

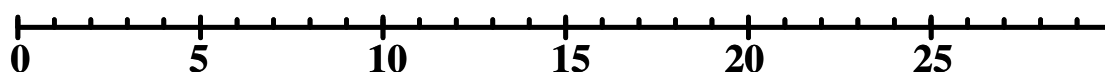
UNIT A
LINEAR MODELLING
CLASS NOTES



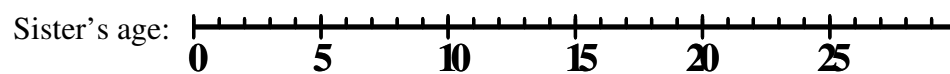
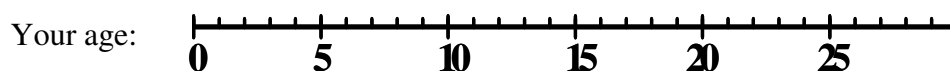
1. Linear modelling is the study of the graphs of **lines**.

GRAPHING DATA

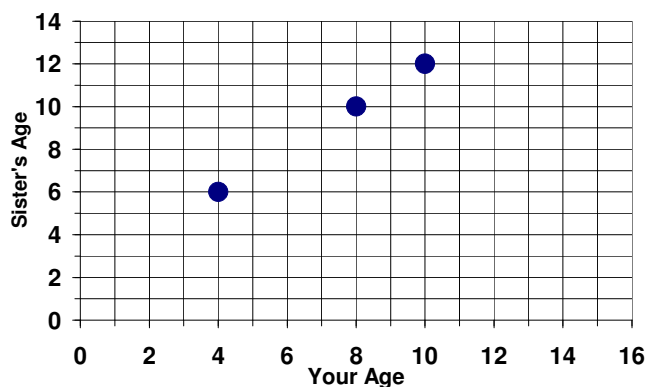
2. To graph means to make a picture of relationships between numbers. A picture is worth a thousand words.
3. Jason has six dollars. Kevin has fourteen dollars. Make a picture on a number line of the situation. Plot the two numbers.



4. Often pairs of numbers are important and they go together in some relationship. *For example:* your age compared to your sister's age. When you were four your sister was six, when you were eight your sister was 10, when you were 10 your sister was 12. To plot the pairs of numbers you need two separate axis. Plot your ages and your sisters corresponding ages, and connecting the corresponding pairs with a arrows.



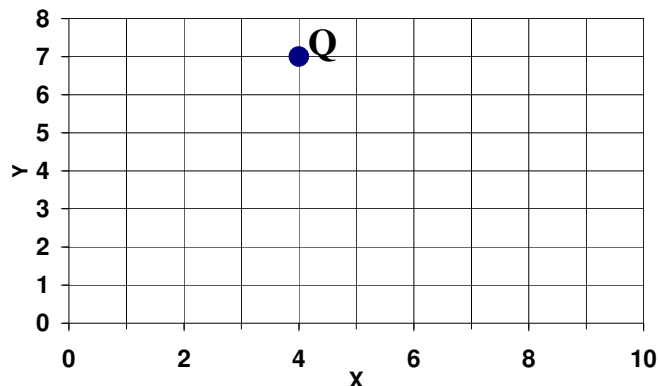
5. It turns out there is a better way to plot the given ages that gives us a better picture. Just turn one of the number line axis on its side. Now the two axis are at right angles. Use the bottom horizontal axis to find your age and use the side vertical axis to find your sisters age.



PLOTTING POINTS ON A CARTESIAN COORDINATE SYSTEM

6. We plot pairs of numbers using a *Cartesian* 'coordinate system' named after the French Mathematician 'Descartes'.

7. Pairs of numbers make a point on the grid. If we have a point that we call **Q** and say it is the pair of numbers **(4, 7)** we could plot it as at right.



8. The first number in the pair we call '*x*', the second we call '*y*' in the absence of knowing what the numbers really are meant to represent.

They are called '*ordered pairs*' because the order is important, the first number is always the '*x*' or the value plotted on the bottom horizontal axis, the second number, '*y*', after the comma is always the value that is plotted using the vertical side axis.

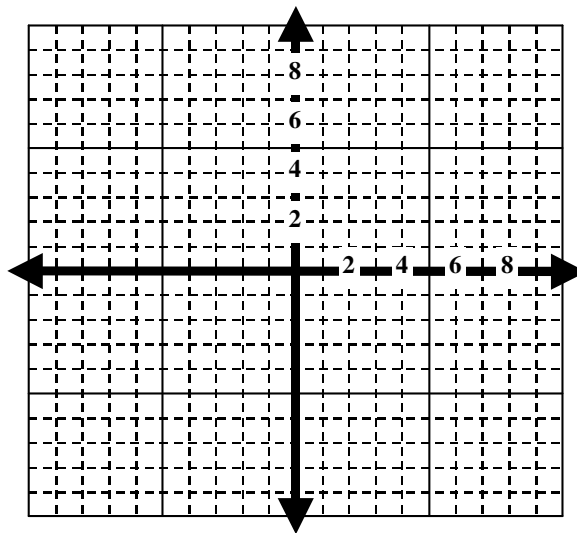
9. You plot above ↑ the points on the grid formed by the following named '*ordered pairs*' (*x*, *y*).

A(4, 5)	B(0, 6)	C(7.5, 0)	D(3, 3.2)	E(8, 2)	F(10, 7.3)
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MORE PLOTTING

10. Of course you can use negative values as well on a number line so you can on a Cartesian grid too. Plot and label these points:

- A(-4, 2)**
- B(5, 9)**
- C(-6, -6)**
- D(0, 7)**
- E(0, 0)**
- G(6, 0)**

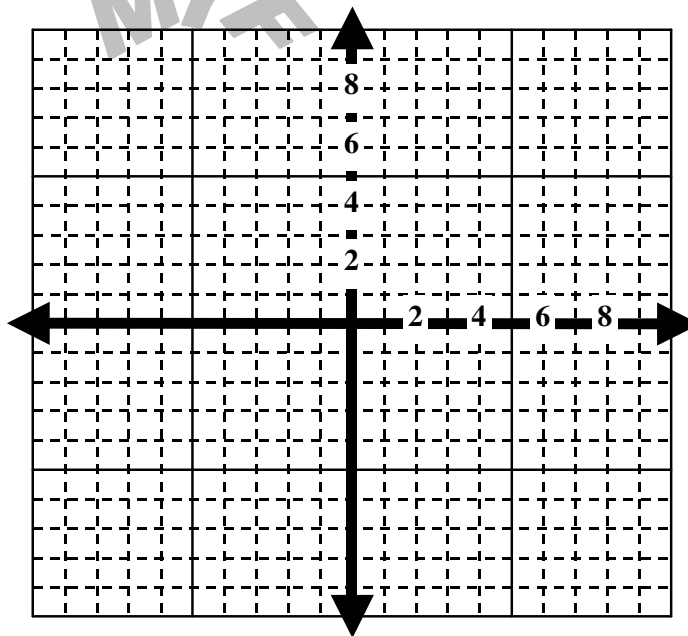


These are just '*random*' points. *Do not try to connect them or anything, they do not make a recognizable pattern.*

11. Let us select some points that do make a pattern, see if you can what the pattern is. Below is a table of some points to plot.

