### GRADE 10 ESSENTIAL UNIT D – 2-D GEOMETRY

#### **CLASS NOTES**

#### **INTRODUCTION**

1. How much paint do you need to paint your bedroom? How many paving stones do you need to buy to make your aunt's garden patio? How many boards do you need to buy to make your child's sand box.

2. These type of questions are frequent in your life. Of course as a helpful spouse, nephew/niece, or parent it is your job to know how to calculate the answer to these types of everyday problems.

3. In this unit we learn about area and perimeter of flat two-dimensional shapes. Much of this may be a worthwhile refresher from previous studies.

#### PERIMETER

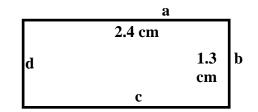
4. **Perimeter of a Rectangle**. Perimeter is the total distance around the outside edge(s) of a flat object. The *perimeter* highway goes around the city! '*Peri*' means close or around; '*meter*' means measure in ancient languages.

5. What is the distance around this rectangle.

This is just a picture of the problem so it likely is not the real measurement. But what is the unknown value of the perimeter; let's call it '**P**'.

6. Value of the Perimeter equals length a plus length b plus length c plus length d.

or more concisely:  $\mathbf{P} = \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ 



*caution: not drawn to scale, not necessarily the correct measurement, do not actually measure it with a ruler!* 

7. There is **your first geometric formula** (!), the perimeter, **P**, of this four-sided figure with straight sides (ie: a *quadrilateral*) is  $\mathbf{P} = \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ ; where *a*, *b*, *c*, and *d* are the lengths of the sides.

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8. *But wait*!! Sides **a** and side **c** the same length! And sides **b** and **d** the same length too? So why not give them the same letter if their values are the same? So we could say:

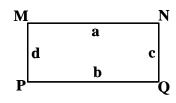
P = a + b + a + b; or you could say: P = 2a + 2b; or you could say: P = 2 (a + b);where and b are the lengths of opposite sides.

Pretty fancy formulae to calculate the distance around a rectangle!

**Labelling 2D Figures**. Be aware how we normally label and identify parts of figures when it is necessary.

a. little letters are used to identify sides;

b. capital letters are used to identify corners.



Some teachers might ask for the length of side **a** *or* they might as for the length of line segment segment **MN**. Similarly, side **MP** could also be called length **d**.

FYI

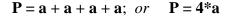
#### **Perimeter of a Square**

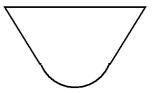
9. A square is a special rectangle where all the edges happen to be the same length. So, we give them the same symbol for the value of the length of the edge,  $\mathbf{a}$  in this case.

a a a a

some books call the 'a' an 's' for side. Does it matter what letter you give it?

10. Shapes can have three to any number of sides. Sides can be straight edges (polygons) or even irregular shaped edges (circular). See the Appendix to these notes for a refresher on the naming of shapes.





#### **Perimeter of a Trapezoid**

11. A trapezoid is a 4-sided polygon where one pair of sides are parallel. Notice the arrow head indicated the pair of sides that are going in the same direction.

 $\mathbf{P} = \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ 

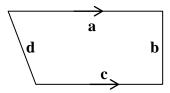
12. **Perimeter of a Triangle**. Did you notice that a trapezoid is really a rectangle with a triangle stuck on one side (could even be two sides)

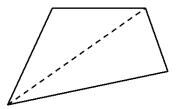
**Triangles are magic**! Every shape can be made of triangles. We therefore must do an extensive study of the perimeter of triangles now.

13. A triangle is a polygon with three angles. [ '*tri*' + '*angle*' ]. Consequently, it has three sides as well.

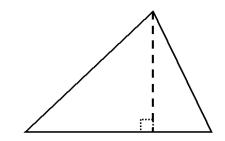
You might recall all the different kinds of triangles.  $\rightarrow$ 

15. Regardless of the types of triangles, the **most important triangle** is the **right triangle**. *Every* kind of triangle can be made of two right triangles and every shape can be made of some triangles, it is important to be rather familiar with a right triangle and all its magic.





