array}$$

SOLVING QUADRATIC EQUATIONS BY FACTORING

2. Solve each quadratic equation by factoring.

A $x^2 + 2x = 8$

$$\begin{aligned} x^2 + 2x &= 8 \\ \underline{-8} &\ \ \underline{-8} \\ x^2 + 2x - 8 &= 0 \end{aligned}$$

The equation must be written in standard form, so subtract 8 from both sides.

$(x + 4)(x - 2) = 0$ *Factor the trinomial.*

$x + 4 = 0$ or $x - 2 = 0$ *Use the Zero Product Property.*

$x = -4$ or $x = 2$ *Solve each equation.*

The solutions are –4 and 2.

B $x^2 + 2x + 1 = 0$

$x^2 + 2x + 1 = 0$

$(x + 1)(x + 1) = 0$ *Factor the trinomial.*

$x + 1 = 0$ or $x + 1 = 0$ *Use the Zero Product Property.*

$x = -1$ or $x = -1$ *Solve each equation.*

Both factors result in the same solution. There is only one solution, –1.

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Completing the Square (for Holt Algebra 1, Lesson 9-8)

- An expression in the form $x^2 + bx$ is not a perfect square. But you can use a method called **completing the square** to form a trinomial that is a perfect square.
- To solve a quadratic equation by completing the square:
  - Write the equation in the form $x^2 + bx = c$.
  - Find $\left(\frac{b}{2}\right)^2$.
  - Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
  - Factor the perfect-square trinomial.
  - Take the square root of both sides.
  - Write two equations, using both the positive and negative square root, and solve each equation.

Examples

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Solve by completing the square.

1. $\boldsymbol{x^2 + 14x = 15}$

Step 1 $x^2 + 14x = 15$ — *The equation is already in the form $x^2 + bx = c$.*

Step 2 $\left(\frac{14}{2}\right)^2 = 7^2 = 49$ — *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 + 14x + 49 = 15 + 49$ — *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

Step 4 $(x+7)^2 = 64$ — *Factor and simplify.*

Step 5 $x + 7 = \pm 8$ — *Take the square root of both sides.*

Step 6 $x + 7 = 8$ or $x + 7 = -8$ — *Write and solve two equations.*

$x = 1$ or $x = -15$

The solutions are 1 and –15.

2. $\boldsymbol{-2x^2 + 12x - 20 = 0}$

Step 1 $\frac{-2x^2}{-2} + \frac{12x}{-2} - \frac{20}{-2} = \frac{0}{-2}$ — *Divide by –2 to make a = 1.*

$x^2 - 6x + 10 = 0$ — *Write the equation in the form $x^2 + bx = c$.*

$x^2 - 6x = -10$

$x^2 + (-6x) = -10$

Step 2 $\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$ — *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - 6x + 9 = -10 + 9$ — *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

Step 4 $(x-3)^2 = -1$ — *Factor and simplify.*

There is no real number whose square is negative, so there are no real solutions.

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The Quadratic Formula and the Discriminant
(for Holt Algebra 1, Lesson 9-9)

- The Quadratic Formula is the only method that can be used to solve *any* quadratic equation.

Quadratic Formula
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

- If the quadratic equation is in standard form, the **discriminant** of the quadratic equation is $b^2 - 4ac$, the part of the equation under the radical sign. You can determine the number of solutions of a quadratic equation by evaluating its discriminant.

The Discriminant of a Quadratic Equation $ax^2 + bx + c = 0$
If $b^2 - 4ac > 0$, the equation has **two** real solutions. If $b^2 - 4ac = 0$, the equation has **one** real solution. If $b^2 - 4ac < 0$, the equation has **no** real solutions.

Examples

USING THE QUADRATIC FORMULA

1. Solve $2x^2 + 3x - 5 = 0$ using the Quadratic Formula.

$2x^2 + 3x + (-5) = 0$ — *Identify a, b, and c.*

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — *Use the Quadratic Formula.*

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$ — *Substitute 2 for a, 3 for b, and –5 for c.*

$x = \dfrac{-3 \pm \sqrt{9 + 40}}{4} = \dfrac{-3 \pm \sqrt{49}}{4} = \dfrac{-3 \pm 7}{4}$ — *Simplify.*

$x = \dfrac{-3 + 7}{4}$ or $x = \dfrac{-3 - 7}{4}$ — *Write as two equations.*

$x = 1$ or $x = -\dfrac{5}{2}$ — *Solve each equation.*

USING THE DISCRIMINANT

2. Find the number of solutions of each equation using the discriminant.

A $\mathbf{3x^2 + 10x + 2 = 0}$

$b^2 - 4ac$

$10^2 - 4(3)(2)$

76

$b^2 - 4ac$ is positive.