Data Table	
x	y
-5	-2
-1	2
0	3
3	6
6	9

We used a table of data here instead of listing the ordered pairs $(-5, -2)$, $(-1, 2)$, etc.



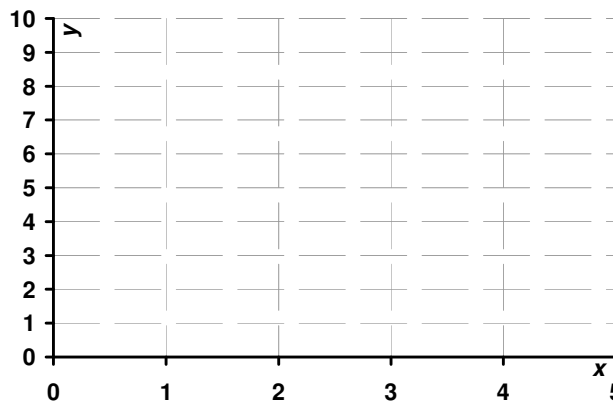
12. What simple pattern do the points appear to make?

NON-LINEAR RELATIONSHIPS

13. Try graphing these data to see what sort of pattern they make:

x	y
0	45
1	40
2	25
3	0

14. Smoothly connect with a curve your dots to see the pattern.



14. The pattern or 'curve' above is *not* a line. It is actually called a 'parabola'. You see this every day, dozens of times a day! Where have you seen this type of curve made by this data before?

In Grade 11 you will learn lots more about these parabolas.

GRAPHING ON A GRAPHING TOOL

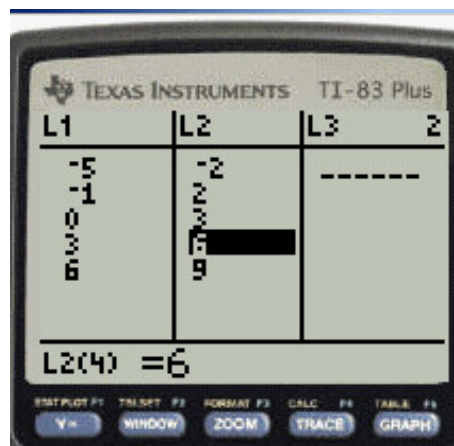
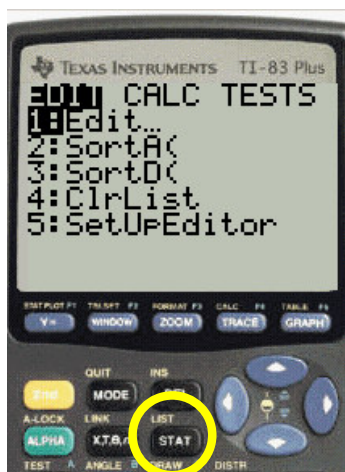


15. Modern technology allows us to use machines to graph for us. There are several graphing tools on-line. In class we tend to use a graphing calculator to do our graphing for us. You likely have used the TI-83 a bit in Grade 9, it has been around for almost 20 years now and every math teacher uses it!

PLOTTING POINTS TO MAKE A GRAPH USING THE 'TI-83 GRAPHING CALCULATOR'

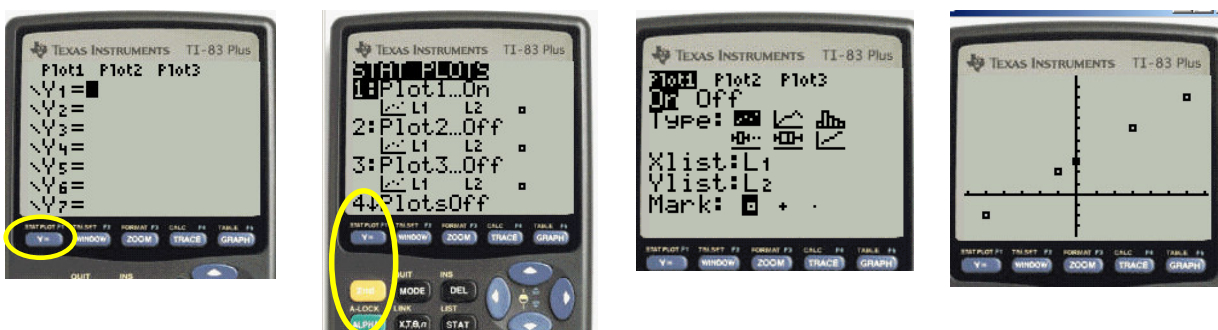
16. *Entering Plot Data in TI83.* Enter data points as follows.

- Press **STAT** Select **1:EDIT** by hitting **ENTER** when positioned on top of it. This brings you to the Statistics Edit Screen. Clear Lists 1 and 2 (**L₁** and **L₂**) (Cursor to the top of each column and press **CLEAR ENTER**) .
- Enter **independent** data (the 'x') in **L₁** . Enter **dependent** data (the 'y') in **L₂**. Enter the data from paragraph 11 above.



17. Plotting Data on the screen.

- Select **Y=** and **CLEAR** any formulas in there.
- Select **STAT PLOT** by pressing **2nd Y=**. Select **Plot 1**. Turn on Plot 1 by highlighting **ON** and selecting it with **ENTER**. Put plot into the *Scatter Plot* mode (the first one with the multiple-dots). Make sure the data is being taken from lists **L₁** for the *x* and **L₂** for the *y*. Select the largest *mark* possible.
- Press **GRAPH**. You should have a *graph* of your data! You will likely need to use **ZOOM 9:ZOOMSTAT** to have the computer perfectly scale the plotting window so you can see all the points.

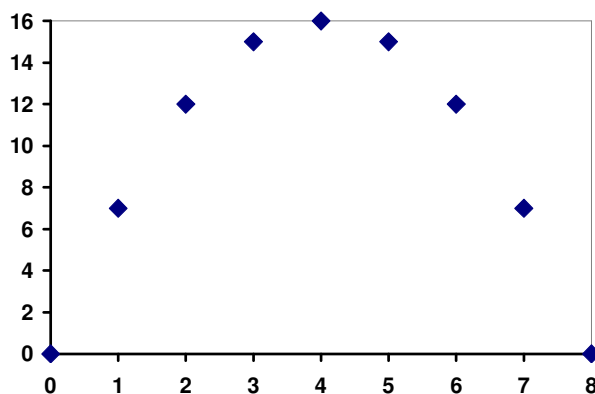


NON-LINEAR PLOT

18. Try a plot using the TI-83 for the following data

x or [L ₁]	y or [L ₂]
0	0
1	7
2	12
3	15
4	16
5	15
6	12
7	7
8	0

Your graph should look like this (don't forget to hit **ZOOM** and select **9:STATS** to make it a perfect fit in the window).

