GRADE 10 MATHEMATICS
(MA20SA)
UNIT C – MEASURMENT
CLASS NOTES

INTRODUCTION

1. These notes are designed to guide the student through the Measurement Unit of Grade 10 Mathematics. They are written in a note frame form, so students are expected to fill in some of there own notes as the course progresses.

2. **Curriculum**. Here is what you will learn

General Outcome:

Develop spatial sense and proportional reasoning.

Specific Outcomes. It is expected that students will:

. Solve problems that involve linear measurement, using SI (Système International) and imperial units of measure, estimation strategies, and measurement strategies.

. Apply proportional reasoning to problems that involve conversions within and between SI and imperial units of measure.

. Solve problems, using **Système International (SI)** and **Imperial Units (British)**, that involve the surface area and volume of 3-D solid objects, including: right cones, right cylinders, right prisms, right pyramids, and spheres.

Linear Measurement

3. Measuring the lengths of lines! Linear means: ‘lines, or one dimension’. Knowing how to measure lengths and distances is part of everyday existence. A line has no width, it is like a skinny thread between two points in the world. A distance is measured along a line.

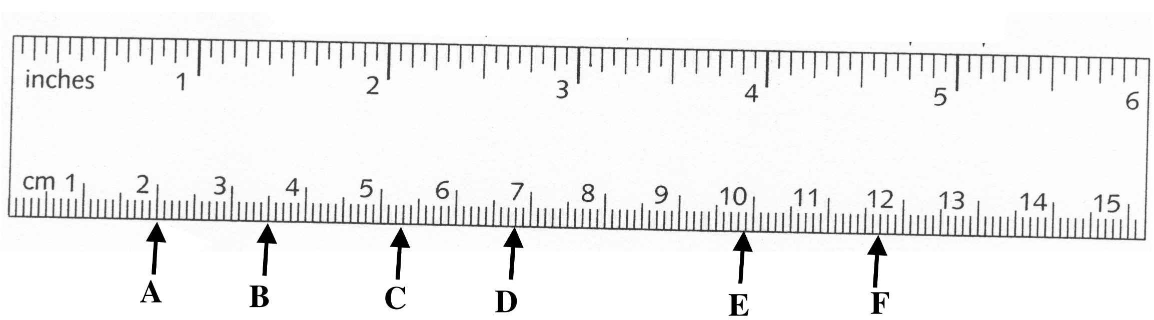
4. Name a few different ‘units’ of measurement (*eg*: inch) that you know and give a typical example of something having that length.

5. Can you apply paint to a line or length? __________ . Can a line hold some milk? __________

You will learn more about ‘areas’ and ‘volumes’ later.

6. **Reading A Metric Ruler**. Reading metric measures is easy, all units can be broken into tenths and tenths again if necessary. So measurements are all decimals (eg: 6.8 cm or 6.82cm).

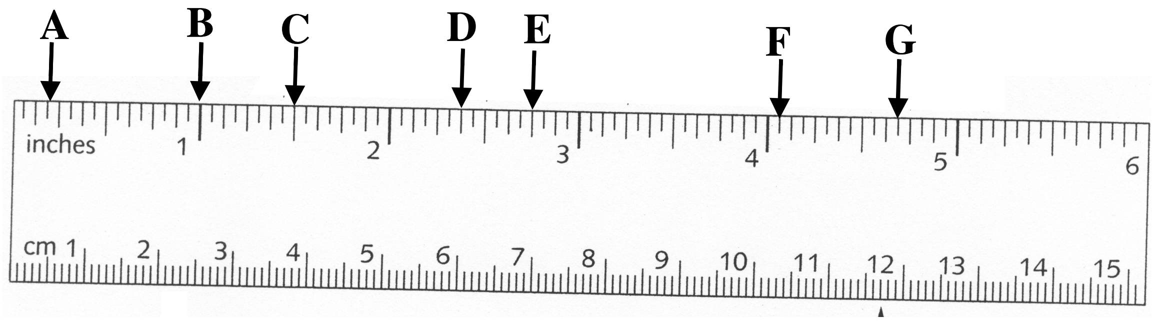
What are the measurements on the ruler indicated by the letters.



Caution: this ruler may not be accurate, its size was changed for image purposes

Letter	A	B	C	D	E	F
Measure [cm]						

7. **Reading an Imperial Ruler (inches)**. The imperial system with inches though is done with fractions of one half. So ½, ¼, 1/8th, 1/16th, etc. The 'Imperial' (or 'English') system does not use decimal numbers, you *cannot* say:'1.2 inches'!



Letter	A	B	C	D	E	F	G
Measure [in]							

8. **Larger Linear Units**. There are many other units used to measure distance. Obviously you would not measure the distance to Higgins and Main in inches. You ***could*** measure it in inches, but you would likely choose a unit that was larger so that you got a smaller number for the measurement. Other linear measurement include: metres, feet, yards, kilometres, miles, ….. Being able to estimate a distance is important too. With experience you become familiar with all these units and can estimate pretty well.

9. Complete the table, estimating the lengths (distances) from your own experience with units of measurement. There is no right answer, estimating is a very personal skill.

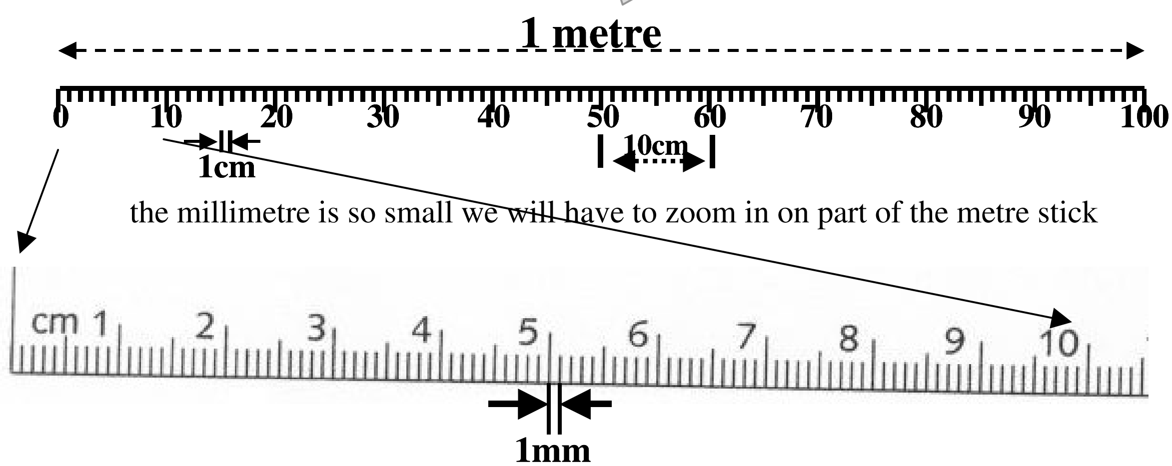
A	Classroom to MMF	Metres	Yard	Feet	
B	Classroom To Higgins and Main	Metres	Yards	Feet	
C	Width of classroom Window	Metres	Centimetres	Feet	Inches
D	Height of the classroom Door	Metres	Cm	Feet	Inches
E	Winnipeg To Dauphin	Miles	Km		
F	Classroom to Richardson Tower	Meters	Km	Miles	
G	Length of Main St to Perimeter	Miles	Km		

Now accurately measure C and D above with an actual instrument (ie: ruler), see how close you got.

C	Width of classroom Window	Metres	Centimetres	Feet	Inches
D	Height of the classroom Door	Metres	Cm	Feet	Inches

10. **Measuring With a Metre Stick**. The metre is a practical and common unit of measure in all the world (except the USA). It is a metric length unit of the '**Système International**' or **SI** or more commonly the '**Metric System**'. Originally the length of the metre was selected so it was close to an arm's length and close to the English 'yard'. It was originally defined as being such that *ten million* of them would go from the North Pole to the Equator. The real length of a metre is now determined by the number of wavelengths of a certain colour of light beam.

The metre is broken into smaller parts. The *centi*metre [cm] and the *milli*metre [mm].



these rulers not to scale

11. **The Metric Prefixes**. The metric system is based on ***10's***. Units keep getting multiplied or subdivided by ***tens***. The '*prefixes*' (the preceding words) mean something specific and you should already have them memorized by now.

