GRADE 10 ESSENTIAL UNIT D – 2-D GEOMETRY

CLASS NOTES

INTRODUCTION

1. How much paint do you need to paint your bedroom? How many paving stones do you need to buy to make your aunt's garden patio? How many boards do you need to buy to make your child's sand box.

2. These type of questions are frequent in your life. Of course as a helpful spouse, nephew/niece, or parent it is your job to know how to calculate the answer to these types of everyday problems.

3. In this unit we learn about area and perimeter of flat two-dimensional shapes. Much of this may be a worthwhile refresher from previous studies.

PERIMETER

4. **Perimeter of a Rectangle**. Perimeter is the total distance around the outside edge(s) of a flat object. The *perimeter* highway goes around the city! '*Peri*' means close or around; '*meter*' means measure in ancient languages.

5. What is the distance around this rectangle.

This is just a picture of the problem so it likely is not the real measurement. But what is the unknown value of the perimeter; let's call it '**P**'.

6. Value of the Perimeter equals length a plus length b plus length c plus length d.

or more concisely: $P = a + b + c + d$

caution: not drawn to scale, not necessarily the correct measurement, do not actually measure it with a ruler!

7. There is **your first geometric formula** (**!**), the perimeter, **P**, of this four-sided figure with straight sides (ie: a *quadrilateral*) is $P = a + b + c + d$; where a, b, c, and d are the lengths of *the sides.*

Revised: 20190223

8. *But wait*!! Sides **a** and side **c** the same length! And sides **b** and **d** the same length too? So why not give them the same letter if their values are the same? So we could say:

> $P = a + b + a + b$; *or you could say*: $P = 2a + 2b$; *or you could say:* $P = 2 (a + b)$; *where and b are the lengths of opposite sides.*

Pretty fancy formulae to calculate the distance around a rectangle!

Labelling 2D Figures. Be aware how we normally label and identify parts of figures when it is necessary.

> a. little letters are used to identify sides;

b. capital letters are used to identify corners.

Some teachers might ask for the length of side **a** *or* they might as for the length of line segment segment **MN**. Similarly, side **MP** could also be called length **d.**

FYI

Perimeter of a Square

9. A square is a special rectangle where all the edges happen to be the same length. So, we give them the same symbol for the value of the length of the edge, **a** in this case.

a a a a

some books call the 'a' an 's' for side. Does it matter what letter you give it?

10. Shapes can have three to any number of sides. Sides can be straight edges (polygons) or even irregular shaped edges (circular). See the Appendix to these notes for a refresher on the naming of shapes.

 $P = a + a + a + a$; *or* $P = 4 * a$

Perimeter of a Trapezoid

11. A trapezoid is a 4-sided polygon where one pair of sides are parallel. Notice the arrow head indicated the pair of sides that are going in the same direction.

 $P = a + b + c + d$

12. **Perimeter of a Triangle**. Did you notice that a trapezoid is really a rectangle with a triangle stuck on one side (could even be two sides)

Triangles are magic! Every shape can be made of triangles. We therefore must do an extensive study of the perimeter of triangles now.

13. A triangle is a polygon with three angles. ['*tri*' + '*angle*']. Consequently, it has three sides as well.

You might recall all the different kinds of triangles. \rightarrow

15. Regardless of the types of triangles, the **most important triangle** is the **right triangle**. *Every* kind of triangle can be made of two right triangles and every shape can be made of some triangles, it is important to be rather familiar with a right triangle and all its magic.

THE 'RIGHT' (90 DEGREE) TRIANGLE

16. We will start to study the right triangle now. If you understand the right triangle you seriously understand every polygon shape in the universe. Every polygon can be made of triangles. Even circles can be made of triangles!

Pythagorean Theorem (The Law of Pythagoras)

This is very important! If there were 10 things you just had to know about math for the rest of your life this is one of them for sure.

17. **Experiment 1**. Take three different length slurpee straws or tooth picks and try to make a triangle of any kind with sides those lengths. You might find there are some combinations of lengths that cannot make any triangle!

18. **Experiment 2**. Take three different length slurpee straws and try to make a right triangle! This will be very difficult. How can it be so difficult if there are right triangles everywhere??!!

Obviously, there is some **secret** to making right-triangles.

19. **Pythagorean Theorem**. One of the more ancient and critical concepts of all. Many cultures had discovered 2500 years ago that "*given any right-triangle, the square on the longest side equals the sum of the squares on the other two sides*". The law has the Greek mathematician Pythagoras's name because he proved it and recorded it. But *ancient bead work* in some North American indigenous cultures suggests they knew it too.

This is one of those math concepts you really do need to know for the rest of your life! Honestly!

It's a law!

20. Given **any** right-triangle, the square on the longest side equals the **sum** of the squares on the other two [shorter] sides.

In 'algebraic' form:

 $c^2 = a^2 + b^2$; ******where** *c* **is the longest side****** *c* is *alone by itself* in the equation!!!!!! ****

The longest side is called the *hypotenuse* and is **always** *across* from the **right angle**.