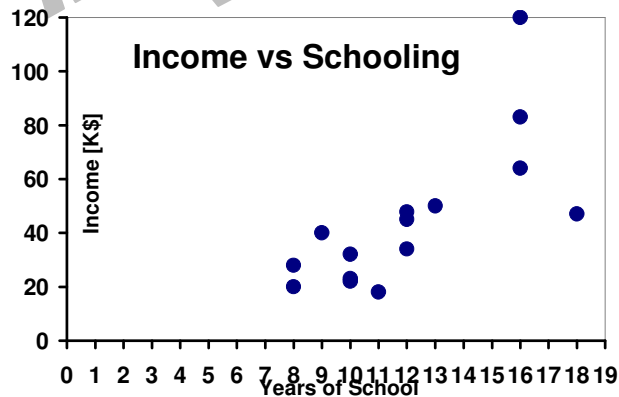


You will study this graph and the relationships between sets of numbers in Grade 11.

GRAPHING STATISTICAL DATA

19. Graphs are important for statistics too; to show trends! You have likely used them many times. Here is some statistical data.

20. Several people were asked their years of schooling and their annual income. (every dot plotted here is an actual person)



21. Does it look like there is trend that the more school you have the more income you have? The trend is also called a '*correlation*'.

DEPENDENT AND INDEPENDENT VARIABLES.

22. Graphs are made from *ordered pairs* of related numbers. We often call the pair (x , y). The ' x ' value relates to something special (such as a time), the ' y ' value relates to something else such as amount of money. And we always give them in that order.

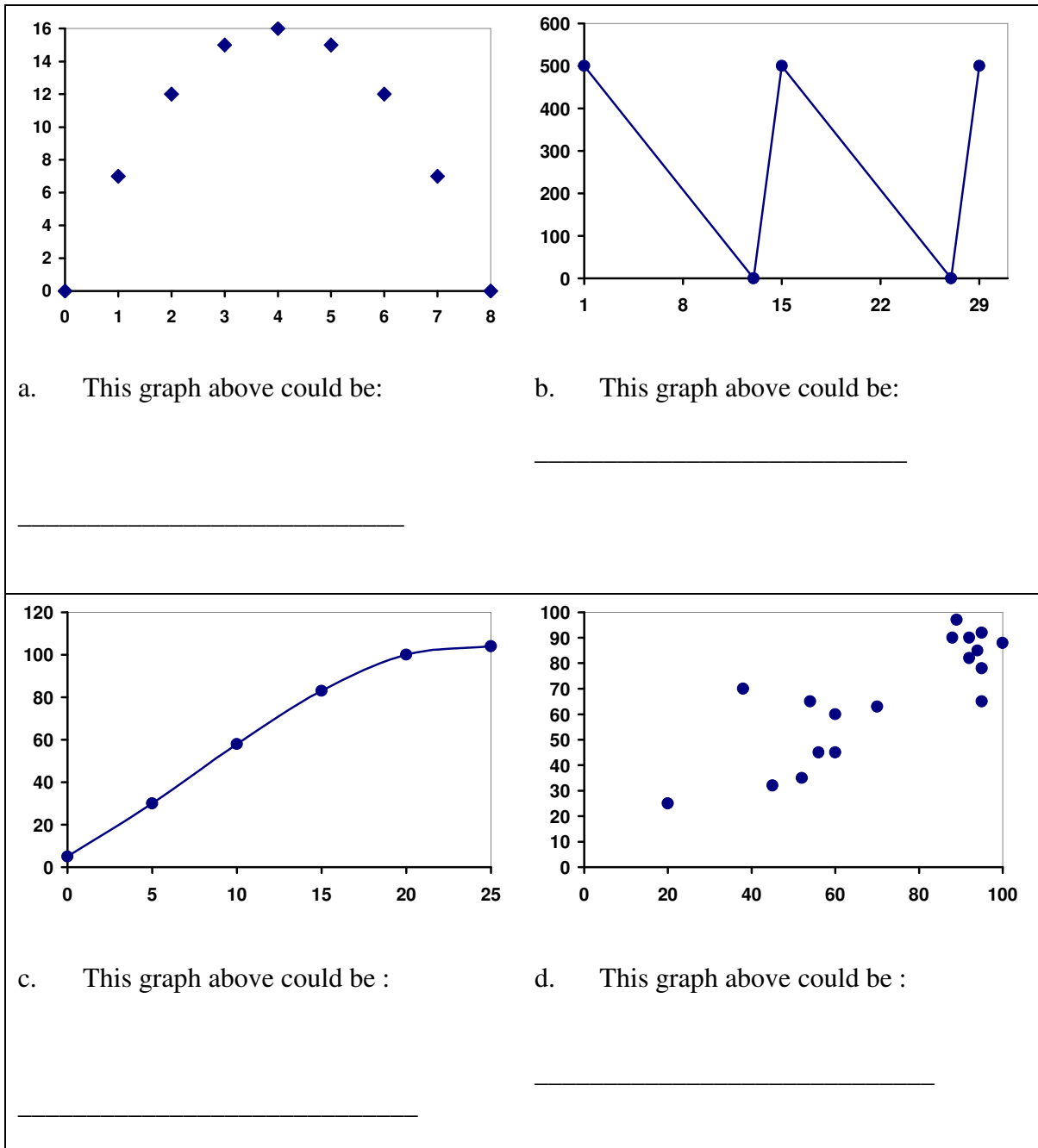
23. The bottom axis of a graph is called the '*independent variable*'. These are the values of numbers that we select when measuring things or doing statistics. Sometimes in statistics this axis is called the '*controlling variable*'. Generally, time is always an independent variable since we cannot control time.

24. The side vertical axis is called the '*dependent variable*'. These are values that we measure for every particular '*independent value*'. So for example: we may be measuring the height of a falling ball. The time is the independent variable, and height is the dependent variable. The dependent variable is sometimes called the '*controlled variable*'.

EXAMPLE GRAPHS

Mr T

25. What might the following graphs represent?





DOMAIN AND RANGE

26. The **Domain** and **Range** of data are important.

- Domain:** the values that the independent variable of the 'x' can take on.
- Range:** the values that the dependent or 'y' variable can take on.

27. **Example:** if we were to make pairs of numbers, (your age, your sister's age). The domain would be your age. Say your age, the *domain*, goes from 0 to 100. Your sister's age, the *range*, goes from 2 to 102 because she is two years older than you. For every number in the domain, there is a number in the range that is two more.

28. What is your guess at the typical Domain and Range of the following situations. What reasonable values can the data take on?

<p>a. Car Speed vs Gas Pedal depress Independent Variable: Gas Pedal Distance [cm] Dependent Variable: Speed [km / h]</p> <p>Domain = _____</p> <p>Range = _____</p>	<p>b. Math Mark vs Attendance Independent Variable: Attendance % Math Mark: %</p> <p>Domain = _____</p> <p>Range = _____</p>
<p>c. Height of a ball thrown up on earth</p> <p>Independent Variable: _____ [units: _____]</p> <p>Dependent Variable: _____ [units: _____]</p> <p>Domain = _____</p> <p>Range = _____</p>	<p>d. Amount of money in your bank account (paid every two weeks).</p> <p>Independent Variable: _____ [units: _____]</p> <p>Dependent Variable: _____ [units: _____]</p> <p>Domain = _____</p> <p>Range = _____</p>

Mr T

29. Which of these domain and range questions match up with previous graphs earlier?

LINEAR RELATIONSHIPS

30. We return to linear relationships. We will explore the other relationships in further grades.

A linear relationship is one in which a **change** in the independent variable has a **proportional change** in the dependent variable.