Metric Prefix	Multiply by	Meaning
Mega-	**1, 000, 000**	One million
Kilo-	**1, 000**	One thousand
Centi-	**1/100**	One one-hundredth
Milli-	**1/1, 000**	One one-thousandth

12. Of course there are several other less common prefixes to encounter later such as *Nano* and *Pico* and *Giga*. You should the know the main ones above though!

Nano: __________ Pico: __________

Giga: ____________ Deka: _____________

….and others.

13. **The Imperial Linear Measurements**. The *Imperial System* (the old English system only really used officially in the USA anymore) has some pretty bizarre measurements.

a. A **foot** (like the length of a normal man's foot) is 12 inches.

Inches are subdivided into 1/4ths and 1/8ths, and 1/16ths, etc.

b. Three feet is a **yard**. A yard is conveniently pretty close to a metre. Of course we all know yards from football and golf!

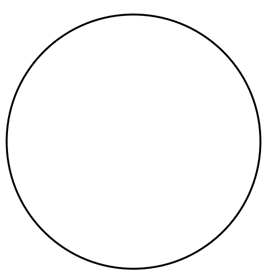
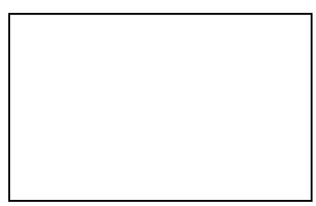
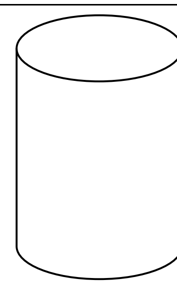
c. 1760 yards is a **mile** (well a statute mile since there are other kinds of miles too). And of course 1760 yards is really **5280** feet (if you multiply by 3). So a **mile** is really **5280** feet.

14. The **Imperial Units** are *crazy* compared to the **SI** metric units. You need to be rather good with fractions with imperial units. Further, Imperial system will mix units, so some one might be 5 feet 10 inches, or a baby might be 7 pounds and 9 ounces, etc. Crazy!

MEASURING DISTANCES AROUND SHAPES

15. Not all linear measurements involve straight lines. Often you need to measure the distance around the '*perimeter*' of an object. The perimeter of a circle has a special name called a '*circumference*'.

16. How could you measure the distance around these objects?

a. 	b. 	c. 

MrF

CONVERTING BETWEEN UNITS

17. Being able to convert between different units of measure is very important. You could just walk around with all different kinds of rulers and measure yourself, but why? We will examine only linear measurements for now. For example: four feet is how many meters? Of course being able to picture in your head a foot and a meter will give you a rough answer; your rough answer is: ____________

18. There are two main methods to do conversions. The two methods are :

    a. conversion by **proportion**; and

    b. conversion by **conversion factor**.

Check the end of these notes for some extensive conversion factors.

CONVERSION BY PROPORTION

19. **You are making muffins**. If *one muffin takes eight raisins*, then how many raisins do you need for four muffins?

20. Your answer: ________________

21. **Proportions**. The idea of proportions is that; if

$\frac{a}{b} = \frac{c}{d}$ then $ad = bc$. It is commonly called 'cross multiplying'.

$\frac{8\ raisins}{1\,muffin} = \frac{x\ raisins}{4\,muffins}$. Therefore: 8*4 = 1 * **x**. So ***x*** = 32. So you would need 32 raisins for four muffins.

22. Try a few proportions:

a. $\frac{12}{1} = \frac{x}{5}$

b. $\frac{2.5}{1} = \frac{x}{5}$

c. $\frac{8}{3} = \frac{x}{12}$

d. $\frac{1760}{1} = \frac{x}{2}$

e. $\frac{8}{3} = \frac{24}{x}$

f. $\frac{2.54}{1} = \frac{x}{9}$

23. Notice how much easier proportions are when:

  a. one of the denominators is a '1'; and/or

  b. the unknown amount is in the numerator.

CONVERTING UNITS USING PROPORTIONS

24. If 1 foot is 12 inches, how many inches is 4 feet?

$\frac{12\,in}{1\,ft} = \frac{x\ in}{4\,ft}$; therefore $12*4 = 1*x$; therefore ($\therefore$) x = 48 inches.

Notice how easy the calculation is when you arrange the proportions with the unknown in the numerator.

25. A table of lots of common conversions is attached at the back of these notes. Check them out.

26. Practice Problems

a. 2.54 cm is the same length as one inch, how many inches is 15 cm?

b. one kilometre is 1,000 metres. How many meters is 4.2 km?

c. one kilometre is1,000 metres. How many km is 840 meters

d. 0.621 miles is the same as1 km. How many miles is 17 km?

e. Some people are taught to use the conversion that eight km is the same as five miles (well fairly closely). So if 8 km = 5 mi, then how many km is 30 miles?

CONVERTING USING CONVERSION FACTORS

See the Conversion Tables attached at the end of these Notes

27. There are several ways to do conversions between units. But the best, and that you will need in science, is the ***conversion factor*** method. Converting between units is a fundamental and essential skill for everyday life.

28. Experience and familiarity with the feel for different measures is important also. If you cannot roughly approximate a metre or have a sense of what a Kilogram weighs or if you do not know how many millimetres are in a metre then you will need to elevate your experience with these units of measure.

CONVERSION FACTOR METHOD

29. A **factor** is simply a number that multiplies another number. Use factors to convert between units of measure.

30. Example, how many inches are there in $3\frac{1}{2}$ feet? Hopefully you have a rough picture of the answer! A foot is longer than an inch so you are expecting a larger number of inches as the answer.

31. Tables or experience will tell you that 1 foot is 12 inches. So to convert $3\frac{1}{2}$ feet to inches first:

a. write down what you are trying to convert including the units!

$$3\frac{1}{2}\text{ feet}$$

b. make a ratio, a fraction, of the known conversion : $\frac{12\ inches}{1\ foot}$. Ensure you keep the units. Ensure the new unit that you want is in the numerator (top).

c. Multiply the given measure by the conversion factor ensuring that the former units cancel and leave you with your new desired unit of measure:

$$3\frac{1}{2}\cancel{feet} * \frac{12\,inches}{1\,\cancel{foot}} = 42\ inches$$

32. **Examples**. Convert: (see tables at the back if you need them)

a. 25 centimetres (cm) to metres (m): ______________________________

b. 25 kilometres to meters: ____

c. 5.6 Kilograms to grams: _____________________________

d. 17 pounds (lb) to Kilograms (Kg): __________________________

e. 4 Imperial Gallon to litres. ______________________________
(caution there are two different gallons, American is smaller that the British Imperial gallon)

f. 8.2 miles to Kilometres: __________________________________

g. 355 millilitres (ml) to litres ('l'): _________________________________

h. 12 cubic centimetres (cc) to ml: _____________________________

i. 74 inches (in) to centimetres (cm): ____________________________

j. 160 acres to hectares: _______________________________

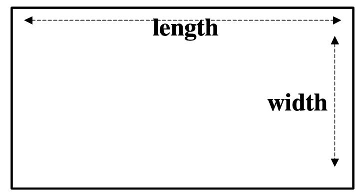
k. 12 ounces (oz) (of volume) to ml: ___________________________ (given that 1 Fluid Ounce is 28.4 ml, unless of course you mean the American Ounce which is 29.6 ml)

***Caution there is also different ounces to measure weight too! Way Confusing!

FORMULAE FOR CALCULATING VARIOUS LENGTHS, AREAS AND VOLUMES OF SELECTED SHAPES

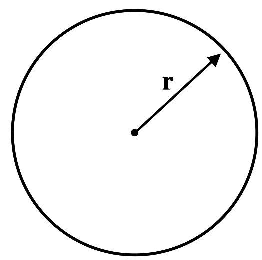
33. **Lengths**:

a. Distance around a rectangle (Perimeter)
Add the length of sides together
Perimeter = 2w + 2l where w = width and
l = length (all measured in the same units)



Of course if it is a square (a special rectangle) then it is 4 * the length of a side

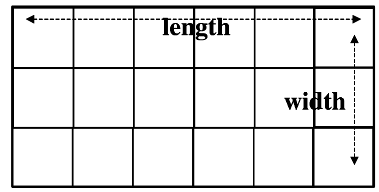
b. Distance around a circle (Circumference)
Circumference is really a perimeter, but a different word is used for circles.