Two real solutions

B $\mathbf{9x^2 - 6x + 1 = 0}$

$b^2 - 4ac$

$(-6)^2 - 4(9)(1)$

0

$b^2 - 4ac$ is zero.

One real solution

C $\mathbf{x^2 + x + 1 = 0}$

$b^2 - 4ac$

$1^2 - 4(1)(1)$

-3

$b^2 - 4ac$ is negative.

No real solutions

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Geometric Sequences (for Holt Algebra 1, Lesson 11-1)

- In a **geometric sequence**, the ratio of successive terms is the same number r, called the **common ratio**.
- The variable a is often used to represent the terms in a sequence. The variable a_4 is the fourth term in a sequence.
- To find the nth term, multiply the first term by the common ratio raised to the power $n - 1$:

$$a_n = a_1 r^{n-1}$$

Examples

EXTENDING GEOMETRIC SEQUENCES

1. Find the next three terms in the geometric sequence 1, 3, 9, 27, ...

Step 1: Find the value of r by dividing each term by the one before it.

$\frac{3}{1} = 3, \quad \frac{9}{3} = 3, \quad \frac{27}{9} = 3$ *The value of r is 3.*

Step 2: Multiply each term by 3 to find the next three terms.

$27 \times 3 = 81, \quad 81 \times 3 = 243, \quad 243 \times 3 = 729$

The next 3 terms are 81, 243, and 729.

FINDING THE *N*TH TERM OF A GEOMETRIC SEQUENCE

2. The first term of a geometric sequence is 10, and the common ratio is 2. Find the 7th term of the sequence.

$a_n = a_1 r^{n-1}$	*Write the formula.*
$a_7 = 10(2)^{7-1}$	*Substitute 10 for a_1, 7 for n, and 2 for r.*
$= 10(2)^6$	*Simplify the exponent.*
$= 10(64)$	*Evaluate the power.*
$= 640$	*Multiply.*

The 7th term of the sequence is 640.

3. For a geometric sequence, a_1 = 8 and r = 3. Find the 5th term of this sequence.

$a_n = a_1 r^{n-1}$	*Write the formula.*
$a_5 = 8(3)^{5-1}$	*Substitute 8 for a_1, 5 for n, and 3 for r.*
$= 8(3)^4$	*Simplify the exponent.*
$= 8(81)$	*Evaluate the power.*
$= 648$	*Multiply.*

The 5th term of the sequence is 648.

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Exponential Functions (for Holt Algebra 1, Lesson 11-2)

- An **exponential function** is a function whose independent variable appears in an exponent.
- An exponential function has the form $f(x) = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.
- Exponential functions have *constant ratios*. As the *x*-values increase by a constant amount, the *y*-values are multiplied by a constant amount. This amount is the constant ratio and is the value of *b* in $f(x) = ab^x$.
- To graph an exponential function, choose several values of *x* and generate ordered pairs. Plot the points and connect them with a smooth curve.
- Depending on the values of *a* and *b*, the graphs of exponential functions will have different shapes.

Examples

EVALUATING AN EXPONENTIAL FUNCTION

1. The function $f(x) = 2(3)^x$ models an insect population after *x* days. What will the population be on the 5th day?

$f(x) = 2(3)^x$ *Write the function.*
$f(5) = 2(3)^5$ *Substitute 5 for x.*
$= 2(243)$ *Evaluate 3^5.*
$= 486$ *Multiply.*

There will be 486 insects on the 5th day.

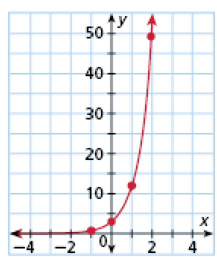
GRAPHING AN EXPONENTIAL FUNCTION

2. Graph $y = 3(4)^x$.

Choose several values of x and generate ordered pairs.

x	$y = 3(4)^x$
–1	0.75
0	3
1	12
2	48

Graph the ordered pairs and connect with a smooth curve.

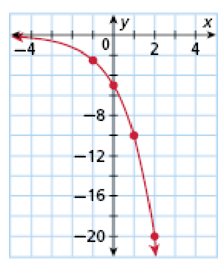


3. Graph $y = -5(2)^x$.

Choose several values of x and generate ordered pairs.

x	$y = -5(2)^x$
–1	–2.5
0	–5
1	–10
2	–20

Graph the ordered pairs and connect with a smooth curve.