The perfect example is a recipe for muffins. If **one muffin** needs **100 grams of raisins**, how many grams of raisins do you need for **12 muffins**?

31. Make a table

Complete the table at right.

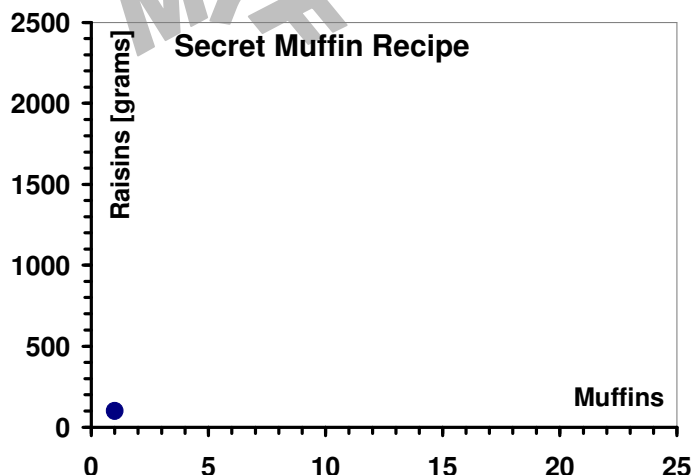
Muffins	Raisins [grams]
1	100
2	
3	
4	
5	
10	
12	
20	
24	

32. Make a graph

Manually plot the data from the table, it will show a simple pattern.

The pattern is called a line!

The relationship is 'linear'



Make An Equation

33. An equation is a *formula* or a *recipe*. An equation makes a relationship between two different sets of numbers. The equation for our secret muffin recipe is:

Grams of Raisins is 100 times the number of muffins

Or: '**Raisins = 100 * muffins**' ('=' means 'is' in math)

Or: **R = 100*M** if you want to be simple about it and provided every one knows that **R** is the amount of *grams of raisins* and **M** is the *number* of muffins.

34. Using this equation you can make a table and a graph of how many raisons you need for how many muffins. Equations are sometimes called '*relations*' or '*functions*'. The grams of raisins you need is dependent on the number of muffins you decide to make.

Equation Example

35. Your age is a function of your mom's age. We can make an equation that explains how old you are compared to your mom. A typical equation might be:

$$\text{You} = \text{Mom} - 20$$

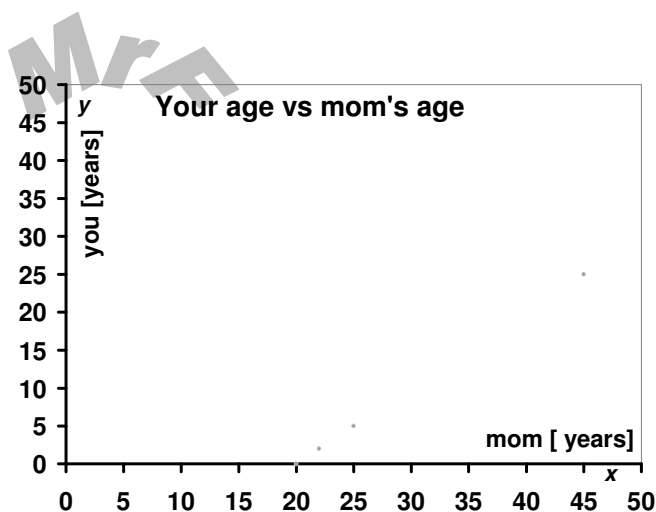
$$y = x - 20$$

of course if you are just going to use **y** and **x**, you must make it clear what the letters represent and the units (days, years, decades, feet, meters, etc) that are used

36. Make a table and a graph of the equation that is your age and your mom's age:

Complete the table for ' $y = x - 20$ ':

x	y
20	0
22	2
24	
30	
35	
50	



Make a graph of the equation above from the table

INTERCEPTS

37. Notice your mom has a head start on you in age! She started at 20, then every year you each got one year older. Where the line crosses the ' x -axis' (ie: when you are born; age 0) is called the ' x -intercept'.

38. What do you think the Domain and Range are of the function for your mom's and your age?

Domain: $0 \leq x \leq 100$ Range: : $-20 \leq x \leq 80$

some teachers write it like this:

Domain: $[0, 100]$ Range: : $[-20, 80]$

SLOPE

39. This relationship, or function, has a '*slope*'. The line has a slope that is a measure of how fast it changes. For every year your mom gets one year older, you get one year older. For every 5 years *your mom* gets older, **you** get 5 years older. This is called a slope of one. Slope is calculated by the change in the dependent variable for every change in the independent variable. When you calculate how many of something for every other something, it is called a *ratio*, a comparison of two numbers by dividing.

$$\text{slope} = \frac{\text{change in dependent}}{\text{change in independent}} = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{1} \text{ or } \frac{5}{5} \text{ or } \frac{20}{20} = 1$$

40. A slope shows how quickly something changes. Curiously it also shows the direction a line is going!



FINDING SLOPE FROM A TABLE

41. Find the slope of a line from this table of values:

x	y
0	3
1	5
2	7
5	13

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

so selecting any two ordered pairs.

$$\text{slope} = \frac{5-3}{1-0} = \frac{2}{1} = 2 \text{ or}$$

$$\text{slope} = \frac{13-3}{5-0} = \frac{10}{5} = 2;$$

a line has the neat feature that its slope is the same everywhere on it.

Y-Intercepts

42. A *y-intercept* is where a graph touches the *y-axis*. The *y-axis* runs right through where the *x-axis* is zero.

ENTERING EQUATIONS IN A GRAPHING TOOL

43. There are many on-line tools and Iphone Apps to graph equations (more correctly called 'functions' actually). An EXCEL spreadsheet is very good for graphing also. The TI-83 has a special ability to graph also.

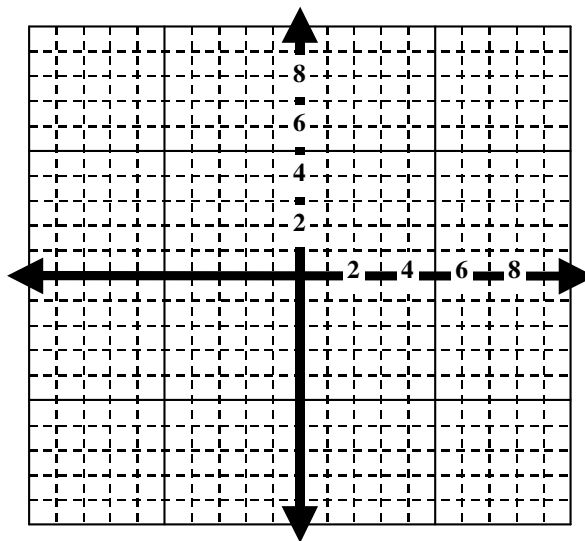
44. Let's graph the function $y = 2x - 4$.

Plot the data to see the pattern.

First use the function to make a table.