C = 2πr; where r is the radius measurement
or since 2*r is a diameter ; or
C = πd where d is the diameter measurement

34. **Areas**: How many 'squares' would fit onto a surface.

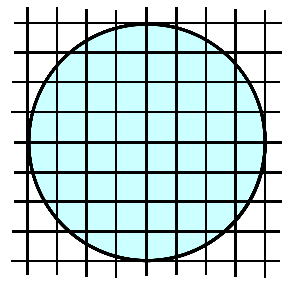
Rectangles and squares



Area = l * w

This area is 6 units* 3 units = 18 square units or **18 units^2**

Circles

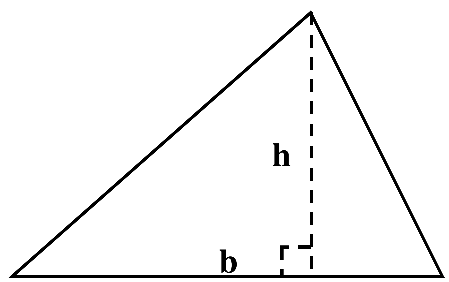


Area = πr^2

Where **r** is the radius (or half the diameter)

How many square cm are in a circle of radius **4 cm**? _______________

Triangles



Area = ½ * base * height

Note that 'height', h , is always measured perpendicular to the base! When you measure your kid's height they stand up straight I hope!

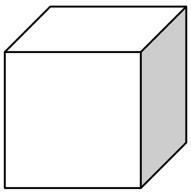
35. What is the area of a triangle having a **base** of 4 meters and a **height** of 2 meters? Your Solution:

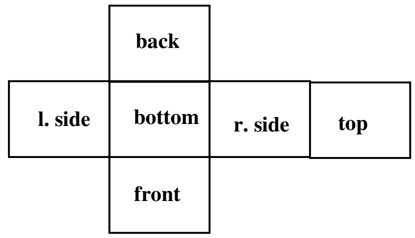
36. **Memorize.** All of these formulae to this point are so basic and fundamental that is important they be memorized for life. Combined with a couple basic trigonometry formulae and you will have all the basics of shape and design to last you a lifetime.

37. **Rhombuses, Trapezoids, Parallelograms**: You would want to consult other references and prior studies for these formulae and for other uncommon shapes.

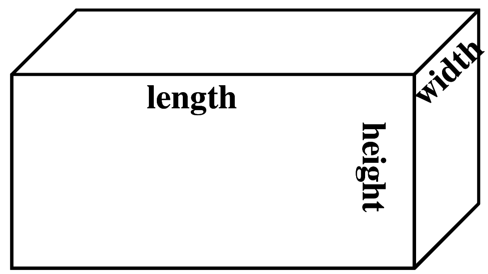
38. **Surface Area** (**SA**). The area of all the surfaces of a three-dimensional object if you covered the outside in squares of some size (m^2, ft^2, etc.). Sometimes when finding the surface areas of solid figures it is easier to draw the 'net' of the figure:

NET



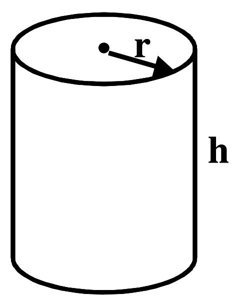


a. **SA** of a rectangular box or cube (called a rectangular '*prism*' really)
(A cube is just a rectangular box but all sides are the same length)
The front and back = l*h*2
The sides = w*h*2
The top and bottom = l*w*2
Total: 2*l*h + 2*w*h + 2*l*w



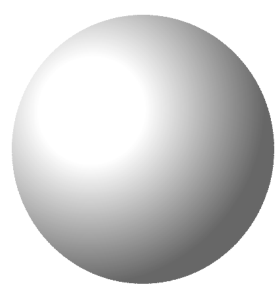
b. **SA** of a cylinder
The surface area of a cylinder if you were to cover its outside in squares of some size would be:

Top and Bottom: $\pi r^2 * 2$
Lateral sides: $2\pi r*h$
Total: $2\pi h + 2\pi r^2$.



c. **SA** of a sphere (a ball)

The surface area of a sphere if you were to cover its outside in squares is:
$\mathbf{SA_{sphere} = 4\pi r^2}$
This is really remarkable! Why?



What is the surface area of half a sphere (a dome)? (like the top 'half' of a cylindrical grain silo with a radius of 4 metres?)

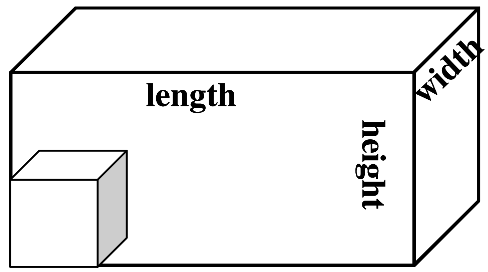
____

39. **Volume**. Volume is how many '***cubes***' of some size you could fit inside a three-dimensional object. Picture how many ***sugar cubes*** could fit into your shoe for example.

a. Volume of a rectangular prism or cube (a box)

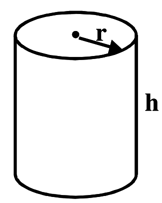
V = l*w*h

If your 'rectangular prism' is :
2cm * 3cm * 4 cm
your volume is 24 cubic cm or 24 cm^3



b. Volume of a cylinder:

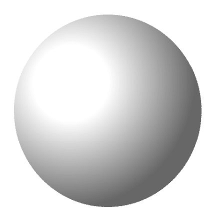
$\mathbf{V_{cyl} = \text{area of base} * \text{height} = \pi r^2 h}$



What is the volume of a cylinder of radius **8 cm** and height **125 mm**? Your Solution:

c. There is a formula for the volume of a sphere:

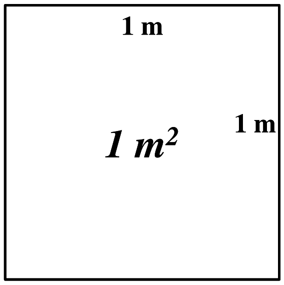
$$V_{sphere} = \frac{4}{3}\pi r^3$$

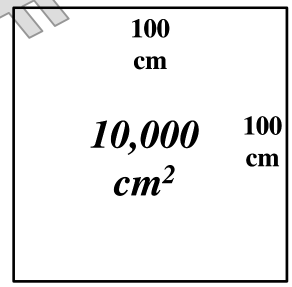


40. So what is the volume of sphere of radius 4 cm? ___________. (Your Solution:)

Converting Areas and Volumes using Square and Cube Dimensions

41. Be very careful when computing areas and volumes in **square** and **cube** types of units. For example; **1 m^2** is **not the same** as **100 cm^2;** it is *actually* another **100** times that or **10,000 cm^2**.

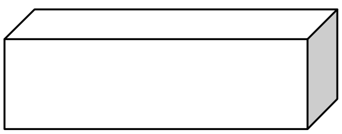




42. **Example**: Convert **5.2 m^2** into **ft^2**. (Given that 3.28 feet is the same length as one meter)

$$5.2m^2 * \frac{3.28ft}{1m} * \frac{3.28ft}{1m} = 55.9ft^2$$

43. **Example**: Your gas bill shows you used **200 cubic meters** [**m^3**] of gas to heat your house, but your meter is in **cubic feet [ft^3]**. So *how many* cubic feet of gas did you use if you want to check the company's readings.

