GRADE 10 (MA20S) REFERENCE NOTES

Gr10Math_Reference_Notes.doc | **Revised**:20121029

UNITS A AND D *'LINEAR MODELLING' and 'LINEAR FUNCTIONS'*

Plot Points. (***x, y***). Along ***x***-axis first, then up down along ***y***-axis. Eg: point **Q (–4, +6)**

Slope & Intercept form of line: $y = mx + b$.

Slope ≡ m ≡ $\frac{Rise}{Run}$. Positive **m** is up and right.

Y-intercept ≡ b ≡ where line crosses ***y***-axis.

Standard form of Line*:* *ax + by = c*

Finding Slope from two Points:

$$m \equiv \frac{rise}{run} \equiv \frac{change\,in\,y}{change\,in\,x} \equiv \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines: y = c. ***Vertical Lines: x = c.***

Graph Standard Form Line*:* make data table or plot intercept and then slope.

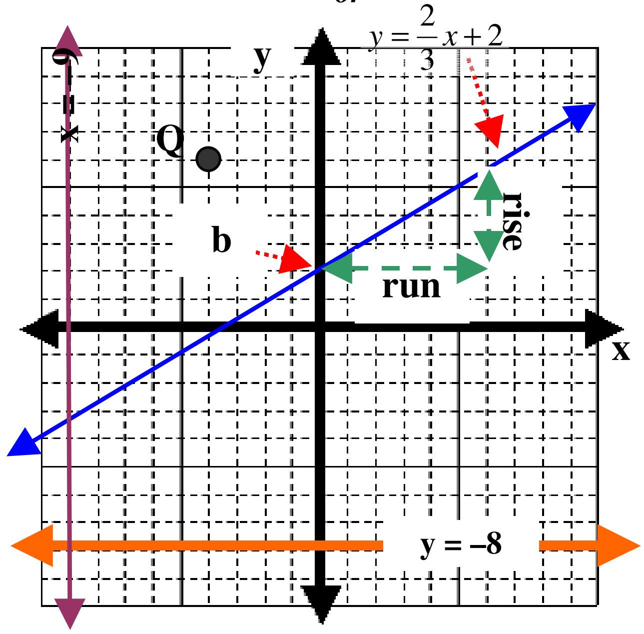
Graphing Lines in general form: ax + by = c.
Make Intercept Table by Evaluating for **y = 0** to *find* **x-intercept**, and **x = 0** to *find* **y-intercept**.

Graph on TI-83 **Y =** . **Zoom 6: Standard.**

Parallel Lines A and B. : $l_A \parallel l_B \leftrightarrow m_A = m_B$.

Perpendicular Lines: $l_A \perp l_B \leftrightarrow m_A = -\frac{1}{m_B}$

$3y - 2x = 6$
or
$y = \frac{2}{3}x + 2$



Converting between Standard and Slope Intercept form; just use Algebra.
Eg: 3y –2x = 6. ∴ 3y = 6 + 2x. ∴ $y = \frac{6+2x}{3} = 2 + \frac{2}{3}x$

UNIT B – NUMBER SENSE

Prime Nbrs: ***2, 3, 5, 7, 11, 13, 17****, cannot be made of two other numbers multiplied together (factors) except itself and one.*

GCF: The largest factor of two numbers**.** Factor to prime; multiply all shared 'common' factors**. GCF(12, 20):** 2*2*3 , 2*2*5 so GCF is 2*2 = 4

LCM: The lowest number that two different numbers can make when used as factors. Factor to primes, find the **greatest degree** or 'gaggle' of each prime factor**.** Multiply the gaggles together. **Eg. LCM(12, 20):** 2*2*3**,** 2*2*5 two 2's, one 3 , and one 5; so 2*2*3*5 = 60. ∴ LCM(12, 20) = 60. Button on calculator does it too!

Squares and Cubes: x*x = x². x*x*x = x³. $\sqrt{x^2} = x$. $\sqrt[3]{x^3} = x$. '*Roots*' undo '*exponents*'.

Rational numbers. Can be made into fractions or terminate in repeat decimal pattern: $3/8 = 0.375000\overline{0}$, $4\,{}^{5}\!/\!{}_{9} = 4.5555\overline{5}$.