Select a few sensible x 's and find the y that goes with them from the function. Complete the one below.

x	$2x - 4$	y
-3	$2(-3) - 4$	-10
0	$2(0) - 4$	-4
2		
5		
7		
3.5		
$\frac{7}{2}$		

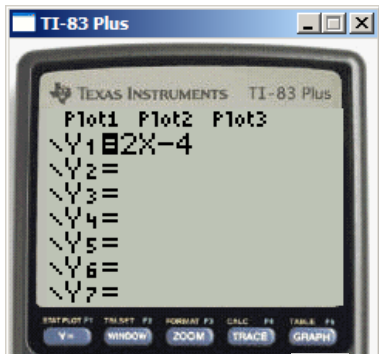


45. The equation above tells us the relationship between all the points on a line. Since we could choose any ' x ' we want, including fractions of x 's like 1.724 or -45.378 we can connect the few calculated dots with a line. So connect through the dots with a straight ruler. Put arrowheads on the ends of your line to show it goes forever in both directions!

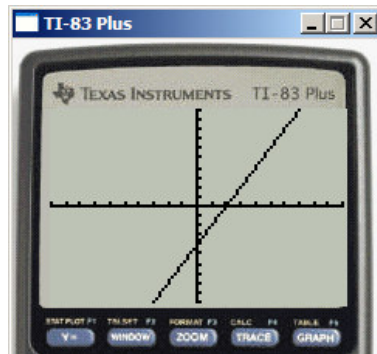
46. Now **graph** using the **TI-83**. The TI-83 can graph 10 different equations at a time! Most the buttons to do with graphing are the **Blue Ones** under the screen. To graph using the TI-83:

47. Press **Y=** to get to the *Equation Editor Screen*. Clear any equations you do not want by cursoring to them and hitting **CLEAR**.

48. Type in the expression that is the function. ie: type in “**2X – 4**” after the equals symbol. The ‘**X**’ is pasted to the equation using the variable key **X, T, θ , n**. You are done! Press the blue **GRAPH** key.



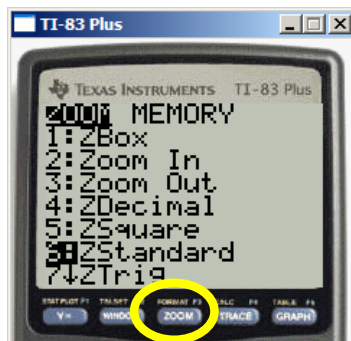
Equation Editor **Y =**



GRAPH

EASY!

49. **Adjusting the window with a special ZOOM.** It may be that the graph ‘**window**’, the part of the graph we are looking at, is set up wrong. So always start with the standard size window by selecting **ZOOM** **6:Standard**. That will give you a standard window with a domain of x’s of -10 to $+10$, and a range of y’s from -10 to $+10$.

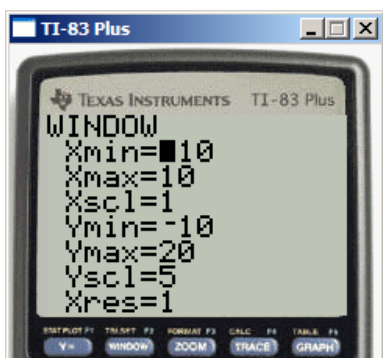


50. **Explore some of the blue graphing buttons.** These notes are not a manual on how to use the **TI-83** Graphing Calculator. The best way to learn is to explore on your own. Experiment with some of the blue graphing buttons:

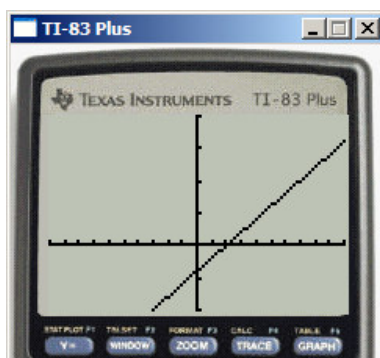
- WINDOW** . Allows you to change the part or size of the graph you want to display. Say you want to make the display show y 's that are bigger up to say 20 instead of 10; change the **Ymax** to 20 by cursoring to it, typing 20, and pushing **ENTER** . The **Yscl** tells you where you want 'tick marks' on your graph. You can select a tick mark every five instead of every one if you want for example. Do not mess about with **Xres** .
- ZOOM** . The zoom button is like a special WINDOW tool. It takes lots of exploration!
 - Try the **ZOOM 3:Zoom Out** selection. It will step you out a bit to see more of the graph. To use Zoom Out select it by pressing enter. You will notice a little '*cross hair*' in the middle of the screen. Press ENTER when you have moved the *cross hair* to where you want to Zoom Out around. The display will zoom out around that cross hair when you select ENTER again the second time.
 - 0:ZoomFit** . You need to cursor down to find this one. It will try to adjust the window in the y -direction so that any special peaks in the equation will show. Usually just use this if you are desperate and can't find a proper Window setting.
- TRACE** . This is a fun button. Press **TRACE** you will notice a little flashing 'spider' on the graph, located on your curve. On the bottom of the display will be a readout like:

$$x = 1.234567 \quad y = 4.56789$$

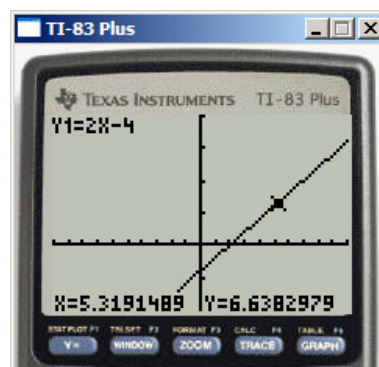
now use the blue arrow cursor buttons to make the spider *trace* (well it actually hops like a flea) along your curve. The readout will change, telling you the ' x ' and ' y ' position of the spider as it traces the curve of your equation.



Adjusting Window



Adjusted Window



Tracing Spider

d. **TABLE** button. To get any button to do the *alternate yellow feature* overtop of a *main button* you need to press **2nd** key first. This is an essential button. The **TABLE** Button will display a table of values for the equation. Exactly the same as any table that you would calculate, but quicker and forever. So press **2nd** **GRAPH**. It will give you a table of values; some x 's and the y 's that are calculated go with them to make the graph.

e. **TBL SET**. The **TBL SET** button allows changes to the way the table is displayed. You can make the table start wherever you want at **TblStart**. Also the **ΔTbl** entry can be adjusted to count by twos or tenths or however finely you want to make your table. So if you want to compute your displayed table every step of 0.1 instead of every step of 1 for x then enter 0.1.

X	Y1
0	-4
1	-2
2	0
3	2
4	4
5	6

X=0

The Function Table

TABLE SETUP
TblStart=0
ΔTbl=1
Indent: Auto Ask
Depend: Auto Ask

Table Setup

X	Y1
0	-4
.1	-3.8
.2	-3.6
.3	-3.4
.4	-3.2
.5	-3
.6	-2.8

X=0

Smaller x Steps Table

LINEAR EQUATIONS

51. Linear equations will always have a plain ' x ' and/or a ' y ' in them; or at least one or two plain unknowns; they could of course be other letters like Amount, A , vs time, t . The following equations are linear:

$$y = 2x + 4, \quad 2x + 5y = 10; \quad 2x - 4y + 3 = 0; \quad C = G + 2, \quad y = 5; \quad A = 250*t$$

52. If the equations have any other funky things happening then they are **not lines**. The following are not lines: (you may want to graph them with a graphing tool to see they are not lines).

$$y = x^2 - 4; \quad y = \sqrt{16 - x^2} \text{ and } y = -\sqrt{16 - x^2}; \quad y = x^3, \quad y = 1/x,$$

$E = m*c^2$ are not linear.