Types of Triangles		
Based on angles Based on		
	comparing sides	
right	equilateral	
acute	isosceles	
obtuse	scalene	



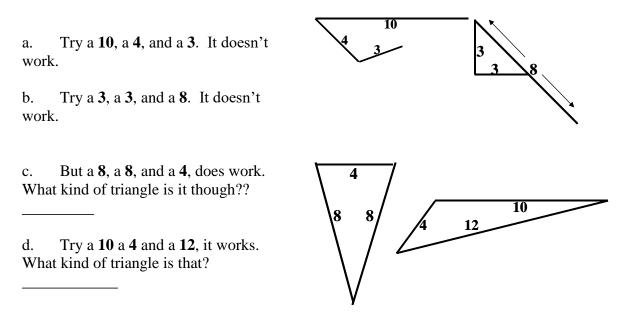
### THE 'RIGHT' (90 DEGREE) TRIANGLE

16. We will start to study the right triangle now. If you understand the right triangle you seriously understand every polygon shape in the universe. Every polygon can be made of triangles. Even circles can be made of triangles!

Pythagorean Theorem (The Law of Pythagoras)

# This is very important! If there were 10 things you just had to know about math for the rest of your life this is one of them for sure.

17. **Experiment 1**. Take three different length slurpee straws or tooth picks and try to make a triangle of any kind with sides those lengths. You might find there are some combinations of lengths that cannot make any triangle!



18. **Experiment 2**. Take three different length slurpee straws and try to make a right triangle! This will be very difficult. How can it be so difficult if there are right triangles everywhere??!!

Obviously, there is some secret to making right-triangles.

19. **Pythagorean Theorem**. One of the more ancient and critical concepts of all. Many cultures had discovered 2500 years ago that "*given any right-triangle, the square on the longest side equals the sum of the squares on the other two sides*". The law has the Greek mathematician Pythagoras's name because he proved it and recorded it. But *ancient bead work* in some North American indigenous cultures suggests they knew it too.

This is one of those math concepts you really do need to know for the rest of your life! Honestly!

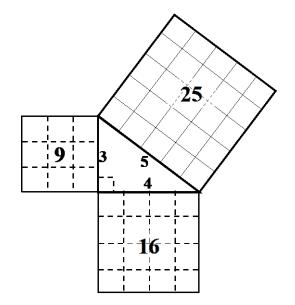
#### It's a law!

20. Given **any** right-triangle, the square on the longest side equals the **sum** of the squares on the other two [shorter] sides.

In 'algebraic' form:

 $c^2 = a^2 + b^2$ ; \*\*\*\* where *c* is the longest side \*\*\*\* *c* is <u>alone by itself</u> in the equation!!!!!! \*\*\*\*

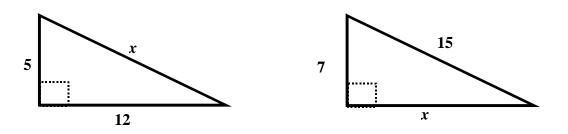
The longest side is called the *hypotenuse* and is **always** *across* from the **right angle**.



21. You most definitely need to add this idea to your two-page cheat sheet [reference notes]

#### Pythagoras Examples: You try!

22. Given the two right-angle triangles below; find length x. (The triangles are **not** '*drawn to scale*', so you can't just measure them!)



Ans: 13 exactly

Ans: about 13.27 if you round to two decimal places.

#### **WORKED EXAMPLE - PYTHAGORAS**

23. The most difficult problem is to find a short side when you are given the other short side and the hypotenuse.

Write down the formula (you only have one!)