Irrational Numbers. Not fractions. No computer can show them**.** $\sqrt{5}, \pi, etc$. No repeat pattern in decimal; not a fraction**.**

Which is bigger? $\sqrt{14}$ *or* 4? Square both! Is 14 **>** 16?

Simplifying Radicals. Factor out **perfect squares**. Eg: $\sqrt{50} = \sqrt{25} * \sqrt{2} = 5\sqrt{2}$

UNIT C – MEASUREMENT

Converting between units: See separate conversion chart for factors, equivalents. See separate chart of geometric formulae.

Example: 1 km = __??__ feet. $1\,km = 1\,\cancel{km} * \frac{0.6214\,\cancel{mi}}{1\,\cancel{km}} * \frac{5280\,ft}{1\,\cancel{mi}} = 3281\,ft$

Sphere	Volume Cylinder	Volume Cone	Volume Pyramid
$V = \frac{4}{3} * \pi * r^3$ $SA = 4 * \pi * r^2$	$V = \pi * r^2 * h$ $SA = 2\pi * r^2 + 2\pi * r * h$	$V = \frac{1}{3} * \pi * r^2 * h$	$V = \frac{1}{3} * base * height$

UNIT E – ALGEBRA

Exponent Rules:

Product: $a^m * a^n \equiv a^{(m+n)}$

Quotient: $\frac{a^m}{a^n} \equiv a^{(m-n)}$

Power: $(a^m)^n \equiv a^{(m*n)}$

More Rules:

$(xz)^n \equiv x^n z^n$

$\left(\frac{x}{z}\right)^n \equiv \frac{x^n}{z^n}$

Negative Exponent: $a^{-n} \equiv \frac{1}{a^n}$

Zero Exponent:

$x^0 \equiv 1$ (***anything*** to the zero exponent is **1**)

Fractional Exponents:

$x^{1/2} \equiv \sqrt{x}$, $x^{1/3} \equiv \sqrt[3]{x}$, $x^{1/n} \equiv \sqrt[n]{x}$

$x^{m/n} = \sqrt[n]{x^m} \equiv \sqrt[n]{x}^{\,m}$. Eg: $4^{3/2} = \sqrt[2]{4}^{\,3} = 8$

Polynomial: the sum of one or more monomial terms. **Monomials**: are variables and *whole number* powers of variables, and coefficients all multiplied together. **Constant**: a term that has no variables, just a number. Example of polynomial: $x^2y - \frac{1}{2}x - 4$. 1 and (–1/2) are '***coefficients***', **-4** is a '***constant***'.

Adding Polynomials: Add '*like terms*'. *Eg*: $\mathbf{3x^2y + 4xz^2 - 2x^2y + 2xz^2 + 7 = x^2y + 6xz^2 + 7}$

Multiply polynomial by monomial: **2d * (3a + 4b – 5c) = 6ad + 8bd – 10cd** (distributive property of arithmetic)

Evaluate a polynomial: '*Plug in*' the given values of variables. **4x – 2y** *given* **x = 2** and **y = –3** is **4(2) – 2(–3) = 14**

Factor: to turn a polynomial into the product of basic **prime** '***factors***' multiplied together. **Eg**: 7x + 14 = 7*(x + 2).

EG: $\mathbf{9x^2y^3 + 3x^2y = 3*3*x*x*y*y*y + 3*x*x*y}$. The **GCF** in each term is $\mathbf{3x^2y}$. So '*factor it out*' to get the product:

$\mathbf{3x^2y*(3y^2+1)}$. *Check* factoring by multiplying to see if you get what you started with.

****Factoring a Quadratic Trinomial**: A ***trinomial*** has ***three*** terms. A *quadratic* is of '*degree*' 2 (*ie*: the largest exponent is a 2). When we factor a ***trinomial,*** we turn it into the *product* of two *binomials*. ***Trinomials*** are properly written as $\mathbf{ax^2 + b\,x + c}$ in that order. *Eg*: $\mathbf{x^2 - 5x + 4}$. ***Basic Factor Method***: What two factors multiply to give the constant **+4** but add to give **–5** in front of the middle term? They are **–1** and **–4**. So **(x – 1)*(x – 4)** is the same as $\mathbf{x^2 - 5x + 4}$. **Check** the factors using **FOIL**! (**caution**: *this basic factor method* ***only works*** *if the squared '**a**' term has a coefficient of **1***)