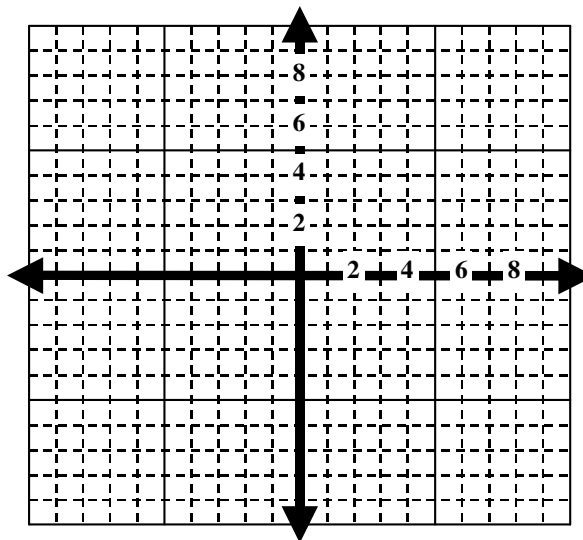
CHARACTERISTICS OF LINEAR EQUATIONS: SLOPE AND INTERCEPTS

53. Manually graph the equation:

$$y = 3x - 9$$

The table of some selected points is completed for you. You can try other points if you want.

x	$3x - 9$	y
0	$3(0) - 9$	-9
2	$3(2) - 9$	-3
3		0
6		9
2.75		-0.75



54. From the graph calculate:

a. slope: $slope = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise of the graph}}{\text{run to right}} = \frac{\Delta y}{\Delta x} =$

b. the x-intercept (when the y is zero):

c. the y-intercept: (when the x is zero)

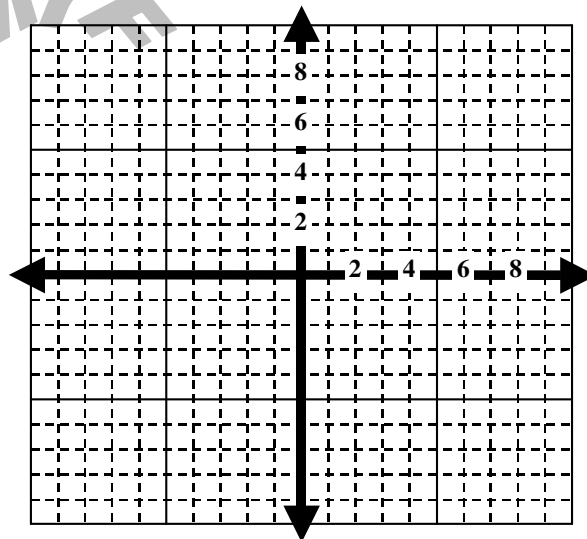
55. Now check it all with a graphing calculator. See if it gives the same table and the same graph. Use the **TRACE** to see if the *x-intercept* and *y-intercept* are about right.

56. Manually graph the function:

$$y = -2x + 10$$

Invent your own table; select a suitable domain of x 's to get a range of y 's that fit on the graph without changing the scale.

x	y



57. From the graph calculate:

a. slope: $slope = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise of the graph}}{\text{run to right}} = \frac{\Delta y}{\Delta x} =$

b. the x-intercept:

c. the y-intercept:

58. Now check it all with a graphing calculator. See if it gives the same table and the same graph. Use the **TRACE** to see if the x -intercept and y -intercept are *about* right.

59. Did you know? There is actually a selection on the **TI-83** that will tell you the x -intercept (where the y is zero). Try using the yellow **CALC** feature [**2nd** **TRACE**]. Select **2:Zero** . Now dance the spider a bit to the left of the x -intercept **ENTER** , a bit to the right of the x -intercept , **ENTER** , then press **ENTER** . The calculator will compute where the x -intercept is and display it on the screen. In this case it should say: $x = 5 \quad y = 0$.

60. And there is actually a feature that will calculate the y -intercept *also*. If you want to know where a line crosses the y -axis then that is where the x is zero. Use the yellow **CALC** feature [**2nd** **TRACE**]. Select **Value** . Tell the calculator you want to know the value of the function for an $X = 0$. So type in '0' **ENTER** .

Of course it might be easier to just look at the table and see what it says the y is when the x is zero. Press **2nd **GRAPH** .

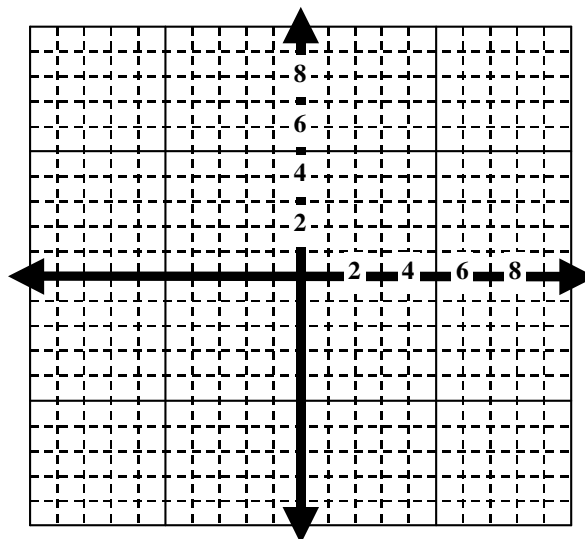
61. There is actually a feature on the calculator that will compute the slope for you too! But you will have to ask the teacher nice to get that information! That would be too easy!

62. Manually graph the function:

$$y = \frac{1}{2}x + 3$$

63. Invent your own table; select a suitable domain of x 's to get a range of y 's that fit on the graph without changing the scale.

x	y



64. From the graph calculate:

a. slope: $\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise of the graph}}{\text{run to right}} =$

b. the x-intercept:

c. the y-intercept:

65. Check your answers with the graphing calculator.

66. When a line is written in the form $y = mx + b$, where m and b are given numbers, the coefficient ' m ' is the *slope* and the constant ' b ', is the *y-intercept*.

67. To graph a line in this Slope and Intercept Form, $y = mx + b$, without making a data table or t-table follow these steps:

a. **Plot y-intercept.** Plot a point on the *y-axis* for the y-intercept; at $(0, b)$. The line goes through this point when the x is zero.

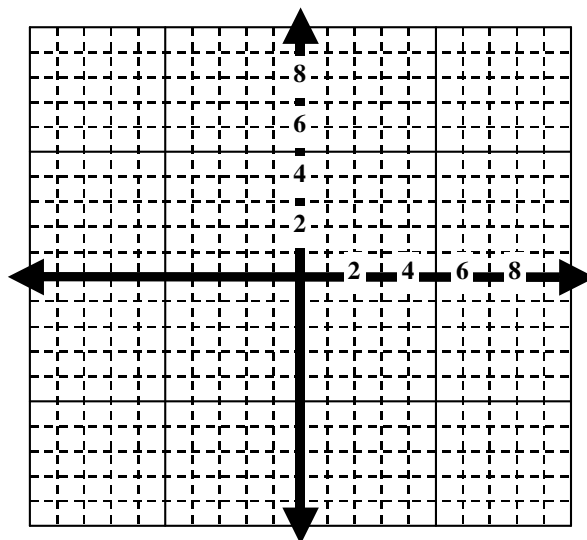
b. **Draw direction of line.** From the y-intercept, draw the direction of the line from the slope, m . Always convert the slope to a fraction ratio. So a slope; $m = -2$ is like $\frac{-2}{1}$ and would be a line where every step of one to the right the line drops down two.

