GRADE 11 ESSENTIAL
UNIT B – INTEREST AND CREDIT

NOTES

INTRODUCTION TO GRADE 11 ESSENTIAL UNIT B INTEREST AND CREDIT

1. Have you ever bought something on credit? (Then likely regretted it later?).
Have you ever received mail offering you a cheque for $3,500 and wondered if you should use it? Are you planning on saving for retirement or for your children's education or wedding?

This unit is all about borrowing money and saving money.

Objectives

2. You will learn about simple interest and compound interest using manual formulas and using technology apps.

You will learn about effective use of different credit options, lines of credit, overdrafts, etc.

Pre-Requisites

3. To readily succeed in this unit you will want to be familiar with decimal arithmetic, percentages and decimals and fractions, exponents, simple algebra equations and to know how to use the internet.

SIMPLE INTEREST

4. Interest is the fee paid for the use of money. The fee is usually at a pre-determined percentage rate of the money borrowed or invested.

Simple Interest Example

5. You **borrow** $1000. You pay back $1100 at the end of some period of time. So it has cost you **$100** to get **$1000**.

The original amount you borrowed (or if you are the bank the amount you loaned) is called **Principal** amount. The *extra* amount you pay back for the use of that money is called the **Interest**. The **Amount Owing** at the end of the period for that initial **$1,000** is **$1,100**.

6. Complete the table:

Principal (P)	**Interest (I)**	**Amount Owing (A)**
$2,500	$200	
$5,800		$6,400
	$1,200	$10, 800
	$1,825.72	$22,352.35
$4,625		$5200.34

A fancy formula for this:

Amount Owing = **P**rincipal plus **I**nterest

or

A = P + I

(so also using algebra you could re-arrange the relationship so that: I = A – P and P = A – I)

Types of Interest. There are two types of interest: **Simple Interest** and **Compound Interest**.

SIMPLE INTEREST

7. **Definition**. **Simple interest** is a percentage paid on only the principal and at fixed intervals of time.

See the table below for an example of a Principal Amount of $1,000 borrowed over a period of three years.

End Year	Principal [$]	Interest Rate [%]	Interest Owing [$]
1	$1,000	10%	=$1,000*10% = $100
2	$1,000	10%	=$1,000*10% = $100
3	$1,000	10%	=$1,000*10% = $100
		Total Interest	**$300.00**
		+ Principal	**$1,000.00**
		=Amount Owing	**$1,300.00**

8. So in the table above if you borrow $1,000 it costs $100 each and every year in simple interest. After three years you owe $300 in interest plus the original principal amount. So your original *principal* of $1,000 has become a final **amount owing** of $1,300 after three years.

You Try: Simple Interest Calculation using a table.

9. You invest **$2,500** in a simple interest bearing product that has an interest rate of **6%**. You keep it in that account for **10** years. What will the total amount owing (to you) be after 10 years?

Time t [units: years]	Principal P [unit: $]	Interest Rate r [%]	Interest Owing for the period I [units: $]
1	$2,500	6%	
2			
3			
4			
10			
		Total Interest	
		+ Principal	
		=Amount Owing	

SIMPLE INTEREST FORMULA

10. Doing a table like we did above is a *perfectly acceptable* way to solve a mathematical problem. Doing a table row by row, year by year, step by step, is called doing an 'iterative' procedure or also as a 'recursive' procedure. However, doing an iterative table for a twenty year or 50 year period might get a bit laborious. A formula is desirable.

11. The **formula** for the interest owing on a principal amount is:

$$\mathbf{I = P*r*t}$$

where **I** is the total Interest owing in units of **$** after a period of time **t**, **P** is the Principal amount in units of **$**, **r** is the rate of interest in amount per hundred per year, and **t** is the time in years.

12. Make sure you understand what all the symbols are in any formula and what units the quantities are measured in, otherwise it is a useless formula.

of course the total amount you have owing at the end of the loan is the Principal plus the Interest you have accrued. So

A = P + I

where **A** is the total amount owing at the end of the loan period.

13. The formula works for monthly or any other interest rate calculation also, but it is law that the yearly rate be always indicated.

Be aware: In some smart phone and on-line Apps you will find that the amount owing is called FV for Future Value and Principal is called the PV for Present Value.

15. Leo borrows $1200 from his uncle to be paid back over 2 years. His uncle charges 10% interest *per annum (latin for 'per year')* simple interest. How much does Leo pay back? How much interest was paid?