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Exponential Growth and Decay (for Holt Algebra 1, Lesson 11-3)

- **Exponential growth** occurs when a quantity *increases* by the same rate r in each time period t. **Exponential decay** occurs when a quantity *decreases* by the same rate r in each time period t.

Exponential Growth and Exponential Decay Functions	
An exponential growth function has the form $y = a(1 + r)^t$, where $a > 0$. An exponential decay function has the form $y = a(1 - r)^t$, where $a > 0$.	
y represents the final amount.	a represents the original amount.
r represents the rate of growth/decay as a decimal.	t represents time.

- An application of exponential growth is **compound interest**. Its function is $A = P(1+\frac{r}{n})^{nt}$, where A is the balance after t years, P is the principal amount, r is the interest rate written as a decimal, and n is the number of times the interest is compounded in one year.
- An application of exponential decay is **half-life**, which is the time it takes for one-half of a substance to decay into another substance. Its function is $A = P(0.5)^t$, where A represents the final amount, P represents the original amount, and t represents the number of half-lives in a given time period.

Examples

EXPONENTIAL GROWTH AND DECAY

1. The original value of a painting is $1400, and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting after 25 years.

Step 1 Write the exponential growth function for this situation.

$y = a(1 + r)^t$ — *Write the function.*
$= 1400(1 + 0.09)^t$ — *Substitute 1400 for a and 0.09 for r.*
$= 1400(1.09)^t$ — *Simplify.*

Step 2 Find the value in 25 years.

$y = 1400(1.09)^t$ — *Write the function.*
$= 1400(1.09)^{25}$ — *Substitute 25 for t.*
$\approx 12{,}072.31$ — *Use a calculator and round to the nearest hundredth.*

The value of the painting in 25 years is $12,072.31.

2. Fluorine-20 has a half-life of 11 seconds. Find the amount of fluorine-20 left from a 40-gram sample after 44 seconds.

$t = \frac{44\text{ s}}{11\text{ s}} = 4$ — *Find t, the number of half-lives in the given time period.*
$A = P(0.5)^t$ — *Write the formula for half-life.*
$= 40(0.5)^4$ — *Substitute 40 for P and 4 for t.*
$= 2.5$ — *Use a calculator to simplify.*

There are 2.5 grams of fluorine-20 left after 44 seconds.

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Square-Root Functions (for Holt Algebra 1, Lesson 11-5)

- A **square-root function** is a function whose rule contains a variable under a square-root sign.

  EXAMPLES: $y = \sqrt{x}$; $y = \sqrt{2x+1}$; $y = 3\sqrt{\dfrac{x}{2}} - 6$

  NON-EXAMPLES: $y = x^2$; $y = \dfrac{2}{x+1}$; $y = \sqrt{3}x$

- Because the square root of a negative number is not a real number, the domain (set of x-values) of a square root function is restricted to numbers that make the value under the radical sign greater than or equal to 0.

Examples

EVALUATING SQUARE-ROOT FUNCTIONS

1. The function $y = 8\sqrt{x}$ gives the speed in feet per second of an object in free fall after falling x feet. Find the speed of an object in free fall after it has fallen 4 feet.

$y = 8\sqrt{x}$	*Write the speed function.*
$= 8\sqrt{4}$	*Substitute 4 for x.*
$= 8(2)$	*Simplify.*
$= 16$	

After an object has fallen 4 feet, its speed is 16 ft/s.

GRAPHING SQUARE-ROOT FUNCTIONS

2. Graph $f(x) = \sqrt{2x} + 3$.

Step 1 Find the domain.

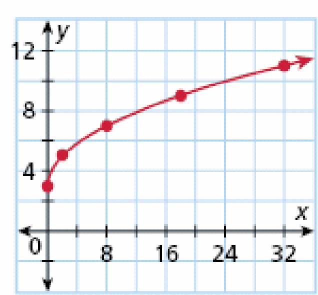
$2x \geq 0$	*2x must be greater than or equal to 0.*
$x \geq 0$	*Solve the inequality by dividing both sides by 2.*

The domain is the set of all real numbers greater than or equal to zero.

Step 2 Choose x-values greater than or equal to zero and generate ordered pairs.

x	$f(x) = \sqrt{2x} + 3$
0	3
2	5
8	7
18	9
32	11

Step 3 Plot the points. Then connect them with a smooth curve.