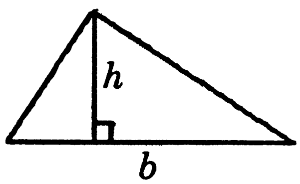


MORE ADVANCED SHAPES

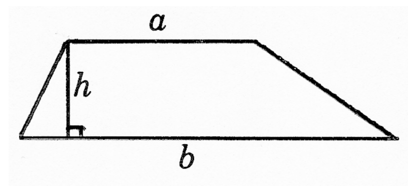
The formulae for finding the areas of other geometric figures in a plane are:

Note: Anytime ***height*** *(h) or* ***altitude*** *is used in a formula it must be measured* ***perpendicular*** *to a base.*

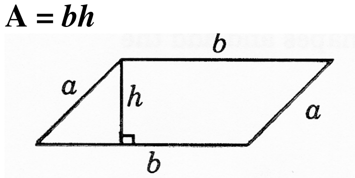
1. Triangles: **A = ½*bh***



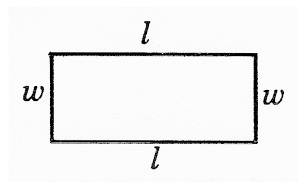
2. Trapezoid (only two parallel sides):
A = ½(*a* + *b*)*h*, where a and b must be the parallel sides



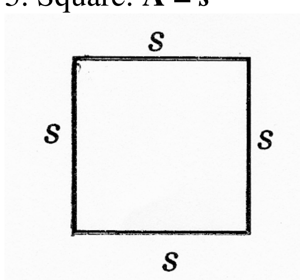
3. Parallelogram (two opposite parallel sides):
A = *bh*



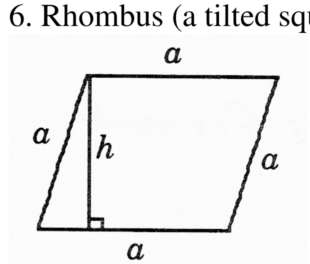
4. Rectangle (4 square corners which necessarily means two opposite parallel sides) : **A = *lw***



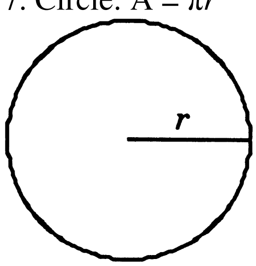
5. Square: **A = s^2**



6. Rhombus (a tilted squared): **A = *ah***

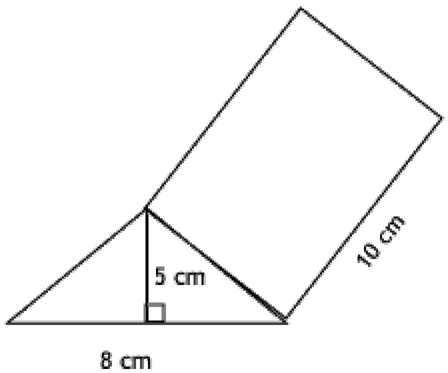


7. Circle: A = πr^2



TRIANGULAR PRISM

Find the volume of the triangular prism.



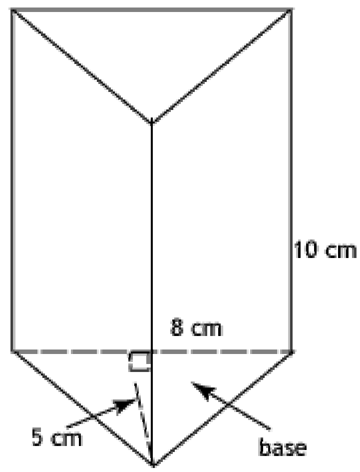


Fig3

Solution

The triangular base of the prism, **B**, is equal to the area of the triangle.

Triangular area of base or ***B*** = ½(8)(5) = 20 cm^2

Volume of prism = *Bh*

V = (20)(10)

= 200 cm^3

PYRAMIDS AND CONES

A pyramid is a 3D figure in which the shape of the base reduces to a single point throughout the height of the figure. Height is always measured straight up from the plane of the base (ie: perpendicular). The volume of these figures is one-third of the volume of its corresponding prism that would contain it. It is easy to get confused with heights because there are two of them, the height of the base when on its side, and the height of the 3D object (perhaps better called altitude in some books).

$$V = \frac{1}{3} Area_{base} h$$

Triangular Pyramid (base is a triangle)

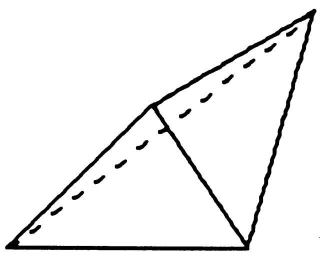


Fig 4

Cone (base is a circle)

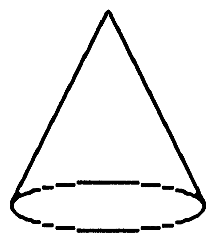


Fig 5

Rectangular Pyramid (base is a rectangle)

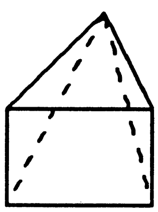
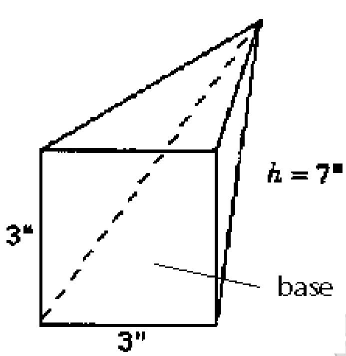


Fig 6

Example: Find the volume of the following:

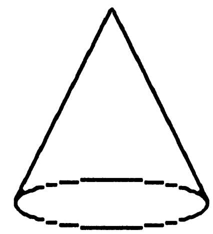


Solution (base = rectangle (square))

$$V = \frac{1}{3} Area_{base} h$$

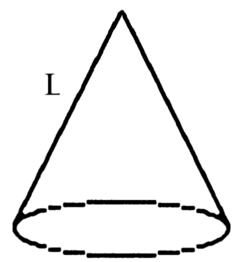
$= 21$ in^3

Example 1: Find the volume of the cone if its **height** is **10** cm, and it the **diameter** of its base is **8** cm.



Solution:

Example 2: Find the volume of the right cone if the diameter of its base is **8** cm. But its '**slant length**', **L**, is **20**.



Will require some 'Pythagoras'.

Solution:

Btw:some formulas elsewhere might call the length of the side 's'. Doesn't matter what you call it, as long as you know what it represents

If a cone inserted within a cylinder is filled with water, what is the volume of air left in the cylinder?

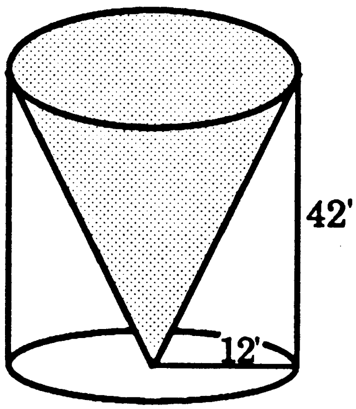


Fig 9

Solution

Volume of Cylinder:

$V = Bh$ (B = circular base of cylinder = πr^2)

$= \pi r^2 h$

$= \pi(12)^2(42)$

$\approx 19000.4 \text{ ft}^3$

Volume of cone:

$V = \frac{1}{3}Bh$ (B = circular base of cone if it were turned over)

$= \frac{1}{3}\pi(12)^2(42)$

$= 6333.5 \text{ ft}^3$

Volume of air left in cylinder

V = 19000.4 – 6333.5

= 12666.9 ft^3

Surface Area of 3D Objects

Another important characteristic of 3D solid objects, other than volume, is **surface area**. Surface area represents the area of all faces of an object, and can be defined in two ways:

Lateral Surface Area

- Pyramids: Area of all faces (triangles) except the base.
- Prisms: Area of all faces (rectangles) except the ends.

Total Surface Area

- Area of all faces including the ends or base.
  (Note: A special case, which will be dealt with in Lesson 4, is the surface areas of spheres.)

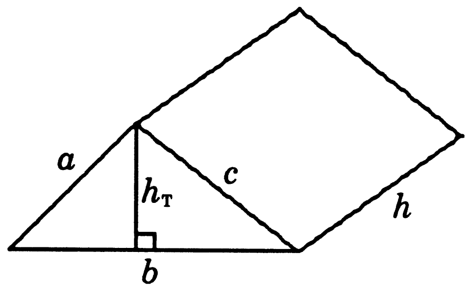
NETS

It sometimes helps to draw the 'net' of an object if you can.

Surface Area of 3D Objects

SURFACE AREA OF Triangular Prism

Draw the net of this object



There are three rectangular sides.