21. You most definitely need to add this idea to your two-page cheat sheet [reference notes]

Pythagoras Examples: *You try*!

22. Given the two right-angle triangles below; find length *x*. (The triangles are **not** '*drawn to scale*', so you can't just measure them!)

Ans: 13 exactly

Ans: ab*out* **13.27** if you round to two decimal places.

WORKED EXAMPLE - PYTHAGORAS

23. The most difficult problem is to find a short side when you are given the other short side and the hypotenuse.

Write down the formula (*you only have one!*)

 $c^2 = a^2 + b^2$; now plug in the numbers in the correct place; (*'c' is the hypotenuse!*)

 $9^2 = 6^2 + b^2$ evaluate the squared numbers

So $81 = 36 + b^2$; get the b^2 by itself

 $81 - 36 = b^2$; evaluate (subtract)

 $45 = b^2$; now calculate the 'b' by 'unsquaring'

6.7082… = b

 $\mathbf{b} = 6.71$ rounded to nearest hundredth

Square and Square Roots of Numbers

25. If you are not overly familiar with the idea of squares and square roots from previous studies try this quick lesson; complete the table here:

Evaluate the following squares and square roots (to nearest hundredth)

If you don't like all the fancy steps to the left (algebra) just sketch the picture \downarrow

Labelling Triangles (*special way***)**

As usual CORNERS are labelled with CAPITAL LETTERS and sides are labelled with small letters.

But triangles have **special labelling**; the small letter across from a corner is the same as the CAPITAL letter at the corner across.

So side '**a'** can also be called side **BC** Some folks label the sides and some the corners that make the sides!

Un-natural Numbers (Irrational Numbers)

26. Often when doing a square root you will end up with a crazy number on your calculator like:

 $5 = 2.236067977$. It actually turns out to be closer to this really:

2.2360679774997896964091736687313……………; *(it goes forever with no pattern)*

but your calculator is too small to show it all!

So normally you would just properly round the answer to 2 or 3 decimal places as directed in an assignment.

PERIMETER OF A CIRCLE (CIRCUMFERENCE) AND π

27. **The Circle**. Another rather important shape! The circle. And another important formula that you should have memorized forever. Seriously!

28. **Experiment**. Make a good circle. Measure the distance across the centre of the circle, call it **d** for diameter. Measure the distance around the circumference of the circle, call it **C** for circumference. Calculate: $C \div d$; or since we are in high school now more properly written **C/d**.

It doesn't matter what you use for units of measure; cm, mm, inches; miles; ….

Your answer had best come out close to a **bit more than 3**. If it did not the universe is collapsing.

30. The real value of **C/d** for any and all circles is:

3.1415926535897932384626433832795…→

We call this number 'Pi' and use the Greek alphabet symbol ' π '.

31. There is a button on your calculator that will tell you the value is **3.14159265** (depending on the precision of your calculator display). Find the π on your calculator.

32. If we are doing manual calculations without a calculator we usually just use **3.14** as a close *approximation* to π . The fraction 22/7 works nicely two if you prefer fractions.

 π is another one of those pesky numbers that has decimal places that go on and on forever with no rational pattern! It is '*irrational*'.

35. **Calculate the Circumference**. Circumference is a specific word for a **perimeter** of a circle. The formula for circumference is:

36. Circumference equals π times diameter; or $\mathbf{C} = \pi \cdot \mathbf{d}$, where '**d**' is the diameter.

Distance around a circle (**Circumference**)

Circumference is really a perimeter, but a different word is used for circles.

 $C = 2\pi r$; where r is the radius measurement or since **2*r** is a diameter; or $C = \pi d$ where **d** is the diameter measurement.

37. **You Try**; calculate the perimeter (circumference) of the following (round to nearest hundredth decimal place):

Working a Formula

40. If you already know the circumference but want the diameter, the formula still works, it just needs to be juggled around a bit. Example: You know the circumference of a bike wheel is 70 inches, what is its diameter?

a. Solve by Simple Proportion. You know that a wheel of 1 in diameter is 3.14 in circumference so since you wheel is just 70 times bigger:

$$
\frac{1}{3.14} = \frac{d}{70}; \quad \therefore \quad d = \frac{70}{3.14} = 22.29 \text{in}
$$

b. Solve by algebra. Math students learn to more quickly manipulate symbols to solve problems using 'algebra'.

if **C = 3.14* d** but you want to know just '**d**' then '*un-multiply*' (ie: divide) both sides of the formula by the 3.14 to find the '**d**' by itself.

If **C** = 3.14***d**, then
$$
\frac{C}{3.14} = d
$$
.

Perimeter of Irregular Shapes 1.

41. Not every shape you encounter is a nice easy shape. Some shapes are combinations of several shapes.

42. Say you are designing a plaque to award the top student. It is a rectangle with a semi-circle on top. You want to put a gold ribbon around the perimeter. What length of gold ribbon do you need?