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Radical Expressions (for Holt Algebra 1, Lesson 11-6)

- A **radical expression** is an expression that contains a radical sign $(\sqrt{\ })$.
- A **radicand** is the expression under the radical sign. A radicand may contain numbers, variables, or both.
- An expression containing a square root is in simplest form when the radicand has no perfect square factors other than 1, the radicand has no fractions, and there are no square roots in any denominator.
- The following properties are used in simplifying radical expressions.

Product Property of Square Roots		
WORDS	**NUMBERS**	**ALGEBRA**
For any nonnegative real numbers *a* and *b*, the square root of *ab* is equal to the square root of *a* times the square root of *b*.	$\sqrt{4(25)} = \sqrt{100} = 10$ $\sqrt{4(25)} = \sqrt{4}\sqrt{25} = 2(5) = 10$	$\sqrt{ab} = \sqrt{a}\sqrt{b}$, where $a \geq 0$ and $b \geq 0$.

Quotient Property of Square Roots		
WORDS	**NUMBERS**	**ALGEBRA**
For any real numbers *a* and *b*, ($a \geq 0$ and $b > 0$), the square root of $\frac{a}{b}$ is equal to the square root of *a* divided by the square root of *b*.	$\sqrt{\frac{36}{4}} = \sqrt{9} = 3$ $\sqrt{\frac{36}{4}} = \frac{\sqrt{36}}{\sqrt{4}} = \frac{6}{2} = 3$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$.

Examples

USING THE PRODUCT AND QUOTIENT PROPERTIES OF SQUARE ROOTS

Simplify. All variables represent nonnegative numbers.

1. $\sqrt{18}$

$= \sqrt{9(2)}$ *Factor using perfect squares.*

$= \sqrt{9}\sqrt{2}$ *Product Property*

$= 3\sqrt{2}$ *Simplify.*

2. $\sqrt{20x^4y^2}$

$= \sqrt{4(5)x^4y^2}$

$= \sqrt{4}\sqrt{5}\sqrt{x^4}\sqrt{y^2}$

$= 2\sqrt{5}x^2y$

3. $\sqrt{\frac{5}{9}}$

$= \frac{\sqrt{5}}{\sqrt{9}}$ *Quotient Property*

$= \frac{\sqrt{5}}{3}$ *Simplify.*

4. $\sqrt{\frac{a^4}{81}}$

$= \frac{\sqrt{a^4}}{\sqrt{81}}$

$= \frac{a^2}{9}$

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Adding and Subtracting Radical Expressions

(for Holt Algebra 1, Lesson 11-7)

- **Like radicals** are square-root expressions with the same radicand.

  LIKE RADICALS: $2\sqrt{5}$ and $4\sqrt{5}$; $6\sqrt{x}$ and $-2\sqrt{x}$; $3\sqrt{4t}$ and $\sqrt{4t}$

  UNLIKE RADICALS: 2 and $\sqrt{15}$; $6\sqrt{x}$ and $\sqrt{6x}$; $3\sqrt{2}$ and $2\sqrt{3}$

- You can combine like radicals by adding and subtracting the numbers multiplied by the radical and keeping the radical the same.
- Sometimes radicals do not appear to be like until they are simplified. Before adding or subtracting, simplify all radicals in the expression to determine if there are like radicals.

Examples

ADDING AND SUBTRACTING SQUARE-ROOT EXPRESSIONS

1. Add or subtract.

A $3\sqrt{7}+8\sqrt{7}$

$3\sqrt{7}+8\sqrt{7}$

$11\sqrt{7}$

B $9\sqrt{y}-\sqrt{y}$

$9\sqrt{y}-1\sqrt{y}$

$8\sqrt{y}$

C $8\sqrt{3d}+6\sqrt{2d}+10\sqrt{3d}$

$8\sqrt{3d}+6\sqrt{2d}+10\sqrt{3d}$ *Identify like radicals.*

$18\sqrt{3d}+6\sqrt{2d}$ *Combine like radicals.*

SIMPLIFYING BEFORE ADDING OR SUBTRACTING

2. Simplify.

A $\sqrt{12}+\sqrt{27}$

$\sqrt{4(3)}+\sqrt{9(3)}$ *Factor the radicands using perfect squares.*

$\sqrt{4}\sqrt{3}+\sqrt{9}\sqrt{3}$ *Product Property of Square Roots*

$2\sqrt{3}+3\sqrt{3}$ *Simplify.*

$5\sqrt{3}$ *Combine like radicals.*

B $3\sqrt{8}+\sqrt{45}$

$3\sqrt{4(2)}+\sqrt{9(5)}$ *Factor the radicands using perfect squares.*

$3\sqrt{4}\sqrt{2}+\sqrt{9}\sqrt{5}$ *Product Property of Square Roots*

$3(2)\sqrt{2}+3\sqrt{5}$ *Simplify.*

$6\sqrt{2}+3\sqrt{5}$ *The terms are unlike radicals. Do not combine.*

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Multiplying and Dividing Radical Expressions