****Special Cases of Factoring:** **Difference of squares**: $\mathbf{a^2 - b^2 = (a + b)*(a - b)}$. *Eg*: $\mathbf{x^2 - 16 = (x + 4)*(x - 4)}$ or just picture the trinomial as: $\mathbf{x^2 + 0x - 16}$. **Perfect squares trinomial**. The *first* and *last* term are ***perfect squares*** of integer numbers and the *middle term* is twice the product of the those two numbers. Eg: $\mathbf{9x^2 + 30x + 25 = (3x + 5)*(3x + 5) = (3x + 5)^2}$

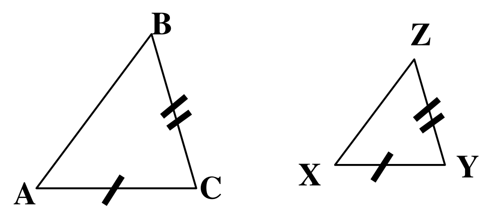
UNIT G – TRIGONOMETRY

SIMILAR TRIANGLES

Given that **ΔABC ~ ΔXYZ (ie:** they are similar) **then:** $\frac{AB}{XZ} = \frac{BC}{YZ} = \frac{AC}{XY}$

Of course you cannot have 2 equal signs in an equation! This really means:

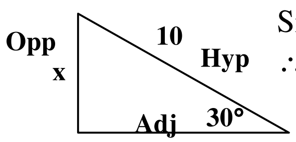
$\frac{AB}{XZ} = \frac{BC}{YZ}, and\ \frac{BC}{YZ} = \frac{AC}{XY}, and\ \frac{AC}{XY} = \frac{AB}{XZ}$



TRIGONOMETRIC RATIOS: **Sine**, **Cosine**, & **Tangent**. *Just comparison ratios of different sides of same right triangle. .*

SOH CAH TOA . **Sin(θ) = Opp / Hyp**, ***Cos(θ) = Adj / Hyp***, **Tan(θ) = Opp / Adj**

Find side x.

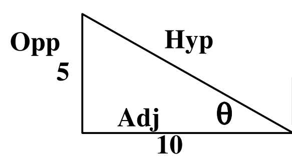


Sin(30°) = x / 10

∴x = 10 * Sin(30°)

x = 5

Find angle θ

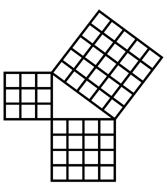


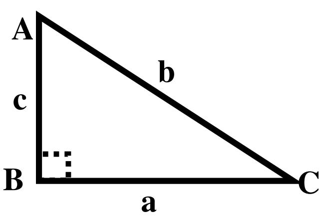
Tan(θ) = Opp/Adj = 5/10

$\mathbf{\theta = Tan^{-1}(5 / 10)}$

θ = 26.6°

Make sure your calculator is in degree mode!





The **square** on the long side of a **RA** triangle is the sum of the squares on the other two sides.

$\mathbf{b^2 = a^2 + c^2}$

!!! ***The long side by itself*** **!!!**

UNIT H: APPLICATIONS OF LINEAR FUNCTIONS

Systems Of Linear Equations.

SOLVING SYSTEMS OF TWO EQUATIONS BY SUBSTITUTION

Label the equations. Express one variable in terms of the other ('isolating' a variable). ***Substitute*** that expression into the ***other*** equation. Solve for the single variable. Substitute the found variable ***back into either equation*** to find the remaining variable. ***Check*** the answer in both original equations

SOLVING SYSTEMS OF TWO EQUATIONS BY ELIMINATION

Put equations into a '*standard*' order:(**ax + by = c**). Label the equations. Multiply either, or both, equations by some number so that one variable can be eliminated when the equations are added or subtracted. Add or subtract the equations to eliminate one variable and find the other. Find the eliminated variable now by substituting the solved variable back into either equation of the system. Check your answer (***x, y***) in both original equations.

SOLVING SYSTEMS OF EQUATIONS BY GRAPHING. Find intersection. On **TI-83**: **2nd TRACE** , **5: intersect** .

UNIT F- COORDINATE GEOMETRY

Distance Between Two Points Given two points $\mathbf{P(x_1, y_1)}$ and $\mathbf{Q(x_2, y_2)}$	**Mid Point between Points** Given two points $\mathbf{P(x_1, y_1)}$ and $\mathbf{Q(x_2, y_2)}$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$midpo\,\text{int} = \left\{\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right\}$