Ans: 91.96 cm

Perimeter of Irregular Shapes 2

43. What is the perimeter length of this irregular shape below?

Ans: 34.28 units

AREA OF 2 DIMENSIONAL SHAPES

44. Say you want to paint your child's bedroom walls! You select the perfect colour that she likes. The can says that a one litre can of paint will cover 12 square metres of wall surface. How many litres of paint do you need to buy?

This section is all about **area of shapes**. How much surface they have measured in little **squares**.

45. **Area: Rectangles and squares**

Area $=$ $l * w$

This area is 6 units $*$ 3 units $= 18$ square units or **18 units²**

46. **Area: Rhombus and Parallelograms**

At right is a parallelogram. Notice how you could rip off the triangle on the left side and add it too the figure on the right side and have a nice easy rectangle. The area would not change!

47. Instead of length times width now, we call the area the *base times the height*.

$$
\mathbf{A} = \mathbf{b} * \mathbf{h};
$$

but notice the '**h**' must be measure perpendicular, at 90 degrees to the base!

48. **Area: [Right] Triangles**

What is the area of the **rectangle** at the right?

__________ square units

Did you notice that a [right] triangle is really half of a rectangle?

So, what is the area of the **triangle**?

49. **Area: Any Triangle (obtuse, right, acute)**

What is the formula for the area of the parallelogram at the right?

A = __________

Did you notice that *any* triangle is really half of a parallelogram?

So, what is the area of any triangle?

$$
A = \boxed{\qquad \qquad }
$$

Fancy formula for area of half a rectangle:

$$
A_{\Delta} = \frac{1}{2} * b * h
$$

*** '*h*' , height, is always ⊥ to base***

50. Calculate the area of the triangles below: (all indicated measurements are in cm) ****Not to scale!****

Area of a Circle

60. How many squares fit inside a circle? You just **know** that 3.14 is involved somehow!

Count, roughly, how many whole or equivalent little unit squares are inside this circle. Include bits of squares too.

Number of squares (approx?) = $__$

What is the diameter, **d**, of this circle?

Try this formula: $\mathbf{A} = \pi^* \mathbf{r}^2 =$

61. So that is the area of a circle!

 $A_{circle} = \pi^* r^2$; where '**r'** is the radius of the circle

62. Calculate the area of each circle below; use as accurate as value of π as you can. Round final answers to two decimal places.

CONVERTING SQUARE MEASURES

65. What if you are painting your son's bedroom and you measure the walls to have an area of 360 square feet [**360 ft 2**]. You get to the hardware store and the paint can says it covers 10 square meters [**10 m²**]. How do you convert? How many square meters is the same as how many square feet.

66. Draw a diagram to see the problem! \rightarrow

See how one square metre is really 10.76 square metres!

 $1m^2 = 10.76 m^2$

Of course you knew this because you knew that one metre was the same as 3.28 ft.

67. So a metre times a metre, 1 square metre, must be 3.28 ft^* 3.28 ft; or what we call 3.28^2 ft²; 10.76 ft²

So the rule is you need to convert the length *and* the width into the new units, so you have to do a double conversion.

- 70. You try
- a. Convert 16 square yards into square ft

b. Convert 10 square meters into square cm.

Ans: 144 ft^2

Ans: $100,000 \text{ cm}^2$

SCALING OF SHAPES

71. What happens to the perimeter and the area of a shape if you just double al the lengths? Check it out! Draw yourself a simple example:

72. Notice that doubling all the lengths doubles the perimeter! *But* Doubling all the lengths also quadruples the area.

73. See why from the geometric formula:

 $P_{small} = a + b + c + d$ $P_{\text{big}} = 2a + 2b + 2c + 2d = 2*(a + b + c + d) = 2*$ P_{small} *but* $A_{small} = b * h$ $A_{\text{big}} = 2 * b * 2 * h = 2^2 * b * h = 4bh = 4 * A_{\text{small}}$

SCALE

74. Scale is a comparison of the lengths of similar shaped objects in some ratio. Big : Little or *Actual Model* ; etc. Doubling the scale doubles the perimeter but quadruples the area. Tripling the

scale will change the perimeter by three times but the area by nine times $(3^2 = 9)$.

75. Doubling the height and the width of Mona's picture gives her **twice** the **length** of hair, twice the length of nose, twice her hat size around her circumference of her head; etc **but**

Doubling the height and the width of Mona's picture gives her **four times the area** of her face (for makeup), four times as much fabric for her blouse, etc.

Think about it!

CONCLUSION

78. That completes your study of the geometry of two-dimensional objects!

79. You will be given all the necessary geometric formulae on tests and exams (although they are rather logical and can readily be memorized or derived without reference to a formulae sheet)

80. Start to add some of these ideas to *your* Grade 10 Reference Notes ('cheat sheet').

81. In Grade 11 you will do three dimensional objects like cubes and cones!

APPENDIX A GRADE 10 ESSENTIAL UNIT D – 2-D GEOMETRY

Add your own extra formulae below.

APPENDIX B TO GR 10 ESSENTIAL UNIT D – 2D GEOMETRY