$I = P \cdot r \cdot t$ $\quad I = 1200 * \frac{10}{100} * 2 = \$240$ So Leo has to pay $240 in interest. And he has to pay back the principal also of course. So the total he pays back is $1440

Notice! A percentage rate is actually a fraction! It is French for 'per hundred'. If you do not write the formula as shown above you will likely mess it up. **If you need a refresher on percents and decimals see Appendix B to these notes.**

16. Simple interest is *not very common*. Neither a bank nor an uncle is likely to actually loan you money and tell you to come back in two years with the amount you owe! But **Canada Savings Bonds** (CSB) use simple interest and so do some **Guaranteed Investment Certificates** (GICs) at your bank.

Simple interest is easy to calculate at the end of any reasonable lengthy period and was the only type used until the advent of computers when the more prevalent compound interest could be readily calculated.

17. You try some Simple Interest problems:

a. Carla borrows $900 at 32% Annual Percentage Rate (APR) for three years. (sort of like buying a TV on credit!). How much does she owe after the three years. (Ans: 1764).

b. Erick finds an old Canada Savings Bond in some old files of his grandfather's. It was a $1500 bond bearing a simple interest rate of 7.5% per annum. The bond is 50 years old. What is the Savings Bond worth now? (Ans: $7,125).

c. Jasmine wants to borrow $700 from a Pay Day Loan company *for 2 months* until her income tax refund comes in. They charge her 48% interest rate per annum plus a $30 Administration Fee. How much does Jasmine end up paying for her $700 loan? (Ans: 786)

COMPOUND INTEREST

20. Compound interest is interest calculated at regular intervals on an amount of money to which interest from previous intervals has been added. So interest is paid on the principal and the accrued interest! Investments that receive compound interest can grow rapidly (exponentially like bunnies breeding).

End Year	Amount [$]	Interest Rate [%]	Interest [$]	Total Amount
1	$1,000	10%	$100	$1,100.00
2	$1,100	10%	$110	$1,210.00
3	$1,210	10%	$121	$1,331.00
4	$1,331	10%	$133.10	$1,464.10
↓	↓	↓	↓	↓
20				$6,727.50

21. **Compound Interest Compared to Simple Interest**. If you compare this Compound Interest calculation to the Simple Interest calculation you will find the Total Amount at the end of the 20 years is only $3,000 for the Simple Interest method. So clearly Compound Interest is a better way to earn interest.

Caution! Presently it is unlikely you would receive 10% interest. You would be lucky to get 2% interest. However, some of your older family may recall in the mid-eighties when interest was at 18%. 18% is good if you are earning interest, bad if you owe it!

22. **Rapid Growth – Exponential**. Notice that compared to simple interest, your savings grow rapidly with compound interest, especially the longer you leave your money in savings. Loans and credit cards also use compound interest though; so your debt can grow more rapidly also! You may have studied this exponential growth relationship before!

Notice the above table can be easily done as a 'recursive' table in **EXCEL** or Google **SHEETS** for those students familiar with really useful financial spreadsheet programs.

23. **Compounding over different Periods**. But what if, instead of calculating the interest annually, the bank calculates how much interest you get paid four times a year (every quarter of a year; ie: 'quarterly'). Now you will need to do four lines of calculation for every year!

End Period	Start Amount	Annual Interest Rate	Interest *Rate* for Period	Interest for Period	Total End Amount
March Yr1	$1,000.00	10%	2.50%	$25.00	$1,025.00
June Yr1	$1,025.00	10%	2.50%	$25.63	$1,050.63
September Yr1	$1,050.63	10%	2.50%	$26.27	$1,076.89
December Yr1	$1,076.89	10%	2.50%	$26.92	
March Yr2		10%			
June Yr2		10%	2.50%		$1,159.69
September Yr2	$1,159.69	10%	2.50%		$1,188.69
December Yr2		10%	2.50%		

Complete the blanks above!

25. The formula for compound interest is:

$$\boxed{A = P\left(1 + \frac{r}{s}\right)^{n*s}}$$

Notice the 's' occurs in two places. Be aware some book may use different letters, the formula is still the same!

A is the final **A**mount of value of the investment ***or*** the total **Amount** paid on a loan at the end of '**n**' years
P is the **Principal** or initial value, i.e., the amount that has been invested or borrowed
r is the **rate** of interest expressed in decimal form
n is the **number** of years of investment
s is the number of times that the interest is calculated per year

26. **Compounding Periods**. The '*s*' in this exponential growth equation for compound interest is the most confusing part. It is the number of times the interest is calculated per year. It occurs **twice** in the formula. Here is how you know what value to use for the '**s**'.

Compounding Frequency	**Annual**	**Semi-Annual**	**Quarterly**	**Monthly**	**Daily**
s =	**1**	**2**	**4**	**12**	**365**
Meaning	on interest calculation per year	two interest calculations per year	four times per year (every three months)	12 times per year	365 times a year

Example formula calculation.