 $c^2 = a^2 + b^2$ ; now plug in the numbers in the correct place; (*'c' is the hypotenuse!*)

 $9^2 = 6^2 + b^2$  evaluate the squared numbers

So

 $81 - 36 = b^2$ ; evaluate (subtract)

 $81 = 36 + b^2$ ; get the b<sup>2</sup> by itself

 $45 = b^2$ ; now calculate the 'b' by 'unsquaring'

6.7082... = b

 $\therefore$  **b** = 6.71 rounded to nearest hundredth

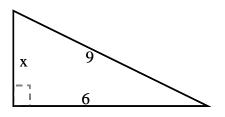
#### **Square and Square Roots of Numbers**

25. If you are not overly familiar with the idea of squares and square roots from previous studies try this quick lesson; complete the table here:

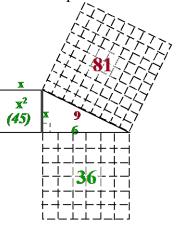
$x \rightarrow$	2	3	8			5		$\leftarrow \sqrt{y}$
$x^2 \rightarrow$				81	100		49	<b>←</b> y

Evaluate the following squares and square roots (to nearest hundredth)

a.	2.5 <sup>2</sup>	b.	√6.25	c.	<b>7.071</b> <sup>2</sup>	d.	$\sqrt{50}$
e.	100 <sup>2</sup>	f.	1000 <sup>2</sup>	g.	7.1 <sup>2</sup>	h.	$\sqrt{7.1^2}$



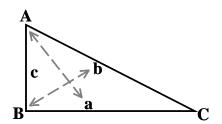
If you don't like all the fancy steps to the left (algebra) just sketch the picture  $\checkmark$ 



# Labelling Triangles (special way)

As usual CORNERS are labelled with CAPITAL LETTERS and sides are labelled with small letters.

But triangles have **special labelling**; the small letter across from a corner is the same as the CAPITAL letter at the corner across.



So side '**a**' can also be called side **BC** Some folks label the sides and some the corners that make the sides!

# **Un-natural Numbers (Irrational Numbers)**

26. Often when doing a square root you will end up with a crazy number on your calculator like:

 $\sqrt{5} = 2.236067977$ . It actually turns out to be closer to this really:

# **2.2360679774997896964091736687313.....;** (it goes forever with no pattern)

but your calculator is too small to show it all!

So normally you would just properly round the answer to 2 or 3 decimal places as directed in an assignment.

# PERIMETER OF A CIRCLE (CIRCUMFERENCE) AND $\boldsymbol{\pi}$

27. **The Circle**. Another rather important shape! The circle. And another important formula that you should have memorized forever. Seriously!

28. **Experiment**. Make a good circle. Measure the distance across the centre of the circle, call it **d** for diameter. Measure the distance around the circumference of the circle, call it **C** for circumference. Calculate:  $\mathbf{C} \div \mathbf{d}$ ; or since we are in high school now more properly written  $\mathbf{C}/\mathbf{d}$ .

It doesn't matter what you use for units of measure; cm, mm, inches; miles; ....

Your answer had best come out close to a **bit more than 3**. If it did not the universe is collapsing.

30. The real value of **C/d** for any and all circles is:

# 3.1415926535897932384626433832795...→

We call this number 'Pi' and use the Greek alphabet symbol ' $\pi$ '.

31. There is a button on your calculator that will tell you the value is **3.14159265** (depending on the precision of your calculator display). Find the  $\pi$  on your calculator.

32. If we are doing manual calculations without a calculator we usually just use 3.14 as a close *approximation* to  $\pi$ . The fraction 22/7 works nicely two if you prefer fractions.

 $\pi$  is another one of those pesky numbers that has decimal places that go on and on forever with no rational pattern! It is '*irrational*'.

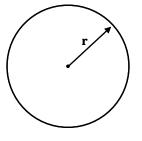
35. **Calculate the Circumference**. Circumference is a specific word for a **perimeter** of a circle. The formula for circumference is:

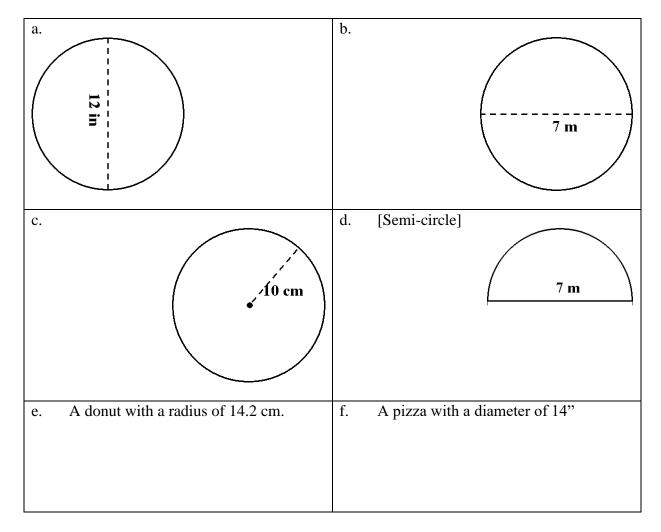
36. Circumference equals  $\pi$  times diameter; or  $\mathbf{C} = \pi \cdot \mathbf{d}$ , where '**d**' is the diameter.

# Distance around a circle (Circumference)

Circumference is really a perimeter, but a different word is used for circles.

 $C = 2\pi r$ ; where r is the radius measurement or since 2\*r is a diameter; or  $C = \pi d$  where d is the diameter measurement.





37. **You Try**; calculate the perimeter (circumference) of the following (round to nearest hundredth decimal place):

# Working a Formula

40. If you already know the circumference but want the diameter, the formula still works, it just needs to be juggled around a bit. Example: You know the circumference of a bike wheel is 70 inches, what is its diameter?

a. Solve by Simple Proportion. You know that a wheel of 1 in diameter is 3.14 in circumference so since you wheel is just 70 times bigger:

$$\frac{1}{3.14} = \frac{d}{70};$$
  $\therefore$   $d = \frac{70}{3.14} = 22.29in$ 

b. Solve by algebra. Math students learn to more quickly manipulate symbols to solve problems using 'algebra'.

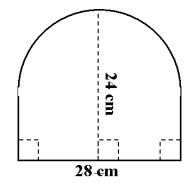
if  $C = 3.14^* d$  but you want to know just 'd' then '*un-multiply*' (ie: divide) both sides of the formula by the 3.14 to find the 'd' by itself.

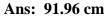
If **C** = 3.14\* **d**, then 
$$\frac{C}{3.14} = d$$
.

#### Perimeter of Irregular Shapes 1.

41. Not every shape you encounter is a nice easy shape. Some shapes are combinations of several shapes.

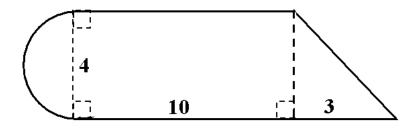
42. Say you are designing a plaque to award the top student. It is a rectangle with a semi-circle on top. You want to put a gold ribbon around the perimeter. What length of gold ribbon do you need?





#### **Perimeter of Irregular Shapes 2**

43. What is the perimeter length of this irregular shape below?



Ans: 34.28 units

#### **AREA OF 2 DIMENSIONAL SHAPES**

44. Say you want to paint your child's bedroom walls! You select the perfect colour that she likes. The can says that a one litre can of paint will cover 12 square metres of wall surface. How many litres of paint do you need to buy?

This section is all about **area of shapes**. How much surface they have measured in little **squares**.

#### 45. Area: Rectangles and squares

Area = l \* w

This area is 6 units\* 3 units = 18 square units or **18 units**<sup>2</sup>

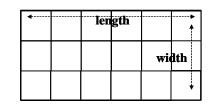
# 46. Area: Rhombus and Parallelograms

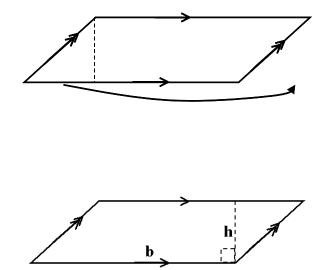
At right is a parallelogram. Notice how you could rip off the triangle on the left side and add it too the figure on the right side and have a nice easy rectangle. The area would not change!

47. Instead of length times width now, we call the area the *base times the height*.

$$\mathbf{A} = \mathbf{b} * \mathbf{h};$$

but notice the '**h**' must be measure perpendicular, at 90 degrees to the base!