(for Holt Algebra 1, Lesson 11-8)

- To multiply and divide expressions containing square roots, you can use the Product and Quotient Properties of square roots. You can also use the Distributive Property and the FOIL method of multiplying binomials with expressions containing square roots.
- A quotient with a square root in the denominator is not simplified. To simplify these expressions, multiply by a form of 1 to get a perfect-square radicand in the denominator. This is called *rationalizing the denominator*.

Examples

MULTIPLYING SQUARE ROOTS

1. Multiply. Write each product in simplest form.

A $\sqrt{3}\sqrt{6}$

$\sqrt{3(6)}$	*Product Property of Square Roots*
$\sqrt{18}$	*Multiply the factors in the radicand.*
$\sqrt{9(2)}$	*Factor 18 using a perfect-square factor.*
$\sqrt{9}\sqrt{2}$	*Product Property of Square Roots*
$3\sqrt{2}$	*Simplify.*

B $\sqrt{3}\left(\sqrt{3}-\sqrt{5}\right)$

$\sqrt{3}\sqrt{3}-\sqrt{3}\sqrt{5}$	*Distribute* $\sqrt{3}$.
$\sqrt{3(3)}-\sqrt{3(5)}$	*Product Property of Square Roots*
$\sqrt{9}-\sqrt{15}$	*Simplify the radicands.*
$3-\sqrt{15}$	*Simplify.*

C $\left(4+\sqrt{5}\right)\left(3-\sqrt{5}\right)$

$12-4\sqrt{5}+3\sqrt{5}-5$	*Use the FOIL method.*
$7-\sqrt{5}$	*Simplify by combining like terms.*

RATIONALIZING THE DENOMINATOR

2. Simplify the quotient $\frac{\sqrt{7}}{\sqrt{2}}$.

$\frac{\sqrt{7}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$	*Multiply by a form of 1 to get a perfect-square radicand in the denominator.*
$\frac{\sqrt{14}}{\sqrt{4}}=\frac{\sqrt{14}}{2}$	*Use the Product Property of Square Roots and simplify.*

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Solving Radical Equations (for Holt Algebra 1, Lesson 11-9)

- A **radical equation** is an equation that contains a variable within a radical.
- You can solve a radical equation by using inverse operations to isolate the variable. For nonnegative numbers, the inverse operation of taking a square root is squaring.
- The Power Property of Equality states that you can square both sides of an equation, and the resulting equation will still be true.
- Squaring both sides of an equation may result in an **extraneous solution**. An extraneous solution is not a solution of the original equation. Therefore, it is important to check the answers using the original equation.

Examples

SOLVING RADICAL EQUATIONS

Solve each equation. Check your answer.

1. $6 = \sqrt{4x}$

$(6)^2 = \left(\sqrt{4x}\right)^2$	*Square both sides.*
$36 = 4x$	*Simplify.*
$9 = x$	*Divide both sides by 4.*

Check $6 = \sqrt{4x}$

6	$\sqrt{4(9)}$
6	$\sqrt{36}$
6	6 ✓

2. $\sqrt{x-5} = 4$

$\left(\sqrt{x-5}\right)^2 = (4)^2$	*Square both sides.*
$x - 5 = 16$	*Simplify.*
$x = 21$	*Add 5 to both sides.*

Check $\sqrt{x-5} = 4$

$\sqrt{21-5}$	4
$\sqrt{16}$	4
4	4 ✓

3. $\sqrt{6-x} = x$

$\left(\sqrt{6-x}\right)^2 = (x)^2$	*Square both sides.*
$6 - x = x^2$	*Simplify.*
$x^2 + x - 6 = 0$	*Write in standard form.*
$(x-2)(x+3) = 0$	*Factor.*
$x - 2 = 0$ or $x + 3 = 0$	*Zero-Product Property*
$x = 2$ or $x = -3$	*Solve for x.*

Check $\sqrt{6-x} = x$

$\sqrt{6-2}$	2
$\sqrt{4}$	2
2	2 ✓

$\sqrt{6-x} = x$

$\sqrt{6-(-3)}$	-3
$\sqrt{9}$	-3
3	-3 ✗

–3 does not check; it is extraneous. The only solution is 2.