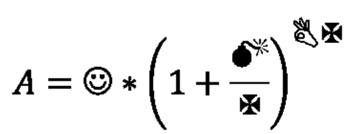
27. Jay borrows \$1,000. He will pay it back after three years with interest. The Annual Percentage Rate (APR) is 10%. The interest on the loan is calculated and compounded once per year. How much does Jay pay back?

$$A = P\left(1+\frac{r}{s}\right)^{n*s} \text{ so } A = 1{,}000\left(1+\frac{10/100}{1}\right)^{3\bullet 1} = 1{,}000(1+0.1)^3 = 1{,}000(1.1)^3 \text{ so}$$

$$A = 1{,}000*(1.1)^3 = 1{,}000*1.331 = \$1331$$

You try some Compound Interest Calculations

Caution: Some references might use different symbols for the same variables. Some references label time as '**t**', not our '**n**' above. Some references label the number of times per year that interest is calculated as '**n**' instead of our '**s**'. It doesn't matter what symbol is used, you could use emojis, as long as it is clear when the formula is presented what each symbol represents.



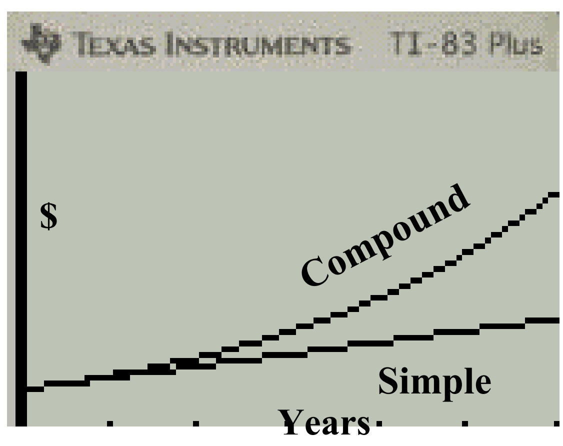
28. **CSB for a Newborn Calculation**. Brandon buys a \$3,000 Canada Savings Bond (CSB) for his newborn son as a way to save for his child's education. The bond pays Compound Interest at a rate of 12% APR compounded annually. What will the CSB be worth when his son turns 18? (Ans: \$23,069.90)

30. **New Credit Card Calculation**. Lyle gets a new credit card in the mail! The terms of the card say he does not need to make any payments for the first six months. However, of course, they still calculate how much he owes every month, he just gets a 'breather' before he has to pay! The interest rate is a somewhat normal 23% APR. Lyle takes a $2,500 advance on his new credit card to buy a nice new plasma TV for his girlfriend. How much will he owe on his credit card when he gets his first statement in six months? Assuming he only bought the TV on his new card! (Ans: $2,801.63)

31. **Unclaimed Balance**. Your great-grandfather had $7,800 squirreled away in a secret bank account back in the 1930s. No one in the family knew this and the money has been sitting in Ottawa for 85 years earning compound interest at 3.5% interest compounded monthly. If no one in the family claims the money within 99 years the government keeps the money in 14 years from now. You are a descendant and put in a claim as per the website. (*Bank of Canada Unclaimed Balance*). How much is the $7,800 worth after those 85 years if you bother to claim it! (Ans: $152,138.85)
[*Note: my family found $13,000 that my grandma had hidden away that we did not know about!*]

32. Notice that Amount Owing or Due (aka: **Future Value)** of a Simple Interest is not that much different than a Compound Interest investment in the short term the first few years, but the *exponential* character of the Compound Interest formula kicks in eventually.

The value of a simple interest situation just grows along a straight line, the value of a compound interest situation takes off like a rocket after a while. Check the Appendix to these notes for how to graph.



35. **Rule of 72**: A quick mental way to calculate the time to double a fixed investment receiving compound interest. Take the interest rate; divide it *into* 72, the result gives the approximate time to double an initial amount of money growing with compound interest. Eg: 8% interest, your money doubles every 9 years. Only works well for rates between 3 to 10%.

36. Do the quick mental math with the **Rule of 72**! Time to double your money. Then test it with the compound interest formula to see how closely it works.

APR (%)	Years to double	Test with the actual formula with $1
2	36	$A = P\left(1+\frac{r}{s}\right)^{n*s} = 1\left(1+\frac{0.02}{1}\right)^{36} \cong 2.04$
6		
12		
	15	
	20	
	6	
	5	
9		

Credit Card Buying (Buying on Credit)

37. When you use a credit card or borrow on credit from stores like MasterCard or Leon's, you receive a statement each month listing your transactions for that month. It tells the new balance owing as well as the minimum amount due to be paid. If you do not pay the entire balance, but pay only the minimum, your lending institution interest will charge you interest on what remains unpaid.