# 48. Area: [Right] Triangles

What is the area of the **rectangle** at the right?

\_\_\_\_\_ square units

Did you notice that a [right] triangle is really half of a rectangle?

So, what is the area of the **triangle**?

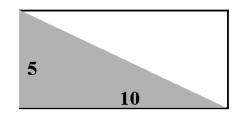
49. Area: Any Triangle (obtuse, right, acute)

What is the formula for the area of the parallelogram at the right?

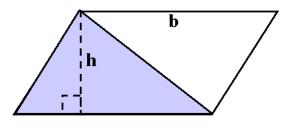
A = \_\_\_\_\_

Did you notice that *any* triangle is really half of a parallelogram?

So, what is the area of any triangle?

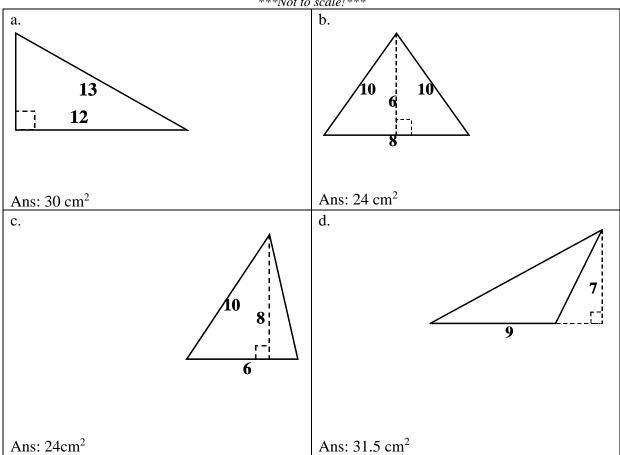


Fancy formula for area of half a rectangle:



$$\mathbf{A}_{\Delta} = \frac{1}{2} * \mathbf{b} * \mathbf{h}$$

\*\*\* 'h', height, is always  $\perp$  to base\*\*\*



#### 50. Calculate the area of the triangles below: (all indicated measurements are in cm) \*\*\*Not to scale!\*\*\*

# Area of a Circle

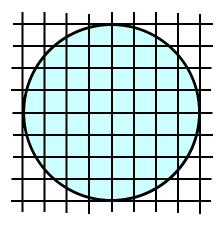
60. How many squares fit inside a circle? You just know that 3.14 is involved somehow!

Count, roughly, how many whole or equivalent little unit squares are inside this circle. Include bits of squares too.

Number of squares (approx?) = \_\_\_\_\_

What is the diameter, **d**, of this circle?

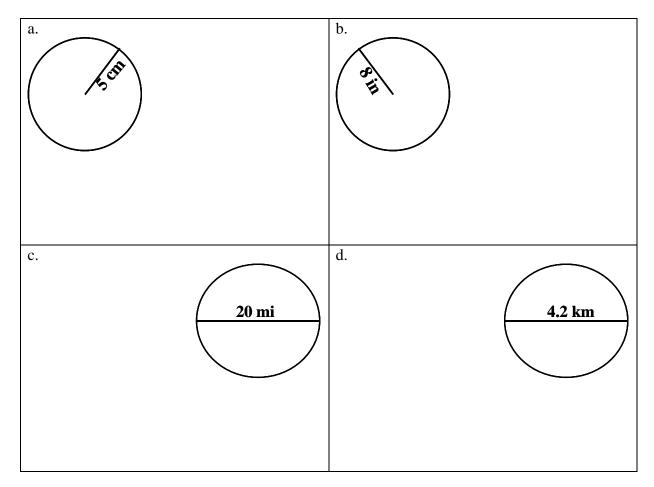
Try this formula:  $\mathbf{A} = \pi^* \mathbf{r}^2 =$ 



61. So that is the area of a circle!

 $A_{circle} = \pi^* r^2$ ; where 'r' is the radius of the circle

62. Calculate the area of each circle below; use as accurate as value of  $\pi$  as you can. Round final answers to two decimal places.