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Rational Functions (for Holt Algebra 1, Lesson 12-2)

- A **rational function** is a function whose rule is a quotient of polynomials in which the denominator has a degree of at least 1. In other words, there must be a variable in the denominator.
- An **excluded value** is an *x*-value that makes the *y*-value undefined. For a rational function, an excluded value is any value that makes the denominator equal 0.

Rational Function	$y = \dfrac{8}{x}$	$y = \dfrac{3}{x+3}$	$y = \dfrac{2}{x-6}$
Excluded Value	0	–3	6

- Most rational functions are **discontinuous functions**, meaning their graphs contain one or more jumps, breaks, or holes. The discontinuity occurs at an excluded value.
- An **asymptote** is a line that the graph gets closer to as the absolute value of a variable increases. The graph of a rational function is discontinuous at a vertical *asymptote*.

Identifying Asymptotes	
WORDS	**EXAMPLE**
A rational function in the form $y = \dfrac{a}{x-b} + c$ has a vertical asymptote at the excluded value, or $x = b$, and a horizontal asymptote at $y = c$.	$y = \dfrac{1}{x+2}$ $= \dfrac{1}{x-(-2)} + 0$ Vertical asymptote: $x = -2$ Horizontal asymptote: $y = 0$

Example

GRAPHING A RATIONAL FUNCTION

Graph the function $y = \dfrac{2}{x+1}$.

Step 1 Identify the vertical and horizontal asymptotes.

vertical: $x = -1$ *Use $x = b$. $x + 1 = x - (-1)$, so $b = -1$.*

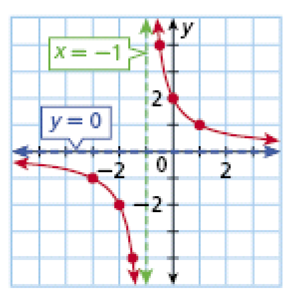
horizontal: $y = 0$ *Use $y = c$. $c = 0$.*

Step 2 Graph the asymptotes using dashed lines.

Step 3 Make a table of values. Choose *x*-values on both sides of the vertical asymptote.

x	–3	–2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	1
y	–1	–2	–4	4	2	1

Step 4 Plot the points and connect them with smooth curves. The curves will get very close to the asymptotes, but will not touch them.



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Simplifying Rational Expressions
(for Holt Algebra 1, Lesson 12-3)

- A **rational expression** is an algebraic expression whose numerator and denominator are polynomials.
- Like fractions, rational expressions are in simplest form when the numerator and denominator have no common factors except 1.
- To simplify a rational expression, factor the numerator and denominator when possible, and then divide out common factors that are in both the numerator and denominator.
- Because division by zero is undefined, rational expressions may have excluded values. Be sure to use the original denominator when finding excluded values because the excluded values may not be "seen" in the simplified denominator.

Examples

SIMPLIFYING RATIONAL EXPRESSIONS

1. Simplify $\frac{3t^3}{12t}$. Identify any excluded values.

$\frac{3t^3}{3(4)t}$ *Factor 12. Note that if t = 0, the expression is undefined.*

$\frac{{}^{1}\cancel{3}t^{\cancel{3}\,2}}{{}_{1}\cancel{3}(4)\cancel{t}^{1}}$ *Divide out common factors.*

$\frac{t^2}{4}$ *Simplify.*

$t \neq 0$ *The excluded value is 0.*

SIMPLIFYING RATIONAL EXPRESSIONS WITH TRINOMIALS

2. Simplify $\frac{k+1}{k^2-4k-5}$. Identify any excluded values.

$\frac{k+1}{(k+1)(k-5)}$ *Factor. If k = –1 or k = 5, the expression is undefined.*

$\frac{\cancel{k+1}^{\,1}}{{}_{1}\cancel{(k+1)}(k-5)}$ *Divide out common factors.*

$\frac{1}{k-5}$ *Simplify.*

$k \neq -1,\ k \neq 5$ *The excluded values are –1 and 5.*

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Multiplying and Dividing Rational Expressions
(for Holt Algebra 1, Lesson 12-4)

- The rules for multiplying and dividing rational expressions are the same as the rules for multiplying and dividing fractions.