68. Graph and label the following lines without making a *t-table*.

a. $y = 2x - 6$

b. $y = -4x + 10$

c. $y = \frac{3}{8}x + 3$



Physics Science Example

69. The velocity of *your car* is given by:

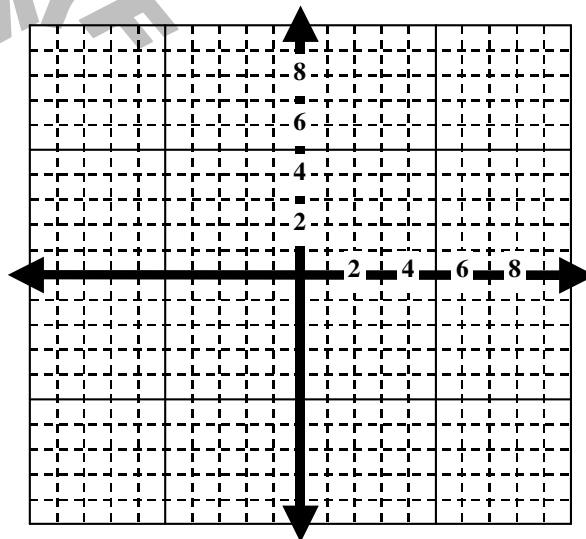
$v = 1.5*t + 5$; where ' v ' is velocity in meters per second [m/s] and ' t ' is time in seconds [s]. Incidentally, the **1.5** '*coefficient*' is in physics what is called the '*acceleration*'.

' t ', time, is obviously the independent variable to be plotted along the x-axis; and ' v ' the dependent variable plotted along the y-axis.

70. Your *sister's* old battle wagon has the velocity equation:

$$v = 0.2*t + 5$$

Graph the performance of both cars.



Horizontal lines

71. If the slope of a line is zero, it neither rises nor falls. It is constant. It has a slope of zero if the rise is zero. So the equation of a horizontal line becomes:

$$y = 0x + b \text{ ; or just}$$

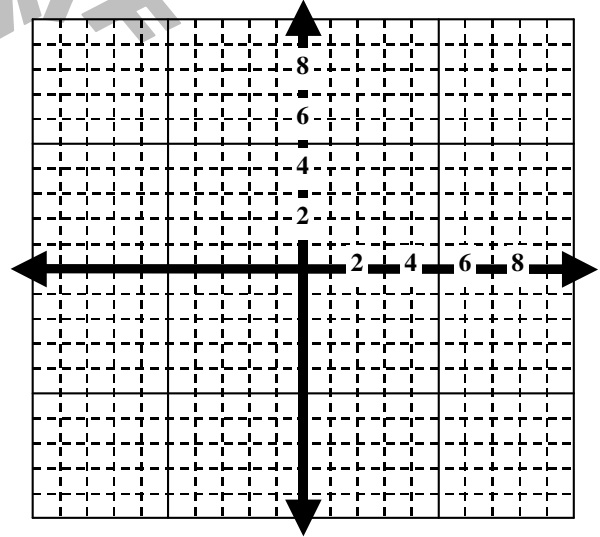
$$y = b.$$

72. A horizontal line is just a function that is constant. The y does not depend on the x, it is just constant.

73. Graph the following **horizontal lines**:

- a. $y = 4$
- b. $y = 8$
- c. $y = -3$
- d. $y = 0$

Check them with a graphing tool.

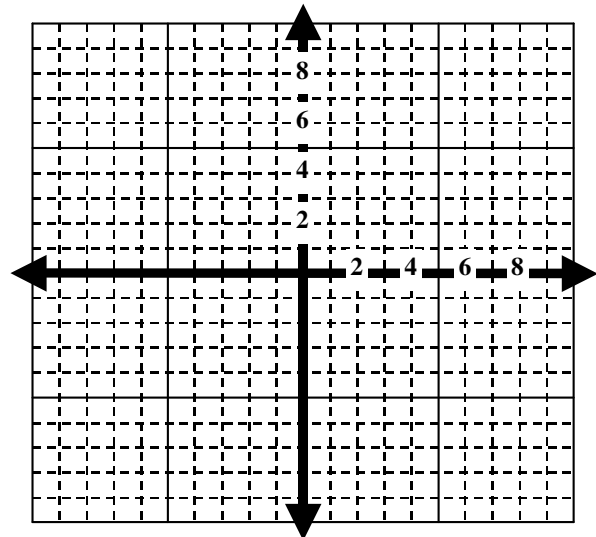


Vertical Lines

74. Vertical lines go straight up and down. The equation is not a well-behaved equation. The y is not a function or dependent on anything. There are many values of ' y ' that go with a single constant ' x '. The idea of slope is non-sensical for a vertical line, since there is no horizontal run involved. It just goes straight up; some might describe the slope as being 'infinite'.

75. Graph and label the **vertical lines**:

- a. $x = 5$
- b. $x = 9$
- c. $x = 0$
- d. $x = -6$



LINES IN THE GENERAL FORM $ax + by = c$

76. The 'Slope and Intercept' form of a line; $y = mx + b$, which we have been using is a simple form and the most popular. It is easy to *evaluate* a t-table, it is easy to *graph* even without a t-table, and it is easy to type into a graphing tool.
77. As long as an equation has a plain 'x' and plain 'y' in it is a line. But who says that the 'y' has to be written as a 'function' or equation in terms of 'x'?
78. If we said that: $you = mom - 20$, then we could also say that:
 $mom - you = 20$. (the difference in your ages is 20 and she is older)
79. Lines written in the form: $ax + by = c$ where both variables are on the same side of the equation and the **a**, **b**, and **c** are real numbers that are given (but **a** and **b** not both zero) are said to be written in the **general form**.
80. There are two ways to graph lines in this general form.
- Make a 'Literal' Expression.** Make the same relationship but put the 'y' in terms of the 'x', in other words, use algebra to make the same relationship but in the form $y = mx + b$ by *subtracting the x term from both sides*.
 - Intercept-Intercept Plot.** Use a simple t-table and plot the *x* and *y-intercepts*.