# **CONVERTING SQUARE MEASURES**

65. What if you are painting your son's bedroom and you measure the walls to have an area of 360 square feet [**360** ft<sup>2</sup>]. You get to the hardware store and the paint can says it covers 10 square meters [**10**  $m^2$ ]. How do you convert? How many square meters is the same as how many square feet.

66. Draw a diagram to see the problem!  $\rightarrow$ 

See how one square metre is really 10.76 square metres!

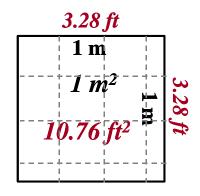
 $1m^2 = 10.76 m^2$ 

Of course you knew this because you knew that one metre was the same as 3.28 ft.

67. So a metre times a metre, 1 square metre, must be  $3.28 \text{ ft}^* 3.28 \text{ ft}$ ; or what we call  $3.28^2 \text{ ft}^2$ ; 10.76 ft<sup>2</sup>

So the rule is you need to convert the length *and* the width into the new units, so you have to do a double conversion.

- 70. You try
- a. Convert 16 square yards into square ft



b. Convert 10 square meters into square cm.

Ans: 144 ft<sup>2</sup>

Ans: 100,000 cm<sup>2</sup>

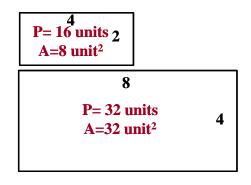
# SCALING OF SHAPES

71. What happens to the perimeter and the area of a shape if you just double al the lengths? Check it out! Draw yourself a simple example:

72. Notice that doubling all the lengths doubles the perimeter!

But

Doubling all the lengths also quadruples the area.



73. See why from the geometric formula:

$$\begin{split} \mathbf{P}_{small} &= a + b + c + d \\ \mathbf{P}_{big} &= 2a + 2b + 2c + 2d = 2*(a + b + c + d) = \mathbf{2*} \ \mathbf{P}_{small} \\ & \mathbf{but} \\ \mathbf{A}_{small} &= b * h \\ \mathbf{A}_{big} &= 2*b*2*h = 2^{2*}b*h = 4bh = \mathbf{4*}\mathbf{A}_{small} \end{split}$$

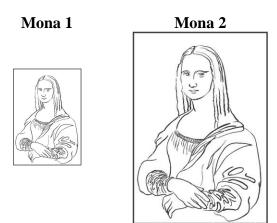
# SCALE

74. Scale is a comparison of the lengths of similar shaped objects in some ratio. Big : Little or  $\frac{Model}{Actual}$ ; etc. Doubling the scale doubles the perimeter but quadruples the area. Tripling the

scale will change the perimeter by three times but the area by nine times  $(3^2 = 9)$ .

75. Doubling the height and the width of Mona's picture gives her **twice** the **length** of hair, twice the length of nose, twice her hat size around her circumference of her head; etc **but** 

Doubling the height and the width of Mona's picture gives her **four times the area** of her face (for makeup), four times as much fabric for her blouse, etc.



Think about it!

# CONCLUSION

78. That completes your study of the geometry of two-dimensional objects!

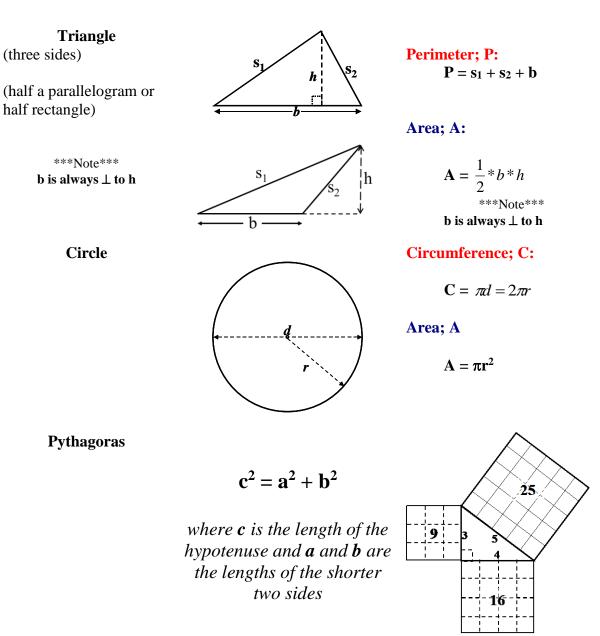
79. You will be given all the necessary geometric formulae on tests and exams (although they are rather logical and can readily be memorized or derived without reference to a formulae sheet)