Area of rectangular sides **= ah + bh + ch = (a + b + c)h**

= Ph (where P = perimeter of base)

There are two triangular ends.
Area of the triangular ends = $\mathbf{2(\tfrac{1}{2}bh_T)}$,

Since $2*\frac{1}{2}=1$ therefore area of triangular ends = $\mathbf{bh_T}$
(where h_T is height of the triangle) (we had to distinguish somehow between the height, **h**, of the prism and the height, $\mathbf{h_T}$, of the triangle that forms its base)

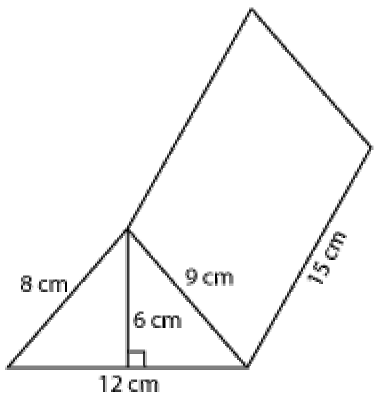
Total Surface Area = **ah + bh + ch +** $\mathbf{2(\tfrac{1}{2}bh_T)}$
Total Surface Area = **Ph +** $\mathbf{bh_T}$

Or Total Surface Area = **Ph + 2B**, if we say that **B** is the area of base

Note: Even though we can create specific formulae for each object, always use common sense when calculating surface area of an object. The triangular prism is simply made up of three rectangles and two triangles. Since we know how to calculate the area of rectangles and triangles one should be able to calculate all parts of the triangular prism and add them together. It is not necessary to memorize a specific formula if you know the few basic ones.

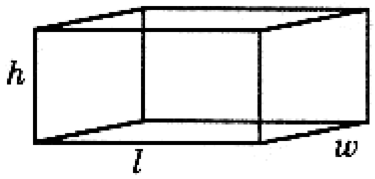
Example:

Calculate the lateral surface area and total surface area for the following triangular prism:



Surface Area of Rectangular Prisms

Rectangular Prism ('rectangular' implies it has all square corners)



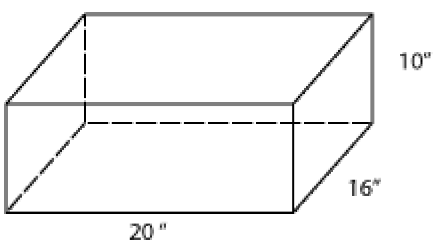
Lateral Surface Area = 2hw + 2hl
lateral surface area = h(2l + 2w)

lateral surface area = Ph (where P = perimeter of base)

Total Surface Area = Ph + 2lw
Total Surface Area = Ph + 2B (where B = area of base)

Note: You could also simply calculate the area of each of the 6 sides and then add them up.
See the example below:

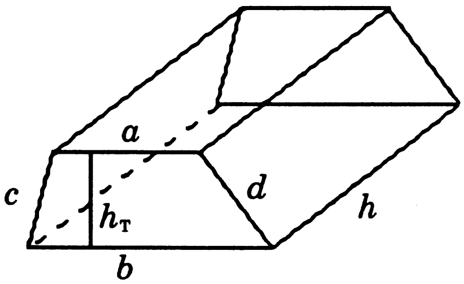
Calculate the total surface area of the following rectangular prism



Solution:
Total surface area = 2hw + 2hl + 2lw = 2(10)(16) + 2(10)(20) + 2(20)(16)
Total surface area = 1360 square inches

Surface Area of Trapezoidal Prisms

Trapezoidal Prism



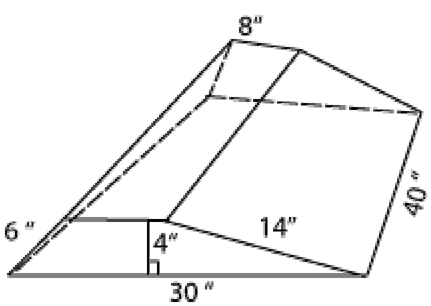
There are four rectangular sides
Lateral Surface Area = ah + bh + ch + dh
= (a + b + c + d)h
= Ph (where P = perimeter of base)

There are two trapezoidal ends.
Area of the trapezoidal ends = $2(\frac{1}{2}(a+b)h_T)$, since $2 \cdot \frac{1}{2} = 1$
Area of the trapezoidal ends = $(a+b)h_T$

The total surface area is the sum of the two calculations.
Total Surface Area = $(a + b + c+ d)h + 2(\frac{1}{2}(a+b)h_T)$

Total Surface Area = $Ph + (a+b)h_T$ or Ph + 2B -- where B = area of base

Example: Calculate the total surface area of the following trapezoidal prism:



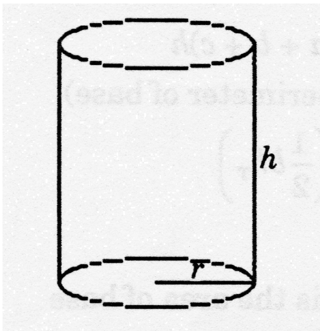
Solution:
Lateral surface area = Ph = (6 + 8 + 14 + 30)(40)
Ph = (58)(40) = 230 in.^{2}

Total surface area = lateral surface area + area of bases
area of bases = 2B = $2(\frac{1}{2}(8 + 38)(4)$ (remember area of a trapezoid = ½ (sum of parallel sides)height of trapezoid
therefore, area of bases = $2(\frac{1}{2}(38)(4))$
area of bases = 2(76)
area of bases = 152 in^2.

Total Surface Area = 2320 + 152 = 1472 in^2

Surface Area of Cylinders

Cylinders



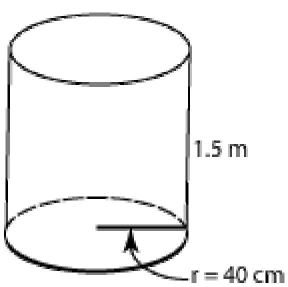
The Lateral Surface area is the area of the cylinder without the ends.

Lateral Surface Area = $(2\pi r)h = Ch$; (where C = circumference)
and the Area of the two top and bottom circles = $\pi r^2 + \pi r^2 = 2\pi r^2$

The total surface area is the sum of the two calculations.
Total Surface Area = $(2\pi rh) + (2\pi r^2)$

Or alternately: Total Surface Area = Ch + 2B (where B = area of base)

Example: Calculate the Total Surface Area of the following cylinder:



Solution:
Be sure to convert all lengths to the same unit. In this case, either cm or m. For this solution we will use centimetres (cm).
1.5 m = 150 cm
Lateral surface area = Ph   (where P is the perimeter of the circumference of the end)   (C is the circumference)
$Ph = Ch = (2\pi r)h$
$Ph = 2\pi(40)(150) = 12000\pi$ or ≈ 37699.11 cm^2

Total Surface Area = Ph + 2B   (where B = area of base or πr^2)
Total Surface Area = $Ph + 2(\pi r^2) = 12000\pi + 2(\pi 40^2) = 12000\pi + 3200\pi$
Total Surface Area = 15200π or   ?.2 cm^2

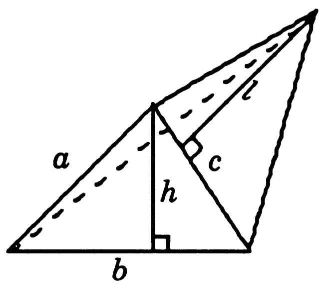
The surface areas of prisms with other regular bases can be found in a similar fashion:
• Lateral Surface Area = Ph, where P is perimeter of one base.
• Total Surface Area = Ph + 2B, where B is area of one base.

Surface Area of Pyramids

Pyramids

(For pyramids and cones, **l** (lower case L) represents the "***slant***" height : the length of the slanted side of the pyramid. This length is measured perpendicular to the base's edge.)