38. **Exampl**e: Complete the following example table to determine the cost of credit for using a credit card. **Minimum Payments** are **5 %** of the **Balance Due** at the end of each month. You (foolishly) make the very **minimum payment** each month. The store credit card charges interest of 2.5% per month, like 30% per year (a bit more actually). Say you buy a big fancy entertainment centre for **$1,800** because the card was so easy to get. You only use the card once, to buy the entertainment centre.

Statement Date **26 Feb 2005**		Payment Due By: **12 Mar 2005**		Credit Limit **$2,000**		
Month	Previous Balance Owing	Payment Made	Purchases Charged	Interest Charges	New Balance Owing	Minimum Payment 5%
Feb 2005			$1,800	+$45.00	$1845.00	$92.25
Mar 2005	$1845.00	–$92.25	+ 0	+$43.81	$1796.56	
Apr 2005			+ 0			
May 2005			+ 0			
Jun 2005			+ 0			
Jul 2005			+ 0			
↓	↓	↓	↓	↓	↓	↓
↓	↓	↓	↓	↓	↓	↓
Jul 2010					$327.40	
TOTALS		**-$2890.66**	$1,800	**$1,418.06**		

39. So by July 2010, 65 months later, (ie: five and a half years later) you have made **$2890.66** in payments and *you still owe* **$327.40**. So this $1,800 entertainment centre has ***so far*** cost you **$3218.06** and you still have not finished paying for it (you still owe $327.40)

40. Interest rates on credit cards are usually anywhere from **18** to **32%** (APR). And if you miss a payment (on this or any other card or loan you have) it goes even higher for a year until your credit report improves! Have you ever checked your credit report?

Personal Loans

42. We had looked at interest rates and loans using compound and simple interest formulas. The examples we had seen were that you borrow a Principal and in a certain time you paid everything back at once. In practice however, loans usually involve a monthly payment. The formulas for calculating the payments are complicated, normally we just use a Loan Payment table. (There are lots of Apps for you smart phone too.)

Amortization Period Monthly Payment Per $1000 Loan Proceeds					
Annual Rate	**1 Year Monthly**	**2 Years Monthly**	**3 Years Monthly**	**4 Years Monthly**	**5 Years Monthly**
6.00%	$86.07	$44.33	$30.43	$23.49	$19.34
6.25%	$86.18	$44.44	$30.54	$23.61	$19.46
6.50%	$86.30	$44.56	$30.66	$23.72	$19.57
6.75%	$86.41	$44.67	$30.77	$23.84	$19.69
7.00%	$86.53	$44.78	$30.88	$23.95	$19.81
7.25%	$86.64	$44.89	$31.00	$24.07	$19.93
7.50%	$86.76	$45.01	$31.11	$24.19	$20.05
7.75%	$86.87	$45.12	$31.23	$24.30	$20.16
8.00%	$86.99	$45.24	$31.34	$24.42	$20.28
8.25%	$87.10	$45.34	$31.45	$24.53	$20.40

Example: Someone borrowing $10,000 over 5 years at 6% would make monthly payments of $19.34. per thousand; or $193.40 per month. For a total payment of:

____

43. The formula for this type of calculation is somewhat onerous:

$$P = \frac{r * M}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} \div n$$

where '**P**' is the monthly payment, ; '**r**' the annual interest rate; '**M'** the mortgage amount; '**t**' the number of years to 'amortize'; and '**n**' is the number of payments per year. So a computer or an App is preferred!

44. Notice most formulas you have ever used only have two or three variables (like A = l*w or I=Prt), this loan payment formula has P, r, M, s, and t ; it is has five variables!! Clearly our simple two-dimensional graphs and tables will not be overly useful if we change all the variables at the same time.

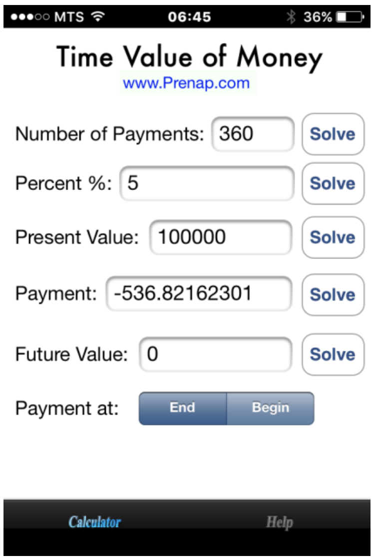
We will generally need a computer application to calculate these types of loan payments or else use tables.