80. Start to add some of these ideas to your Grade 10 Reference Notes ('cheat sheet').

81. In Grade 11 you will do three dimensional objects like cubes and cones!

# APPENDIX A GRADE 10 ESSENTIAL UNIT D – 2-D GEOMETRY

Shape	GEOMETRIC FORMULA Diagram	AE Formulae
	FLAT OBJECTS 2 DIMENSIONAL	
Square (all four sides same length, 90°corners) (a rectangle with all sides same length)		Perimeter, P: P = s + s + s + s = 4*s Area, A: $A = s * s = s^2$
<b>Rectangle</b> (Four sides, square corners)		Perimeter, P: P = l + w + l + w = 2l + 2w Area, A: A = l * w
Parallelogram and Rhombus (a leaning rectangle or leaning square) ***Note*** b is always⊥to h	h s b	Perimeter; P: P = 2b + 2s Area; A: A = b * h ***Note*** b is always + to b
Trapezoid	<b>`</b>	b is always ⊥ to h
(Four sides, only two sides parallel {    })	$\begin{array}{c c} b_1 & \cdots \\ s_2 & h & s_1 \\ b_2 & \cdots \\ b_2 & \cdots \\ b_2 & \cdots \\ b_2 & \cdots \\ b_1 & \cdots \\ b$	Perimeter; P: $P = b_1 + s_1 + b_2 + s_2$ Area; A: $A = b_{avg} * h$
***Note*** b is always⊥to h		$=\frac{1}{2}(b_1+b_2)*h$ ***Note*** b is always $\perp$ to h



Add your own extra formulae below.

# APPENDIX B TO GR 10 ESSENTIAL UNIT D – 2D GEOMETRY

	GLOSSARY		
acute angle	an angle measuring less than 90°		
acute triangle	a triangle with three acute angles		
algebra	the manipulation of values, variables, and expressions in a formula to solve a problem using very basic and logical rules.		
altitude	the perpendicular distance from the base of a figure to the highest point of the figure. Also the height of an object above the earth's surface.		
approximation	a number close to the exact value of a measurement or quantity. Symbols such as $\approx$ or $\cong$ or are used to represent approximate values.		
congruent:	figures that have the same size and shape, but not necessarily the same orientation		
decagon	a polygon with ten sides.		
diameter	distance across a circle (a chord through the centre). A <b>diameter</b> is just twice the measure of the <b>radius</b> which is measured only from the centre to the circumference.		
equi-angular polygon	a polygon where all the angles have the same measure. Also known as a <b>regular</b> polygon.		
evaluate	substitute a value for each of the variables in an expression and simplify the result. (as opposed to solve). Eg: find the value of the expression $2x + 1$ if the x is worth 3. Ans: 7		
formula	a mathematical rule that is expressed as an equation to find values of quantities. <i>Plural</i> : formulae.		
heptagon	a polygon with seven sides.		
hexagon	a polygon with six sides.		

