

Grade 10 Essential Unit A Puzzles and Problem Solving

Problem Solving Strategies

Here are the classic Problem Solving Strategies. Often several of them will work on any particular problem, or occasionally a couple of them can be combined.

Multi-Step. Solve slowly and deliberately in steps, find intermediate answers that give you the final answer.

Draw a Picture. My favourite. You don't even need any numbers! You don't need numbers to do math!! I hate numbers, too many of them!

Look for a pattern. That is what math is fundamentally, the search for patterns!

Guess and Check. We all do this! Guess until you get it right! Play High-Low. Why not if it works?! It might be cumbersome but it *will* work; in fact there do exist many mathematical problems that can *only* be solved by guessing and checking!

Identify Missing Information. You may discover that there is not enough information and you may have to ask for more information. It is hard to solve any problem (in life or in math) if you are not given all the necessary facts. Sometimes there is no solution even!

Make a Table. Calculate lots of possibilities. Can be used especially in conjunction with ‘Look for a Pattern’ method, the ‘Formula’ method, or the ‘Work Backwards’ method.

Make a List. Make a list of all the possibilities and count them! Useful for probability and counting possibilities. Often used in conjunction with ‘Look for a Pattern’ method. Some folks sometimes use a **tree** to make a complete list.

Solve a Simpler Problem. If you can solve a simpler version of the problem then you know you can use the same steps to solve the more difficult version.

Work Backwards. If you know what the result is and want to know what gave you that result use ‘Work Backwards’. **Working Backwards** is actually exactly what *algebra* is! (But I should not scare you with that word!)

Use Estimation. To be honest how often do you really need a perfectly exact answer!!?? Sometimes rounding and estimating is readily sufficient.

Use a Formula. Sometimes you are given a formula or you can figure a formula out! Plug in and calculate! Often used in conjunction with Make a Table method.

Use Logical Reasoning. Girls are especially good at this (unfortunately!). *Example:* Sometimes knowing what something **is not** helps you find what **it is**! Often used with Tables, *truth tables*.

PROBLEM-SOLVING STRATEGIES

Multi-Step

Sandwiches at the cafeteria cost \$1.75, a salad costs \$1.09, and a glass of milk costs \$0.75. Eva and her friends ordered two sandwiches, three salads, and four glasses of milk. How much change should they receive from \$20?

The total cost of the sandwiches was \$3.50.

The total cost of the salads was \$3.27.

The total cost of the glasses of milk was \$3.00.

Eva and her friends spent \$9.77 at the cafeteria.

Eva and her friends received \$10.23 in change.

Find the total cost of the sandwiches, the salads, and the milk.

sandwiches	salads	milk
\$1.75	\$1.09	\$0.75
<u>×2</u>	<u>×3</u>	<u>×4</u>
\$3.50	\$3.27	\$3.00

Next, find the sum of the three individual costs.

$$\$3.50 + \$3.27 + \$3.00 = \$9.77$$

Then, find the change from \$20.

$$\$20.00 - \$9.77 = \$10.23$$

Solve each problem.

SHOW YOUR WORK

1. Sandro must read a total of 375 pages. He read 45 pages each day for 6 days and 25 pages each day for the next 4 days. How many more pages must he read?

Sandro needs to read _____ more pages.

2. Kerri bought three towels that each cost \$7.95. The tax on the purchase was \$1.19. She paid with one \$20 bill, one \$5 bill, and one dime. How much change did Kerri receive?

Kerri gave the clerk _____.

Kerri got _____ in change.

3. On Monday, Flower City sold 14 dozen roses. On Tuesday, half as many roses were sold. On Wednesday, 2 dozen fewer roses were sold than on Tuesday. How many dozen roses were sold in all?

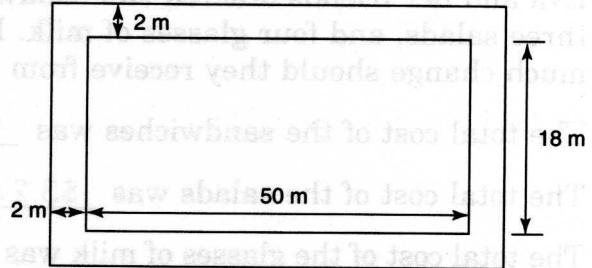
_____ dozen roses were sold in all on Monday, Tuesday, and Wednesday.