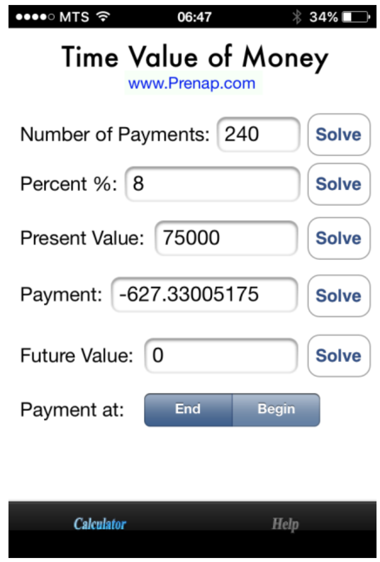
45. Lots of websites have a loan calculator:

a. The Calculator page:
http://www.thecalculatorsite.com/finance/calculators/loancalculator.php

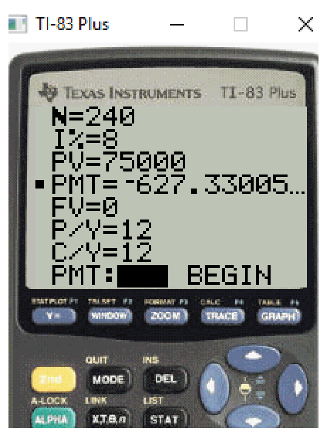
b. Bank of Montreal:
http://www4.bmo.com/popup/loans/Calculator.html#Calc2

Or try an App on your device. Here is a good free one! (of course you know nothing is free!). It is just called TVM (**T**ime **V**alue of **M**oney)





Or Try the App on the Texas Instrument calculator that you use(d) in Applied Math



PV = Present Value (how much you owe)
N = number of payments (20 years * 12 payments per month)
FV = Future Value (what you want it to be after the 240 payments)
P/Y = Payments per period (month)
C/Y = Interest Payments per period

The PMT, payment, is negative because *you* are paying it, money coming out of your account.

46. Try the calculation for a loan of **$75,000** at **8%** interest for **20** years. You should get **$627.33** for the monthly payment. Use the ugly formula, use a website, use an App, … whatever.

So how much in payments would you pay out over the 20 years. ________________

How much extra (ie: Interest) did you end up paying beyond the original Principal Amount of the mortgage? _______________________

47. Try the calculation(s) for a more realistic principal like a $300,000 mortgage at 5% for 25 years.

Total of All Monthly Payments: ____________________
– Original Principal Borrowed: _________________
= Amount of Interest Paid: ____________

GLOSSARY AND FORMULAE

Accrued. A fancy financial term for 'earned', 'built up'. As in 'accrued interest'.

Amortization. The period of time over which a loan is made.

APR: **Annual Percentage Rate**. By law, all loans must be stated in terms of the equivalent annual rate! APR often includes extra fees also, but it is not a perfect comparison device. (for example Payday Loans will charge Fees that are not included in the APR). You may find reference elsewhere to other slightly different interest rates like 'effective rate', etc. Teacher will explain if you are curious.

Budget: a financial plan involving income, expenditures, and savings.

Canada Savings Bonds (CSB). A type of investment device issued by the government of Canada. In effect you are loaning the government money so they can build a stronger Canada and give you back some profit too after several years. The sale of CSBs were discontinued in 2017, they were no longer popular, but there are still billions of dollars worth sitting in peoples sock drawers!

Compound Interest: interest calculated at regular intervals on an amount of money to which interest from previous calculations has been added. $A = P(1+i)^n$. Where A is the total amount of the value of the investment, ; P is the original Principal, i is interest rate for the period, and n is the number of periods. It is an 'exponential' function.

Exponential. A type of growth that grows on itself. The **rate** at which the investment (or debt) grows gets higher and higher since it grows on itself like mould or breeding bunnies. Exponential functions grow quicker and quicker the longer they grow.

Guaranteed Investment Certificate (GIC). An investment product at your bank. The bank guarantees a certain Interest Rate for a certain time. Usually the more you invest the better rate of interest you get.

Interest. The fee paid for the use of money, usually at a predetermined percentage rate of the money borrowed or invested.

Principal: the original amount of a loan or investment. Also known as **Present Value** in many Apps.

Quarterly. Something that happens ever quarter of a year. That is every three months. If you have a quarterly payment of \$300 it is like \$100 per month.

Semi-Annually. Something that happens twice per year. Every six months.

Per Annum: Per year

Formulas:

I=P*r* t

I is interest in units of $. **P** is principal in units of $. **r** is interest rate [%] (no units). **t** is time (in years)

$$\boxed{A = P\left(1+\frac{r}{s}\right)^{n*s}}$$ where

A is the final value of the investment
P is the Principal or initial value, i.e., the amount that has been invested
r is the rate of interest expressed in decimal form
n is the number of years of investment
s is the number of times that the interest is calculated in one year

APPENDIX B - REVIEW FROM PRIOR GRADES

1. In the event you need a quick review of the pre-requisite skills consult the following.

REVIEW PERCENTS, DECIMALS AND FRACTIONS

2. A percent is just a fraction, but *per hundred*. It is actually French for 'per hundred'. There are 100 *cents* in a dollar.