Triangular (if a=b=c)



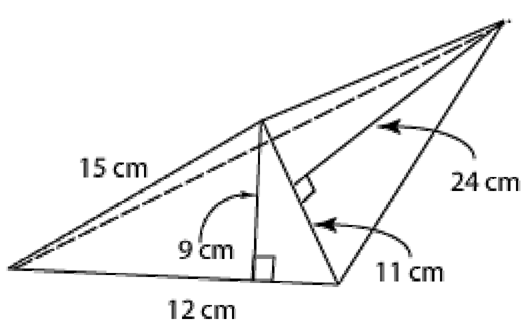
Lateral Surface Area = ½al + ½bl + ½cl
= ½l(a+b+c)
= ½Pl

Base Area = ½bh

Total Surface area = ½ l(a+b+c) + ½bh

Total Surface area = ½Pl + B

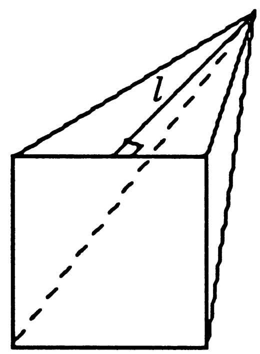
Example: Calculate the total surface area of the following figure



Solution:
Lateral surface area = ½Pl
= ½ (15 + 11 + 12)(24)
=½ (38)(24)
= 456 cm^2

Area of triangular base = ½ bh
=½ (12)(9)
= 54 cm^2

Square

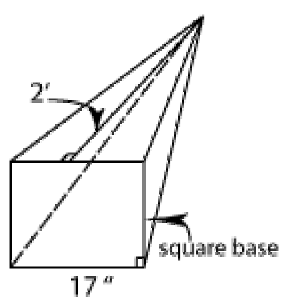


Lateral Surface Area = 4(½)sl
= 2sl
= ½Pl

Base Area = s^2

Total Surface Area = Lateral Surface Area + Base Area
= 2sl + s^2

Example: Calculate the total surface area of the following figure



Solution:
Lateral surface area = ½Pl
= ½ (4(17))(24) (2 ft = 24 inches)
= ½ (68)(24)
= 816 in^2

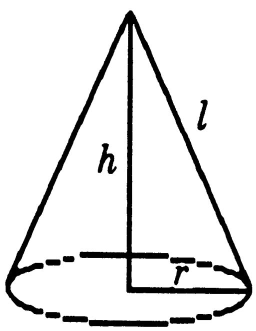
Area of square base = s^2
= (17)(17)
= 289 in^2

Total Surface Area = 816 + 289

Surface Area of Cones

Cone

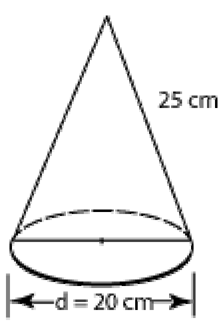
Can you draw the net of this?



Lateral Surface Area = ½Pl
Lateral Surface Area =½Cl (C = circumference = Perimeter of circular base)
= ½(2π r)l
= π rl

Base Area = πr^2
Total Surface Area = $\pi rl + \pi r^2$
= ½Pl + B

Example: Calculate the total surface area of the following cone



Solution:

Lateral surface area = ½Pl
= ½(2π r)l
= ½(2π(10))(25)
= 250 π

Area of Base = πr^2
= $\pi(10)^2$
= 100 π
≈ 314.2 cm^2

Total Surface Area = 250π + 350π
≈ 1099.6 cm^2

Calculating Surface Area for other Pyramids

Surface Area For other Pyramids:

Lateral Surface Area = $\frac{1}{2}Pl$; (where P is perimeter of base and l is the slant height)

Total Surface Area = $\frac{1}{2}Pl + B$; (where B is area of base)

In conclusion, the algebraic models for lateral surface area and total surface area of prisms and pyramids are as follows:

Prisms:

Lateral Surface Area = Ph
Total Surface Area = Ph + 2B
(where P = perimeter of base, h = height of prism, and B = area of the base)

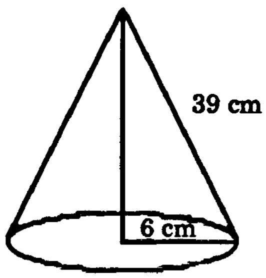
Pyramids:

Lateral Surface Area = $\frac{1}{2}Pl$

Total Surface Area = $\frac{1}{2}Pl + B$
(where P = perimeter of base, l = slant height of the side, and B = area of the base)

Additional Example 1 - Calculating Surface Area

Find the lateral surface area of the following:



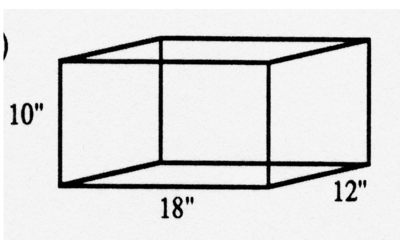
Solution:

Lateral Surface Area = $\frac{1}{2}Pl$

= ½(2πr(6))(39)
= 234π
≈ 735.1 cm^2

Example 2 - Calculating Surface Area

Find the lateral surface area of the following:



Solution:

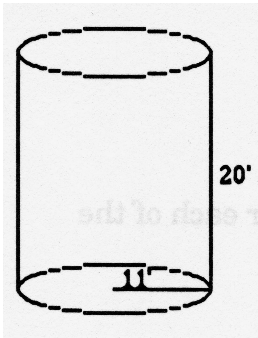
(Let l = 10, w = 18, h = 12) or (Let l = 18, w = 12, h = 10) both will work!

Lateral Surface Area = Ph
= (2l + 2w)h
= [2(10)+2(18)]12
=[20+36]12
= 672 in^2

MrF

Example 3 - Calculating Surface Area

Find the total surface area of the following:

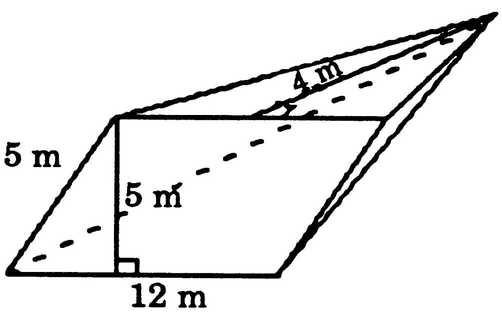


Solution:

Total Surface Area = Ph + 2B
$= (2\pi\ r)\ h + 2(\pi r^2)$
$= (2\pi\ (11))\ (20) + 2(\pi(11^2)$
$= 440\ \pi + 242\ \pi$
$= 682\ \pi$
$\approx 2142.6\ ft^2$

Example 4 - Calculating Surface Area

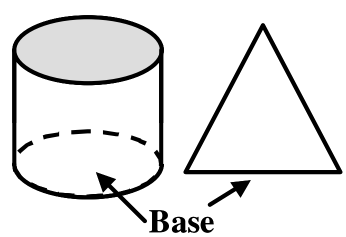
Find the total surface area of the following

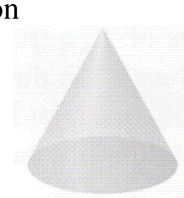
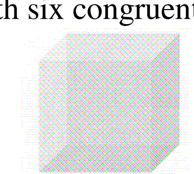


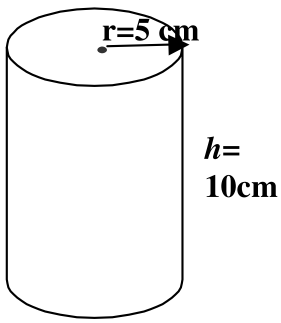
Solution:

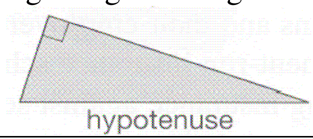
Total Surface Area = ½Pl + B
= ½(perimeter of parallelogram base)(slant height) + (area of parallelogram base)
= ½[2(5) + 2(12)](4) + 12(5)
= 2(10 + 24) + 60
= 68 + 60
$= 128\ m^2$

GLOSSARY
GRADE 10 UNIT A
MEASUREMENT

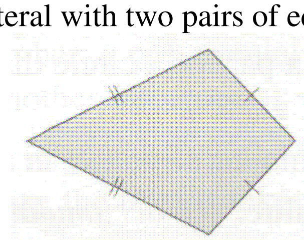
accuracy	how close a measurement is to what is believed to be the true value.
acute angle	an angle measuring less than 90°
acute triangle	a triangle with three acute angles 
altitude	the perpendicular distance from the base of a figure to the highest point of the figure. Also the height of an object above the earth's surface.
angle of depression	the angle between the horizon and the line of sight to an object that is above the horizon.
angle of elevation	the angle between the horizon and the line of sight to an object that is below the horizon.
approximation	a number close to the exact value of a measurement or quantity. Symbols such as ≈ or ≅ or are used to represent approximate values.
area	the number of square units needed to cover a region
base:	(1) the side of a polygon, or the face of a solid, from which the height is measured.  (2) the factor repeated in a power. Eg: In the expression of a power 5^3, the 5 is the base, the ^{3} is the exponent.