Multiplying and Dividing Rational Expressions	
MULTIPLICATION	**DIVISION**
If *a*, *b*, *c*, and *d* are non-zero polynomials, then $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$.	If *a*, *b*, *c*, and *d* are non-zero polynomials, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$.

Examples

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Multiply or divide. Simplify your answer.

1. $\frac{5x^2}{2y^3} \bullet \frac{2y^5}{3x^2}$

$\frac{10x^2y^5}{6y^3x^2}$ — *Multiply the numerators and the denominators.*

$\frac{5y^2}{3}$ — *Divide out common factors. Use properties of exponents.*

2. $(x^2 + 8x + 15)\left(\frac{4}{2x+6}\right)$

$\frac{x^2 + 8x + 15}{1} \bullet \frac{4}{2x + 6}$ — *Write the polynomial over 1.*

$\frac{(x+3)(x+5)}{1} \bullet \frac{4}{2(x+3)}$ — *Factor the numerator and the denominator.*

$\frac{{}^{1}\cancel{(x+3)}(x+5)\cancel{4}^{2}}{{}_{1}\cancel{2}\cancel{(x+3)}_{1}}$ — *Divide out common factors.*

$2x + 10$ — *Multiply remaining factors.*

3. $\frac{1}{x} \div \frac{x-2}{2x}$

$\frac{1}{x} \bullet \frac{2x}{x-2}$ — *Multiply by the reciprocal. (Invert and multiply.)*

$\frac{2\cancel{x}^{1}}{{}_{1}\cancel{x}(x-2)}$ — *Multiply. Divide out common factors.*

$\frac{2}{x-2}$ — *Simplify.*

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Adding and Subtracting Rational Expressions

(for Holt Algebra 1, Lesson 12-5)

- The rules for adding and subtracting rational expressions are the same as the rules for adding and subtracting fractions. If the denominators are the same, add or subtract the numerators and keep the common denominator.
- If the rational expressions do not have the same denominator, multiply each rational expression by a form of 1 so that each term has the LCD as its denominator.

Examples

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Add or subtract. Simplify your answer.

1. $\dfrac{\boldsymbol{x^2 - 8x}}{\boldsymbol{x - 4}} + \dfrac{\boldsymbol{2x + 8}}{\boldsymbol{x - 4}}$

$\dfrac{x^2 - 8x + 2x + 8}{x - 4} = \dfrac{x^2 - 6x + 8}{x - 4}$ *Combine like terms in the numerator.*

$= \dfrac{(x-2)(x-4)}{x-4}$ *Factor.*

$= \dfrac{(x-2)\cancel{(x-4)}^{1}}{\cancel{x-4}^{1}}$ *Divide out common factors.*

$= x - 2$ *Simplify.*

2. $\dfrac{\boldsymbol{3x}}{\boldsymbol{6x^2}} + \dfrac{\boldsymbol{2x}}{\boldsymbol{4x}}$

$6x^2 = 2 \bullet 3 \bullet x \bullet x$

$4x = 2 \bullet 2 \bullet x$ *Identify the LCD.*

$\text{LCD} = 2 \bullet 2 \bullet 3 \bullet x \bullet x = 12x^2$

$\dfrac{3x}{6x^2}\left(\dfrac{2}{2}\right) + \dfrac{2x}{4x}\left(\dfrac{3x}{3x}\right)$ *Multiply each expression by an appropriate form of 1.*

$\dfrac{6x}{12x^2} + \dfrac{6x^2}{12x^2}$ *Write each expression using the LCD.*

$\dfrac{6x + 6x^2}{12x^2}$ *Add the numerators.*

$\dfrac{\cancel{6}^{1}\cancel{x}^{1}(1+x)}{\cancel{12}^{2}x^{\cancel{2}^{1}}}$ *Factor and divide out common factors.*

$\dfrac{1+x}{2x}$ *Simplify.*

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Dividing Polynomials (for Holt Algebra 1, Lesson 12-6)

- To divide a polynomial by a monomial, write the division as a rational expression. Then divide each term in the polynomial by the monomial.
- To divide a polynomial by a binomial, you can sometimes factor the numerator first and then divide out any common factors and simplify.
- Another method for dividing a polynomial by a binomial is long division.