hexahedron	a polyhedron with six faces. A regular hexahedron is a cube.	
hypotenuse	the longest side of a right-angle triangle. The side opposite the right angle in a right triangle.	
irrational number	a number that is <b>not rational</b> . ( <i>think: irresponsible means NOT responsible</i> ). It cannot be calculated to perfect accuracy, it cannot be fully written down. It has <b>no</b> rational pattern of repetition in its decimal places. Examples of irrational numbers are $\pi$ and $\sqrt{5}$ , etc. It exists in your head but cannot be written down.	
isosceles acute triangle:	a triangle with two equal sides and all angles less than $90^{\circ}$	
isosceles obtuse triangle:	a triangle with two equal sides and one angle greater than 90°	
isosceles right triangle:	a triangle with two equal sides and a 90° angle	
isosceles triangle:	a triangle with [at least] two equal sides	
kite	a quadrilateral with two pairs of equal adjacent sides	
legs	the sides of a right triangle that form the right angle	
parallelogram	a quadrilateral with both pairs of opposite sides parallel	
octagon	a polygon with eight sides. A 'regular' octagon has all sides and angles the same.	

octahedron	a polyhedron with eight faces.	
parallel	describes two lines that are going the same direction and do not intersect.	
parallelogram	a quadrilateral with opposite sides parallel.	
pentagon	a five sided polygon.	
perpendicular	two lines are perpendicular if the angle between them is 90 degrees. We use the math symbol: ' $\perp$ '.	
polygon	the union of three or more line segments that are joined together so as to completely enclose an area.	
polyhedron	a solid 3-dimensional object that is bounded by plane polygons. We study polyhedrons like boxes and pyramids in Grade 11.	
proportion	a statement that two ratios are equal. Eg: if one muffin takes 6 raisins, then 10 muffins takes 60 raisins. $\frac{1}{6} = \frac{10}{60}$	
Pythagorean Theorem	or symbolically: $b^2 = a^2 + b^2$ ; provided that c is the hypotenuse length.	

Pythagorean triple	Three natural numbers that satisfy the Pythagorean theorem. One example is: 3, 4 and 5.
	5, 12, 13 also works.
quadrilateral:	a four-sided polygon
rectangle	a quadrilateral with four 90 degree [right] angles.
rectangular pyramid	a pyramid with a rectangular base
rectilinear polygon	a shape made up of straight line segments where all the corners are 90° or square.
regular polygon	a polygon in which all the angles have the same measure and all of the sides are equal in length.
repeating decimal	a decimal in which the decimal digits endlessly repeat a pattern. A repeating decimal can always be re-written as a fraction. Eg: <b>3.566566566566</b> is <b>3563/999</b>
rhombus	a quadrilateral with four equal sides. Really just a square that is leaning. A rhombus is a special case of a parallelogram.
right angle	an angle whose measure is 90 degrees.
right triangle	a triangle that has a right angle.

scale	the scale of an object is a comparison in a ratio or a fraction of its drawing or model size to its actual size. Scale $\rightarrow$ Model: Actual or $\frac{Model}{Actual}$ Many doll houses are 1:40 scale so 1 cm (or unit) on the doll house is 40 cm (or units) on the real house. Boys' model airplanes are often 1/72 scale. A common scale for a city map is 1:100,000, so that 1 cm on the map is 1 km in the actual city.
similar figures	figures with the same shape, but not necessarily the same size
square of a number	To multiply two of the same number together. 3 * 3 = 9; $6 * 6 = 36$ ; $25 * 25 = 625$ . We often use a little 'two exponent' to write a square, eg: $6^2 = 36$ or $9^2 = 81$ , etc. You have a button on your calculator that does a square, the $x^2$ button.
square root	Square root of some number, <b>x</b> , is the desired number that, when multiplied by itself gives that number, <b>x</b> . for example, $\sqrt{9} = 3$ because $3^2 = 9$ . To do a square root is really just to 'un-square' a number. The $$ is called the radical sign. The number inside it is called the radicand. In $\sqrt{10}$ , 10 is the radicand. Find that button on your calculator too.
theorem	a statement that has been proven.
trapezoid	a quadrilateral with one pair of opposite sides parallel but not equal in length.

triangle	a three sided polygon.
triangle types	Triangles have certain named characteristics depending on
	their lengths and also their angles.
	Length Characteristic:
	all three sides the same : equilateral
	at least two sides the same: Isosceles
	no two sides the same : scalene
	Angle Characteristic:
	all acute angles: acute triangle
	a 90 degree square corner: right triangle
	on obtuse angle (>90°): obtuse triangle