PROBLEM-SOLVING STRATEGIES

Draw a Picture

A garden plot is shaped like a rectangle. The length of the garden is 50 m and the width is 18 m. There is a 2-m-wide footpath around the entire garden. What is the total area of the garden and footpath?

Draw a picture of the garden and footpath.



Find the dimensions of the garden, including the footpath.

Then find the area.

$$\text{length: } 50 + 2 + 2 = 54$$

$$\text{width: } 18 + 2 + 2 = 22$$

$$\text{area} = \text{length} \times \text{width}$$

$$\text{area} = 54 \times 22 = 1188$$

The total length of the garden and footpath is 54 m.

The total width of the garden and footpath is 22 m.

The total area of the garden and footpath is 1188 m².

SHOW YOUR WORK

Solve each problem.

1. A bird is flying 1500 m below the clouds. The top of a building is 300 m above the ground. The base of the clouds is 800 m above the ground. How many metres above the building is the bird?

The bird is _____ m above ground.

The bird is _____ m above the building.

2. A tree trunk divides into three branches. Each smaller branch divides into two branches. Each of those divides into three branches. How many branches are on the tree in all?

There are _____ branches on the tree trunk.

SHOW YOUR WORK

PROBLEM-SOLVING STRATEGIES

Look for a Pattern

Awan has a credit card balance of \$4211. He has made payments of \$25, \$50, and \$75. If his payments continue this same pattern, how much will his sixth payment be?

The difference between the first and second payment is \$25.

The difference between the second and third payment is \$25.

Awan will pay \$100 on his fourth payment.

Awan will pay \$125 on his fifth payment.

Awan will pay \$150 on his sixth payment.

Look for a pattern as the payment number increases.

Payment number:	Amount of payment:
1	\$25
2	\$50
3	\$75
4	\$100
5	\$125
6	\$150

The pattern is + \$25.

Solve each problem.

SHOW YOUR WORK

1. A tennis ball is dropped from a height of 96 cm.

On bounce one, it rebounds to a height of 48 cm. On bounce two, it rebounds to a height of 24 cm. How high does the tennis ball rebound on bounce four?

The pattern is _____.

On bounce four, the tennis ball rebounds _____ cm.

2. Lucy's faucet has a leak. The longer it goes unrepaired, the more water it leaks. In the first four days, her faucet leaked 90 mL, 180 mL, 360 mL, and 720 mL of water. If this pattern continues, how many ounces of water will the faucet leak on the sixth day?

The pattern is _____.

On day six, the faucet leaks _____ mL of water.

PROBLEM-SOLVING STRATEGIES

Guess and Check

At the cafeteria, Yancy used an equal number of quarters and nickels to buy his lunch. His lunch cost \$1.80. How many of each coin did he use?

One quarter has a value of \$0.25.

One nickel has a value of \$0.05.

Yancy used six quarters and six nickels.

Guess the possible numbers of each coin.

Guess: 4 quarters and 4 nickels

Value: $\$0.25 \times 4 = \1.00

$\$0.05 \times 4 = \0.20

Total

Value: $\$1.00 + 0.20 = \1.20

Incorrect.

Guess: 6 quarters and 6 nickels

Value: $\$0.25 \times 6 = \1.50

$\$0.05 \times 6 = \0.30

Total

Value: $\$1.50 + 0.30 = \1.80

Correct.

Solve each problem.

SHOW YOUR WORK

1. Ellis drove 495 km from his home to a resort at the beach. He returned home at a 10 km/h slower average speed. He drove 20 h total. Find his average speed in each direction. Give your answer in *kilometres per hour (km/h)*.

Ellis drove at a speed of _____ from home to the beach.

Ellis drove at a speed of _____ from the beach to home.

2. In a basketball game, Debbie scored 38 total points on two-point baskets and three-point baskets. She scored 4 more two-point baskets than three-point baskets. How many of each did she score?

Debbie scored _____ two-point baskets.

Debbie scored _____ three-point baskets.

PROBLEM-SOLVING STRATEGIES

Identify Missing Information

PROBLEM-SOLVING

Mr. Walton bought shirts that cost \$12.95 each, and ties that cost \$9.75 each. He gave the clerk six \$10 bills. How much change did he receive?