Examples

DIVIDING POLYNOMIALS

1. Divide $(6x^3 + 8x^2 - 4x) \div 2x$.

$$\frac{6x^3 + 8x^2 - 4x}{2x}$$

Write as a rational expression.

$$\frac{6x^3}{2x} + \frac{8x^2}{2x} - \frac{4x}{2x}$$

Divide each term in the polynomial by the monomial 2x.

$$3x^2 + 4x - 2$$

Divide out common factors and simplify.

2. Divide $\dfrac{c^2 + 4c - 5}{c - 1}$.

$$\frac{(c+5)(c-1)}{c-1}$$

Factor the numerator.

$$\frac{(c+5)\cancel{(c-1)}^{1}}{\cancel{(c-1)}^{1}}$$

Divide out common factors.

$$c + 5$$

Simplify.

3. Divide $(x^2 + 3x + 2) \div (x + 2)$.

Step 1 $x+2\overline{\smash{)}\,x^2 + 3x + 2}$

Write in long division form with expressions in standard form.

Step 2
$$\begin{array}{r} x \phantom{{}+3x+2} \\ x+2\overline{\smash{)}\,x^2 + 3x + 2} \end{array}$$

Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.

Step 3
$$\begin{array}{r} x \phantom{{}+3x+2} \\ x+2\overline{\smash{)}\,x^2 + 3x + 2} \\ \underline{x^2 + 2x} \phantom{{}+2} \end{array}$$

Multiply the first term of the quotient by the binomial divisor. Place the binomial product under the dividend.

Step 4
$$\begin{array}{r} x \phantom{{}+3x+2} \\ x+2\overline{\smash{)}\,x^2 + 3x + 2} \\ -(\underline{x^2 + 2x}) \downarrow \\ x + 2 \end{array}$$

Subtract the product from the dividend. Bring down the next term in the dividend.

Step 5
$$\begin{array}{r} x + 1 \\ x+2\overline{\smash{)}\,x^2 + 3x + 2} \\ -(\underline{x^2 + 2x}) \phantom{{}+2} \\ x + 2 \\ -(\underline{x + 2}) \\ 0 \end{array}$$

Repeat steps 1 – 4 as needed. The remainder here is 0.

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Solving Rational Equations (for Holt Algebra 1, Lesson 12-7)

- A **rational equation** has one or more rational expressions.
- If a rational equation is a proportion, you can solve it using cross-products. If not, you can solve it using the LCD. Answers should always be checked for extraneous solutions.

Examples

SOLVING RATIONAL EQUATIONS

1. Solve $\frac{3}{x-3} = \frac{2}{x}$ using cross-products.

$3x = 2(x-3)$ *Use cross-products.*

$3x = 2x - 6$ *Distribute 2 on the right side.*

$x = -6$ *Subtract 2x from both sides.*

Check $\frac{3}{x-3} = \frac{2}{x}$

$\frac{3}{-6-3}$	$\frac{2}{-6}$
$\frac{3}{-9}$	$\frac{1}{-3}$
$\frac{1}{-3}$	$\frac{1}{-3}$ ✓

2. Solve $\frac{1}{c} + \frac{3}{2c} = \frac{2}{c+1}$ using the LCD.

Step 1 Find the LCD: $2c(c+1)$.

Step 2 Multiply both sides by the LCD.

$$2c(c+1)\left(\frac{1}{c} + \frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$

$$2c(c+1)\left(\frac{1}{c}\right) + 2c(c+1)\left(\frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$ *Distribute on the left side.*

Step 3 Simplify and solve.

$$2\cancel{c}(c+1)\left(\frac{1}{\cancel{c}}\right) + \cancel{2c}(c+1)\left(\frac{3}{\cancel{2c}}\right) = 2c\,\cancel{(c+1)}\left(\frac{2}{\cancel{c+1}}\right)$$ *Divide out common factors.*

$$2(c+1) + (c+1)3 = (2c)2$$ *Simplify.*

$$2c + 2 + 3c + 3 = 4c$$ *Distribute and multiply.*

$$5c + 5 = 4c$$ *Combine like terms.*

$$c + 5 = 0$$ *Subtract 4c from both sides.*

$$c = -5$$ *Subtract 5 from both sides.*

Check

$$\frac{1}{-5} + \frac{3}{2(-5)} = \frac{2}{(-5)+1} \longrightarrow \frac{2}{-10} + \frac{3}{-10} = \frac{2}{-4} \longrightarrow -\frac{5}{10} = -\frac{1}{2} \longrightarrow -\frac{1}{2} = -\frac{1}{2} \checkmark$$

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