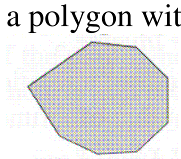
caliper	an instrument used to make precision measurements. Some common calipers may be used to measure to a precision of 0.05 mm.
capacity	the volume of a liquid that can be poured into a container. Two units of capacity are litre and gallon.
complementary angles:	two angles whose sum is 90°
cone	a solid formed by a region and all line segments joining points on the boundary of the region to a point not in the region 
congruent:	figures that have the same size and shape, but not necessarily the same orientation
corresponding angles	in similar triangles two angles, one in each triangle, that are equal.
cosine;	for an acute $\angle A$ in a right triangle, the ratio of the length of the side adjacent to $\angle A$, to the length of the hypotenuse. $\cos(\angle A) = \frac{Length\ of\ Opposite\ Side}{Length\ of\ Hypotensue}$
cube	a solid with six congruent, square faces 
cubic units	units that measure volume.

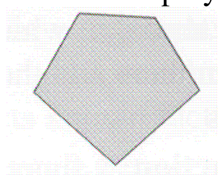
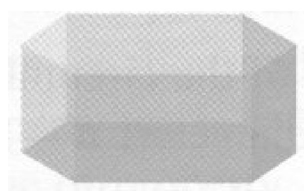
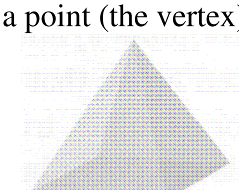
cylinder	a) a 'can' b) a solid with two parallel, congruent, circular bases  c) the union of all line segments that connect corresponding points on congruent circles in parallel planes, where the line segments are perpendicular to the planes of the circles.
cylinder: 	a solid with two parallel, congruent, circular bases $Volume_{cyl} = base * height$ $= \pi * r^2 * h$ $= \pi * 5^2 * 10 = 785\, cm^3$ $SurfaceArea =$ $2 * \pi * r^2 + 2 * \pi * r * h$ $= 471 cm^2$
decagon	a polygon with ten sides.
denominator	the term below the line in a fraction
dodecahedron	a polyhedron with twelve faces.
equiangular polygon	a polygon where all the angles have the same measure.
evaluate	substitute a value for each of the variables in an expression and simplify the result. (as opposed to solve)
formula	a rule that is expressed as an equation

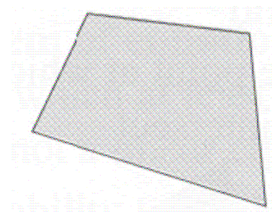
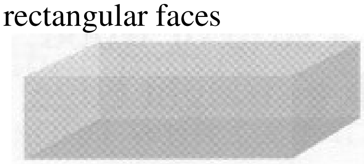
Heron's formula	a formula for the area of a triangle A = $\sqrt{(s)(s-a)(s-b)(s-c)}$ where a, b, and c are the lengths of the sides of the triangle, and s is half the perimeter.
hectare	a unit of area that is equal to 10 000 m^2. A square 100m by 100m would be a hectare. Roughly the amount of surface in a football field if you included end zones and team areas One hectare = 2.47 acres.
heptagon	a polygon with seven sides.
hexagon	a polygon with six sides.
hexahedron	a polyhedron with six faces. A regular hexahedron is a cube.
hypotenuse	the longest side of a right-angle triangle. The side opposite the right angle in a right triangle. 

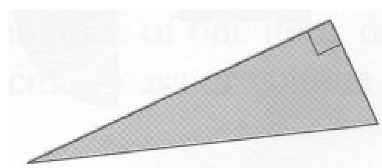
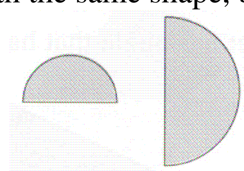
<table>
<tr><td>Imperial system:</td><td>a system of measures that was used in Canada prior to 1976; a variation is still used in the U.S.A..
Measuring devices using this system often have each unit subdivided by halving, then halving the subdivisions, etc.
Eg:½ inch, ¼ inch, two pints to a quart, four quarts to a gallon, etc.

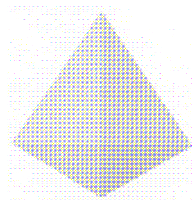
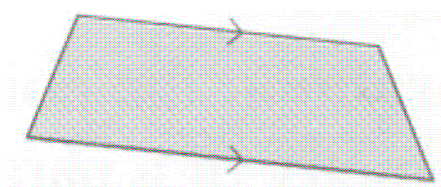
<table>
<tr><th colspan="2">Selected conversions</th></tr>
<tr><th>Imperial to Imperial</th><th>Imperial to Metric (or SI)</th></tr>
<tr><th colspan="2">Length</th></tr>
<tr><td>1 mile=1760 yards</td><td>1 mile=1.609 km</td></tr>
<tr><td>1 yard = 3 feet</td><td>1 yard = 0.9144 m</td></tr>
<tr><td>1 foot = 12 inches</td><td>1 inch =2.54 cm</td></tr>
<tr><th colspan="2">Capacity (volume)</th></tr>
<tr><td>1 gallon = 4 quarts</td><td>1 Gallon = 4.546 l</td></tr>
<tr><td>1 quart = 2 pints</td><td></td></tr>
<tr><th colspan="2">Mass (weight)</th></tr>
<tr><td>1 ton = 2000 lbs</td><td>1 pound = 0.454 kg</td></tr>
<tr><td>1 pound = 16 ounces</td><td>1 ounce = 28.35 g</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">Caution:
US gallons and quarts are different capacities than Imperial</td></tr>
</table>
</td></tr>
<tr><td>irrational number</td><td>a number that cannot be written in the form m/n where m and n are integers ($n \neq 0$). Irrational numbers cannot be written as decimals (decimals are really fractions anyway). Examples of irrational numbers:
π, $\sqrt{2}$, $\sqrt[3]{5}$, …</td></tr>
<tr><td>isosceles acute triangle:</td><td>a triangle with two equal sides and all angles less than 90°</td></tr>
<tr><td>isosceles obtuse triangle:</td><td>a triangle with two equal sides and one angle greater than 90°</td></tr>
<tr><td>isosceles right triangle:</td><td>a triangle with two equal sides and a 90° angle</td></tr>
<tr><td>isosceles triangle:</td><td>a triangle with two equal sides</td></tr>
<tr><td>kite
</td><td>a quadrilateral with two pairs of equal adjacent sides
</td></tr>
</table>

least common denominator	the least common denominator of two fractions, a/b and c/d, is the smallest number that contains both b and d as factors.
least common multiple	the least common multiple of two numbers, a and b, is the smallest number that contains both a and b as factors.
legs	the sides of a right triangle that form the right angle
line	an infinitely long path that has no thickness and no curves.
mass	the amount of matter in an object. Mass is measured in units such as grams or kilograms.
metric system:	also called the SI (Système International) system; based on a decimal system, with each unit subdivided into tenths and prefixes showing the relation of a unit to the base unit; commonly used base units are: Metre (m) for length Gram (g) for mass Litre (l) for capacity Second (s) for time The prefixes include: Mega-: Million, Kilo-: Thousand; centi-:1/100; milli-: 1/1000
micrometer	an instrument used for precision measurement.
natural numbers	the counting numbers. The set of numbers that includes {1, 2, 3, 4, ·,· , ·}
numerator	the top number in a fraction.
parallelogram:	a quadrilateral with both pairs of opposite sides parallel
octagon	a polygon with 8 sides.  A 'regular' octagon has all sides and angles the same.

octahedron	a polyhedron with 8 faces.
parallelogram	a quadrilateral with opposite sides parallel.
pentagon	a five sided polygon. 
perpendicular	two lines are perpendicular if the angle between them is 90 degrees.
polygon	the union of three or more line segments that are joined together so as to completely enclose an area.
polyhedron	a solid that is bounded by plane polygons.
precision	the size of the smallest measurement unit used when doing or reporting a measurement. For example, a measurement of 23.27 cm is more precise than 23.3 cm because 23.27 cm is measured to the nearest hundredth of a centimetre and 23.3 cm to the nearest tenth.
prism	a solid that has two congruent and parallel faces (the *bases),* and other faces that are parallelograms 
protractor	an instrument for measuring angles.
pyramid	a solid with a polygon for a base, and all other sides being triangles that meet at a point (the vertex). 