3. 5% means $\frac{5}{100}$. 12% means $\frac{12}{100}$. In fact, if you do not write it as the fraction when doing math you are being crazy!

Some people like to turn percents directly into decimals;
so 5% is $\frac{5}{100}$ which is $100\overline{)5}$ or 5÷100 or 0.05.

So 12% is 0.12 if you insist on doing decimals.

4. Complete the following table: (first row is done for you)

Percent	**Fraction**	**Reduced Fraction**	**Decimal**
5%	*5/100*	*1/20*	*0.05*
10%			
20%			
			0.30
			0.25
			0.03
	1/2		
	1/4		
	6/8		
	5/12		
	9/12		
32%			
27%			
6.25%			
			0.125
			0.375
			0.25

5. If the above is not a sufficient review of percents, decimals and fractions then you will want to get into some upgrade level of lessons.

MANIPULATING FORMULAS (aka: ALGEBRA)

6. If your sister is 5 years older than you then you must be 5 years younger than your sister!

There! You just did algebra!

If : **Sis age** = your age plus 5 years then can also be said as: **Your age** = sis age less 5 years

If **a = b + 5** then → **b = a –5**

If some first number is five more than a second number then the second number is five less than the first number. AMAZING! That is an algebra fact.

7. If 12 is 6 times (or bunches of) *some number*, ‘x’. What is the mystery value ‘x’?

12 = 6*x? What is the **x** worth? You want the ***x*** all by itself, not six of them

The **opposite** of making six bunches of some amount **x** into one pile (multiplying) is to break it into six piles. Breaking something into six equal piles is called dividing! l.o.l.

So divide everything by 6. That will tell what you had in one pile of x!

$$\frac{12}{6} = \frac{6 * x}{6}; \quad \therefore \quad \boxed{\mathbf{2 = x}} ;$$

and of course you can check you are right since 12 = 6 * (2)

That was the easiest Algebra Review you ever had I bet! If you do not follow that then some considerable Upgrade exploration is required on your behalf.

USING ALGEBRA TO SOLVE SIMPLE INTEREST

I = P*r*t

Example 1: $200 = 2000 * \frac{5}{100} * t$; find the vale of t

$200 = \frac{10000}{100} * t$; 2,000 * 5 was 10,000 so simply that

$200 = 100 * t$; $\frac{10000}{100}$ was 100 so we simplified that

$\frac{200}{100} = t$; isolate the unknown ‘t’ by dividing the 100 t’s by 100.

Solution: $\boxed{\mathbf{t = 2}}$

Example 2: $200 = 2{,}000 * r * 2$; so need to find r; the **rate** of interest.

Of course right away you know a **rate** is a fraction, a comparison of two values! Think: rate of pay: $ per 1 hour; rate of speed: km / hr; etc.

$200 = 2{,}000 * r * 2$; simplify that by multiplying
$200 = 4{,}000 * r$; now get the r by itself by dividing
$\frac{200}{4{,}000} = r$; now convert to a percent.
$\frac{200}{4{,}000} = \frac{50}{1000} = \frac{5}{100} = r$; but of course 5/100 means 5%.

So the rate of interest is 5%.

MONTHLY LOAN PAYMENT TABLE FOR A LOAN OF $1,000

Annual Rate	1 Year Monthly	2 Years Monthly	3 Years Monthly	4 Years Monthly	5 Years Monthly	10 Years Monthly	15 Years Monthly	20 Years Monthly	25 Years Monthly
2%	$84.24	$42.54	$28.64	$21.70	$17.53	$9.20	$6.44	$5.06	$4.24
3%	$84.69	$42.98	$29.08	$22.13	$17.97	$9.66	$6.91	$5.55	$4.74
4%	$85.15	$43.42	$29.52	$22.58	$18.42	$10.12	$7.40	$6.06	$5.28
5%	$85.61	$43.87	$29.97	$23.03	$18.87	$10.61	$7.91	$6.60	$5.85
6%	$86.07	$44.32	$30.42	$23.49	$19.33	$11.10	$8.44	$7.16	$6.44
7%	$86.53	$44.77	$30.88	$23.95	$19.80	$11.61	$8.99	$7.75	$7.07
8%	$86.99	$45.23	$31.34	$24.41	$20.28	$12.13	$9.56	$8.36	$7.72
9%	$87.45	$45.68	$31.80	$24.89	$20.76	$12.67	$10.14	$9.00	$8.39
10%	$87.92	$46.14	$32.27	$25.36	$21.25	$13.22	$10.75	$9.65	$9.09
12%	$88.85	$47.07	$33.21	$26.33	$22.24	$14.35	$12.00	$11.01	$10.53
14%	$89.79	$48.01	$34.18	$27.33	$23.27	$15.53	$13.32	$12.44	$12.04
16%	$90.73	$48.96	$35.16	$28.34	$24.32	$16.75	$14.69	$13.91	$13.59
18%	$91.68	$49.92	$36.15	$29.37	$25.39	$18.02	$16.10	$15.43	$15.17
20%	$92.63	$50.90	$37.16	$30.43	$26.49	$19.33	$17.56	$16.99	$16.78
25%	$95.04	$53.37	$39.76	$33.16	$29.35	$22.75	$21.36	$20.98	$20.88
30%	$97.49	$55.91	$42.45	$36.01	$32.35	$26.36	$25.30	$25.07	$25.02
35%	$99.96	$58.52	$45.24	$38.97	$35.49	$30.12	$29.33	$29.20	$29.17