Multiply to find the amount owed for the shirts and ties.

Not enough information

\$12.95	\$9.75
\times # of shirts	\times # of ties
amount owed for shirts	amount owed for ties

Missing information: _____ the number of ties and _____ the number of shirts purchased

Information on the number of shirts and ties purchased is missing.

SHOW YOUR WORK

Solve each problem.

1. Jim has a job that pays him \$75 per day for 8 h of work. He saves \$15 of his pay each day for repairs on his car. How many days will it take Jim to save enough to repair his car?

Missing information: _____

2. Karen and some of her friends are going on a camping trip. They plan to use a trailer to transport their belongings. Each person is limited to a 1-kg sleeping bag, 8 kg of food, and 5 kg of personal belongings. What is the total mass of their belongings?

Missing information: _____

3. Linda bought 3 kg of chicken for \$7.74 and a package of ground beef for \$8.29. How many more kilograms of chicken than beef did Linda buy?

Missing information: _____

PROBLEM-SOLVING STRATEGIES

Make a Table

Jenna works in a seed packaging plant. For quality control, every 4th seed is checked for colour and every 9th seed is checked for size. How often is a seed checked for both size and colour?

Make a table to determine when a seed is checked for both size and colour.

Every 4th seed is checked for colour.

Every 9th seed is checked for size.

The first seed that is checked for both size and colour is the 36th seed.

Every 36th seed is checked for both colour and size.

Number of Seeds

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

colour ○

size ×

colour and size ⊗

Solve each problem.

SHOW YOUR WORK

- Bucket A contains 250 mL of water and bucket B contains 475 mL of water. Both buckets have a leak. Every 30 min bucket A loses 9 mL of water and bucket B loses 54 mL of water. When will each bucket contain the same amount of water? How much water will each bucket hold when they have the same amount of water?

After _____ min, buckets A and B will contain _____ mL of water each.

- Every 8 min, a washing machine completes one cycle. The machine is turned on at 6:28 A.M. What time is it when the machine completes its fourth cycle?

The fourth cycle is completed at _____.

- A film developer charges \$1.00 for each roll of film developed plus \$0.04 for each picture developed. What is the cost for developing a roll with 12 pictures?

Twelve pictures cost _____.

PROBLEM-SOLVING STRATEGIES

Make a List

Claudia is a T-shirt designer. She wants to have one example of each of the choices from which her clients can choose. Her clients can choose from white or black T-shirts; a blue, red, or green silk-screen design; and designs A, B, or C. How many sample T-shirts must Claudia make?

Claudia will need to make 18 sample T-shirts.

Make a list of all the possible choices.

black, blue, A	black, red, A
black, blue, B	black, red, B
black, blue, C	black, red, C
black, green, A	white, blue, A
black, green, B	white, blue, B
black, green, C	white, blue, C
white, green, A	white, red, A
white, green, B	white, red, B
white, green, C	white, red, C

Count the combinations.

Solve each problem.

SHOW YOUR WORK

1. Riley is playing a grid-game on which he can move his game piece either one or three squares forward, one or three squares to the left, or one or three squares to the right. How many ways can Riley move his game piece?

There are _____ different ways that Riley can move his game piece.

2. Each time Molly jogs, she listens to one CD. She jogs on Monday, Wednesday, and Friday. She rotates in alphabetical order among her four CDs, labelled A, B, C, and D. Which CD will Molly listen to when she jogs the Friday of the third week?

Molly will listen to the CD labelled _____ on the Friday of the third week.

3. Paul has four different cards with one of the letters m , a , t , and h written on them. How many different ways can he arrange the letters?

Paul can arrange the letters _____ different ways.

PROBLEM-SOLVING STRATEGIES

Solve a Simpler Problem

Winona types 70 words in $3\frac{1}{2}$ min. At this rate, how many words can she type in 32 min?

Winona can type 140 words in 7 min.

Winona types at a rate of 20 words per min.

Winona can type 640 words in 32 min.

If you double both the minutes and the number of words, you will have an equal ratio with whole numbers.

$$70 \times 2 = 140$$

$$3\frac{1}{2} \times 2 = 7$$

Find the unit rate.

$$140 \div 7 = 20$$

$$32 \times 20 = 640$$

Solve each problem.