Pythagorean Theorem	for any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides the theorem that relates the three side lengths of a right triangle: $a^2 + b^2 = c^2$
Pythagorean triple	three natural numbers that satisfy the Pythagorean theorem. One example is: 3, 4 and 5. 5, 12, 13 also works.
quadrilateral:	a four-sided polygon 
rectangle:	a quadrilateral that has four right angles
rectangular prism	a prism that has rectangular faces 
rational number	a number that can be expressed exactly as the ratio of two integers. The letter Q (for quotient) is frequently used to represent the set of rational numbers.
rectangle	a quadrilateral with four 90 degree angles.
rectangular pyramid	a pyramid with a rectangular base
regular polygon	a polygon in which all the angles have the same measure and all of the sides are equal in length.
regular polyhedron	a polyhedron whose faces are congruent, regular polygons.
repeating decimal	a decimal in which the digits endlessly repeat a pattern. A repeating decimal may be rewritten in rational form.

rhombus:	a parallelogram with four equal sides
rhombus	a quadrilateral with four equal sides.
right angle	an angle whose measure is 90 degrees.
right circular cone	a cone whose base is a circle located so that the line connecting the vertex to the centre of the circle is perpendicular to the plane containing the circle.
right circular cylinder	a cylinder whose bases are circles and whose axis is perpendicular to its bases.
right triangle	a triangle that has a right angle. 
similar figures	figures with the same shape, but not necessarily the same size 
sine	in a right triangle, the length of a side opposite an angle divided by the length of the hypotenuse of the triangle. The formula may be written as: $\sin \theta = \frac{opposite}{hypotenuse}$
solid	a three dimensional object that occupies or encloses space (i.e. has a volume).
sphere	the set of all points in space that are a fixed distance from a given point. A ball is an example of a sphere.
square root	of a number, x, is the number that, when multiplied by itself gives the number, x. for example, $\sqrt{9} = 3$ because $3^2 = 9$.

tangent	in a right triangle, the length of a side opposite an angle divided by the length of the side adjacent to the angle. The formula may be written as: $\tan\theta = \frac{opposite}{adjacent}$
tetrahedron	a polyhedron with four faces. 
theorem	a statement that has been proven.
three-dimensional	having length, width, and depth or height
transversal	a line that intersects two other lines.
trapezoid	a quadrilateral with one pair of opposite sides parallel but not equal in length. 
triangle	a three sided polygon.
trigonometric function	a function that involves the sin, cos, or tan of the independent variable. One example is y = sin x + 1.
trigonometry	the study of triangles and the relations between the side lengths and the angle measures.
Vernier scale	calipers and micrometers are frequently equipped with a Vernier scale which is required to make precision measurements.
volume	the amount of space occupied by an object. One unit of volume measure is the cubic metre, or m^3.
whole numbers	the set of numbers that includes zero and all of the natural numbers. W = {0, 1, 2, 3, 4, ·, ·, · }

EXPANDED CONVERSION TABLES

Revised: 13 Oct 2008

SI Metric System Conversions

Conversions SI Metric – Length and Distance		
1 *kilo*metre km	=	1,000 metres m
1 meter m	=	100 *centi*metres cm
1 *centi*metre	=	10 *milli*metres mm

Conversions SI Metric – Mass		
1 tonne	=	1,000 kg
1 kilogram kg	=	1,000 grams g
1 gram g	=	1,000 milligrams mg

Conversions SI Metric – Volume		
1 litre l	=	1,000 millilitres ml
1 litre l	=	100 centilitres cl
1 litre l	=	1,000 cc (or 1,000 cm^3)
1 millilitre ml		1 cc (or 1 cm^3)

'cc' stands for cubic centimetre which is really just **cm^3**.

Notice also that a cube of dimensions 10cm by 10 cm by 10 cm is a litre

Conversions SI Metric – Area		
1 square metre	=	10,000 cm^2
1 hectare	=	10,000 m^2

So a square **100 m** by **100 m** is a hectare. Used for measuring land area.

Non-SI System Conversions

Conversions Non-SI (Imperial) – Length		
1 mile mi	=	1,760 yards yd
1 yard yd	=	3 feet ft
1 foot ft	=	12 inches in

Conversions Non-SI Imperial – Mass		
1 ton	=	2,000 pounds lb
1 pound lb	=	16 ounces oz

Conversions Non-SI Imperial – Volume (English)		
1 gallon	=	0.125 bushels
1 gallon	=	160 ounces oz
1 pint	=	0.125 gallons
1 quart	=	0.25 gallons
1 pint	=	0.5 quarts

Conversions Non-SI Imperial – Volume (USA)		
1 gallon (US)	=	0.832 gallons (English)
1 gallon (US)	=	128 ounces oz (US)

Really gets confusing with two different volumes depending on your country!

Caution Ounces of weight are different from ounces of volume.

Conversions Non-SI Imperial – Area		
1 acre	=	43,560 ft^2
1 acre	=	4,840 yd^2
1 foot ft	=	12 inches in
1 square mile	=	640 acres

So a square having sides of 208 feet would be an acre.
An acre originally was supposed to be the amount of land a horse could plow in one day, so it depended on how good your horse was!

Converting between systems

Conversions SI to Non-SI Length		
1 metre m	=	3.2808 feet ft
1 metre m	=	39.370 inches in
1 kilometre km	=	0.6214 miles mi
2.54 cm		1 inch

Conversions Non-SI Imperial – Mass		
1 kilogram kg	=	2.205 pounds lb
1 tonne	=	1.1 ton

Conversions SI to Non-SI Volume		
1 gallon (English)	=	4.546 litres
1 gallon (US)	=	3.785 litres
1 gallon (English)	=	4,546 cc^3
1 gallon (US)	=	3,785 cc^3

Conversions SI to Non-SI Area		
1 sq mile	=	259 hectares
1 sq mile	=	2,589,988 m^2
1 square metre	=	10.76 ft^2
1 square metre	=	1,550 in^2

Examples:

a. To convert 3 miles to kilometres:

$$3miles = \underline{????}km?$$

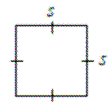
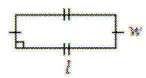
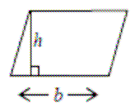
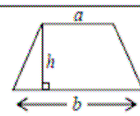
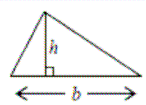
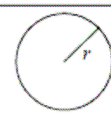
$$3\cancel{mi} * \frac{1\ km}{0.6214\cancel{mi}} = 4.83\ km$$

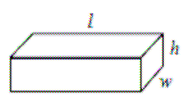
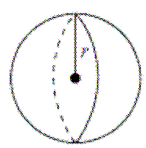
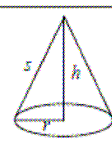
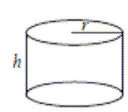
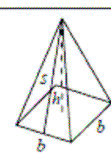
b. Don't forget!: If the conversion factors you want aren't here you can always apply several different factors to make a complicated conversion. Example:
Eg: To convert 1 ton to kilograms:

$$1\cancel{ton} * \frac{2000\cancel{lb}}{1\cancel{ton}} * \frac{1kg}{2.205\cancel{lb}} = 907\ kg$$

c. To convert square feet to square inches (notice you apply the conversion factor twice!):

$$1\,ft^2 = 1\,\cancel{ft}^2 * \frac{12\,in}{1\,\cancel{ft}} * \frac{12\,in}{1\,\cancel{ft}} = 144\ in^2$$

Formula List		
Name of Formula	**Diagram**	**Formula**
area of a square		$A = s^2$
area of a rectangle		$A = lw$
area of a parallelogram		$A = bh$
area of a trapezoid		$A = \frac{1}{2}(a + b)h$
area of a triangle		$A = \frac{1}{2}bh$
area of a circle		$A = \pi r^2$

surface area of a rectangular solid		$SA = 2lw + 2lh + 2wh$
volume of a rectangular solid		$V = lwh$
surface area of a sphere		$SA = 4\pi r^2$
volume of a sphere		$V = \frac{4}{3}\pi r^3$
surface area of a cone		$SA = \pi rs + \pi r^2$
volume of a cone		$V = \frac{1}{3}\pi r^2 h$
surface area of a cylinder		$SA = 2\pi rh + 2\pi r^2$
volume of a cylinder		$V = \pi r^2 h$
surface area of a pyramid		$SA = 2bs + b^2$
volume of a pyramid		$V = \frac{1}{3}b^2 h$

GEOMETRIC FORMULAE