EXAMPLES of loan payments

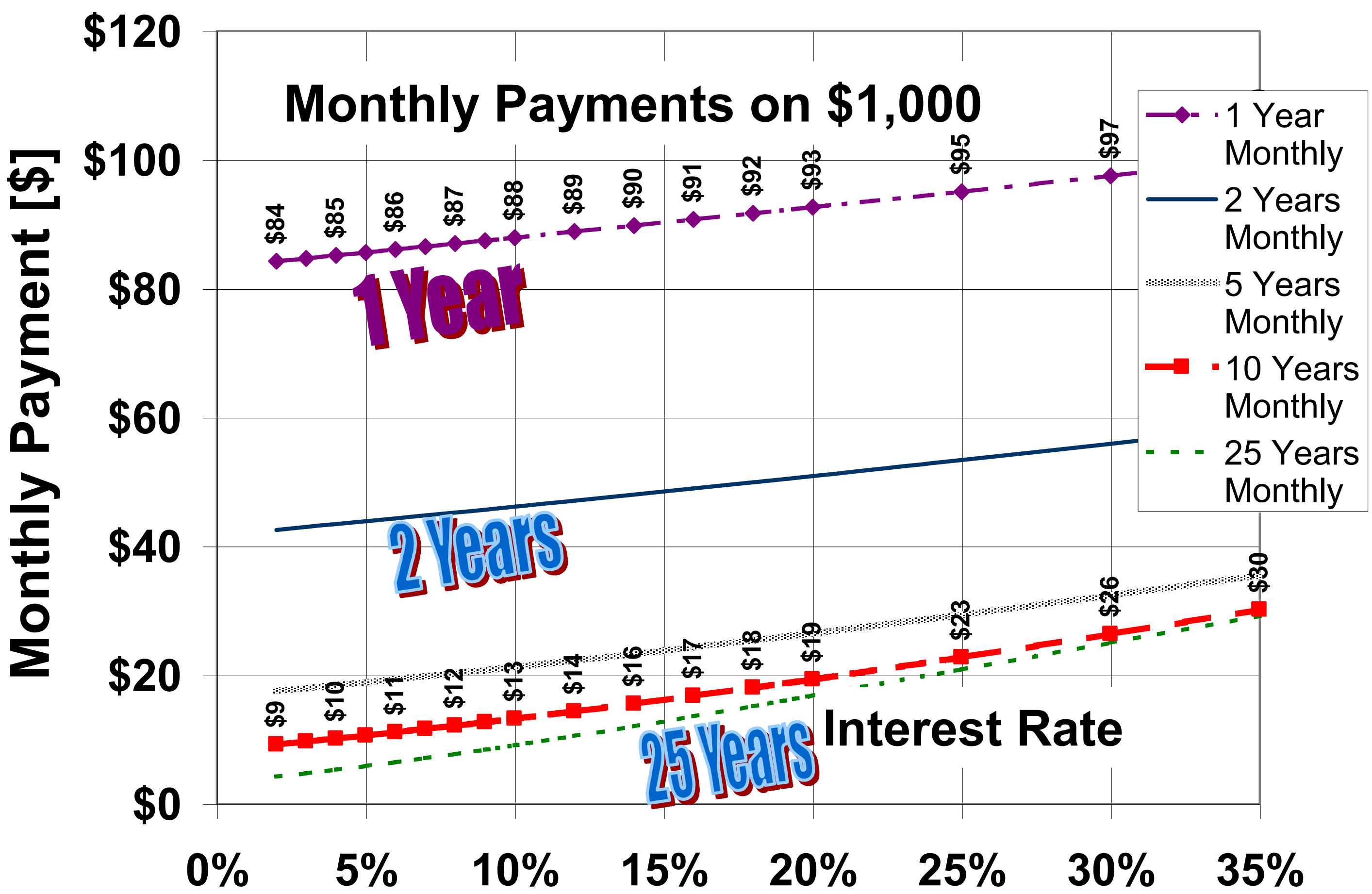
Example A. You borrow $120,000 for 10 years at 14% Annual Rate. Your monthly payments are $13.22 ***for each thousand*** you borrow. So your monthly payment on $120,000 is 120 times as much or $1,586.40 per month. So your loan is paid off after 120 payments of $1,586.40 so a total of $190,368 in payments. So your $120K loan cost you $190K.