LITERAL EXPRESSION CONVERSION

81. Convert $3x + y = 9$ into the '*slope and intercept*' form using basic algebra then graph it.

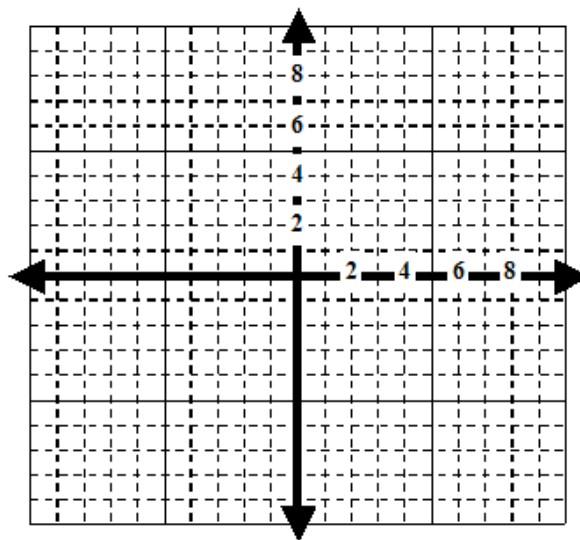
$$3x + y = 9; \text{ subtract } 3x \text{ from both sides}$$

$$y = 9 - 3x$$

$$\text{or } y = -3x + 9 \text{ (by the commutative law)}$$

82. Now graph it.

$$\text{You try with } 6x + 3y = 9:$$



$$\text{Ans: } y = -2x + 3$$

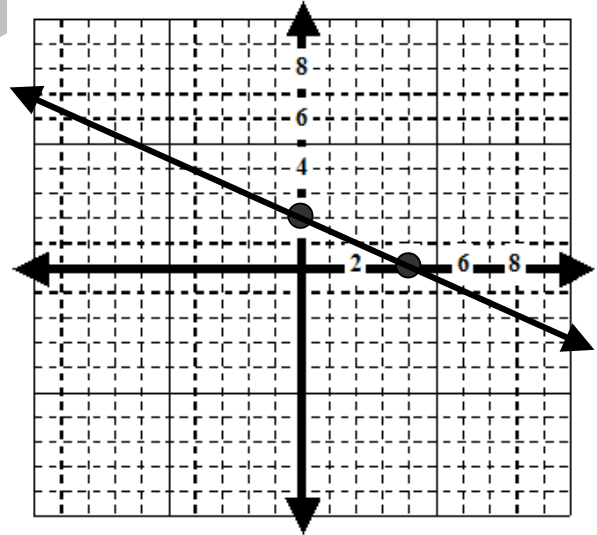
T-TABLE FOR THE GENERAL FORM

83. When the equation is in the General Form $ax + by = c$ a very simple t-table can be made to find the two axis intercepts

Example: $3x + 6y = 12$.

84. Simply find the corresponding values that go with an x of zero and a y of zero.

x	y
$0 \rightarrow$	$3(0) + 6y = 12$ $y = 2$
$3x + 6(0) = 12$ $x = 4$	$\leftarrow 0$



You are just plotting the x- and y-intercepts. Some books call this the intercept method.

85. **You try.** Graph and label the following lines in General Form:

a. $x + y = 10$

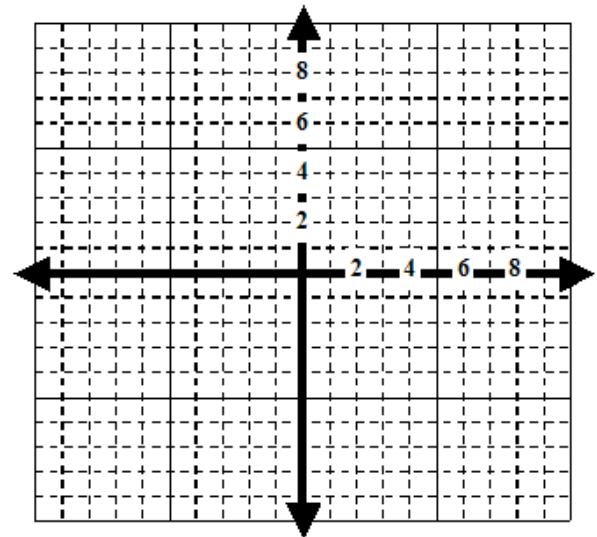
x	y
0	
	0

b. $2x - y = 8$

x	y
0	
	0

c. $\frac{3}{4}x + \frac{2}{3}y = 3$

x	y
0	
	0

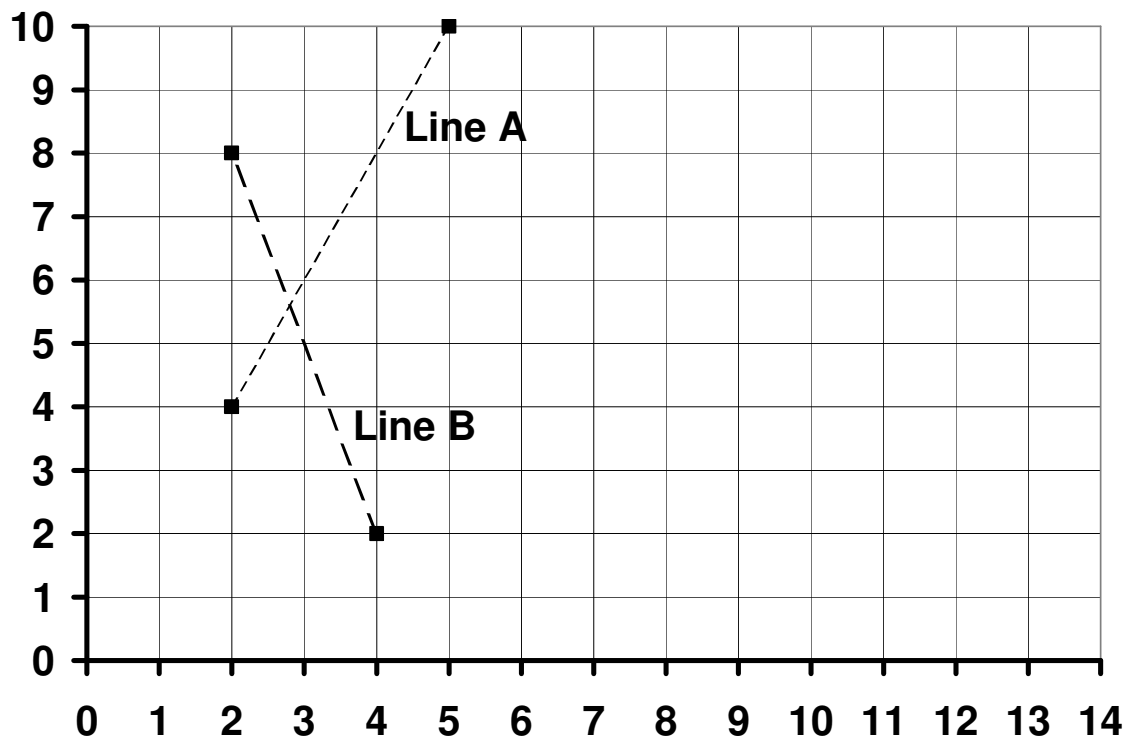




Finding Slope of a Line from its Graph

86. To find the slope of a line from its graph, simply select any two convenient **P**oints, call them **P**₁ at (**x**₁, **y**₁) and **P**₂ at (**x**₂, **y**₂).

87. Then apply the formula for slope: $y = \frac{\text{rise}}{\text{run right}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$



88. Calculations:

a. $\text{slope}_A = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{5 - 2} = \frac{6}{3} = 2$

89. You try for the slope of line B:

b. $\text{slope}_B = \frac{y_2 - y_1}{x_2 - x_1} =$