Example B. You borrow $200,000 for 25 years at 6% Annual Rate to buy a house. Your monthly payments are $6.44 ***for each thousand*** you borrow. So your monthly payment on $200,000 is 200 times as much or $1,288 per month. So your loan is paid off after 300 payments of $1,288 so a total of $386,400 in payments. So your $200K house cost you $386K over 25 years. Of course, hopefully you will be able to sell it for at least $350K, so it really only cost you $36K to live in a house for 25 years. Mind you now you need another place to live… but the kids are gone so you can get a smaller place!

MrF

GRAPH OF MONTHLY LOAN PAYMENTS FOR $1,000 LOAN

Of course many folks like to see a mass of numbers as a **graph**! Easier to see what is happening and to interpolate!



Look how your payments are a lot lower for a 10 year loan than for a 1 year loan! But then again you are making those payments for 10 longer!

APPENDIX D
GRAPHING SIMPLE AND COMPOUND INTEREST

1. Graphing is an important way to represent information instead of just a flurry of numbers! Graphing allows you to readily:

   a. see trends and patterns,
   b. to interpolate (see values in between the data),
   c. extrapolate (see values outside the data),
   d. to solve problems backwards! If you hate algebra!

2. There exist some mathematical problems that can ONLY be solved by graphing! So graphing is more important than algebra! (?)

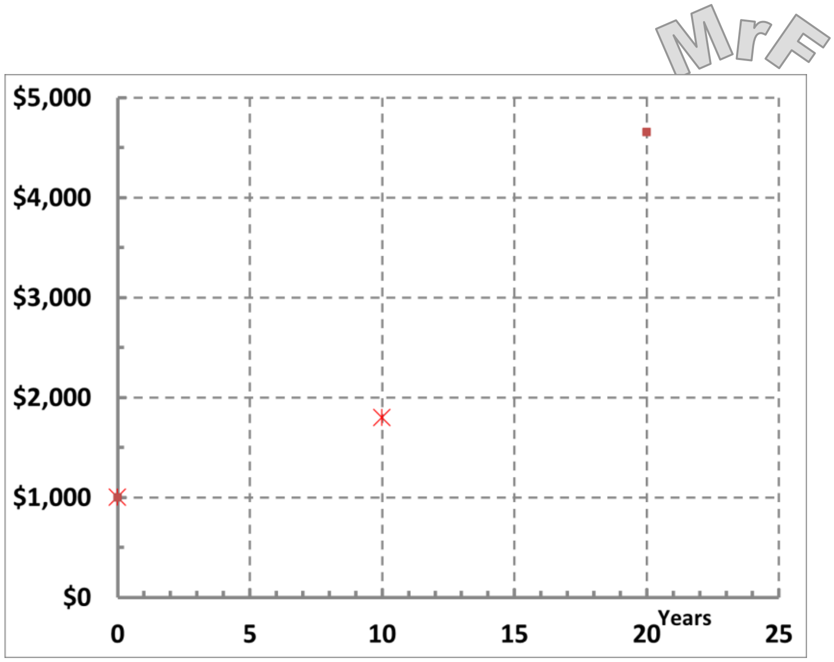
3. Much of this will be covered again in Unit F – Relations and Patterns

4. Complete the following table of Simple Interest Calculations

End Year ‘t’	Principal Amount **‘P’**	Interest at 8% APR **‘I’**	Total Amount **‘A’**
1	$1,000		
2			
3			
4			
5			
10			
20			

Now graph the Simple Interest on the graph below. The principal amount is $1,000.

Follow the years along the bottom (x-axis) and Total Amount along the vertical axis (y-axis).



Now complete the table at the right for compound interest and graph that on the **graph above**!

Assume Annual Compounding of the 8% Interest Rate on an original Principal of $1,000.

Just use the Compounding Interest Formula.

Try it also for a daily compounding (ie: Daily Interest Savings Account).

Connect your ‘dots’ with a smooth curve, not kinky lines!

End Period [Years]	A Compounded Yearly	A Compounded Daily
1		
2		
3		
4		
5		
10		
20		

Have teacher show you how to interpolate. Eg: how much money will your daily compound interest investment be worth after 15 years?

Have teacher show you how to extrapolate. Eg: How much money will your simple interest investment be worth after 50 years.

Have teacher show you how to ‘solve’ using a graph. Eg: how many years do you need to leave your money compounding daily to an amount of $4,000.