SHOW YOUR WORK

1. Leonardo was told that a certain phone card company charges $7\frac{1}{2}$ cents for $1\frac{1}{2}$ min of phone time. At this rate, how much would Leonardo pay for a 15-min phone card?

The phone card company charges at a rate of _____ per minute.

Leonardo would pay _____ for a 15-min phone card.

2. Anna wants to hang new drapes in her room. One window is 1 m 25 cm wide, and the other window is 1 m 75 cm wide. How many metres of fabric should Anna buy? Anna has selected a fabric the same length as the windows.

Anna should buy _____ m of fabric.

3. A best-selling novel sold 24 000 copies in 6 weeks. On average, how many copies were sold each week?

On average, _____ copies were sold each week.

PROBLEM-SOLVING STRATEGIES

Work Backward

Rashida is trying to decide what time to set her alarm so she will be on time for her dental appointment at 9:00 A.M. tomorrow. She needs 45 min to get dressed and eat breakfast. The bus ride to the dentist's office takes 20 min. Rashida would like to be 10 min early so she can give the receptionist her insurance information. What time should Rashida set her alarm?

Rashida should set her alarm for no later than 7:45 A.M. .

List the steps in the order they would occur.

1st: 45 min to get dressed and eat breakfast

2nd: 20 min to get to office

3rd: 10 min early arrival

4th: 9:00 A.M. appointment

Work backward from 9:00 A.M.

$$9:00 - 10 \text{ min} = 8:50$$

$$8:50 - 20 \text{ min} = 8:30$$

$$8:30 - 45 \text{ min} = 7:45$$

Solve each problem.

SHOW YOUR WORK

1. Robert has an 88% average on his Latin tests. He has had four tests, but can only remember what he scored on three of them. Robert knows he scored 95%, 77%, and 82%. What score did he get on the fourth test?

Robert scored _____ % on the fourth test.

2. After deductions, Rafael's paycheque for 12 h of work was \$145. He paid \$1.20 for provincial taxes, \$3.50 for federal taxes, and \$1.50 for pension plan. How much does Rafael get paid per hour?

Rafael earns _____ before deductions.

Rafael gets paid _____ per hour.

3. Ling is 3 cm taller than Lai. Ling is 2 cm taller than Mei. Kwan is 6 cm taller than Mei. Kwan is 173 cm tall. How tall is Lai?

Lai is _____ cm tall.

PROBLEM-SOLVING STRATEGIES

Use Estimation

Sofia is on the basketball team. She scored 261 points during last year's 27-game basketball season. About how many points did she score each game?

Estimate compatible numbers.
Round 261 to 260 and 27 to 26.

$$260 \div 26 = 10$$

Sofia averaged about 10 points per game.

Solve each problem.

SHOW YOUR WORK

1. A tour group of 48 people is flying to Venice. Each person is permitted two suitcases. Each suitcase can have a mass of no more than 16 kg. About how many kilograms of luggage will this group have?

The group will have about _____ suitcases.

Their luggage will have a mass of about _____ kg.

2. There is 750 mL of paint in a can. Julie has 36 cans. Estimate how much paint Julie has.

Julie has about _____ mL of paint.

3. Fred has been offered \$13 259 for his collection of 38 original cartoon cels. He paid \$128 for each cel. About how much profit would he make if he accepted the offer?

Fred paid about _____ for the 38 cartoon cels.

Fred would make a profit of about _____.

4. Tara is helping to load boxes onto a truck. Each box has a mass of 30 kg. The truck cannot exceed a load of 3000 kg. About how many boxes can be loaded onto the truck?

About _____ boxes can be loaded onto the truck.

PROBLEM-SOLVING STRATEGIES

Use a Formula

On a golf course, a rotating valve sprays water a distance of 12 m in all directions. How large is the area that gets wet? Round to the nearest tenth.

The area of the golf course that gets wet has the shape of a circle.

The formula to find the area is $A = \pi r^2$.

The area that gets wet, to the nearest tenth, is about 452.2 m².

Use the formula for area of a circle: $A = \pi r^2$. Use 3.14 for π , and 12 m for r .

$$A = 3.14 \times 12^2$$

$$A = 3.14 \times 144$$

$$A = 452.16$$

To the nearest tenth, 452.16 is about 452.2.

Solve each problem.

SHOW YOUR WORK

1. Joy wants to put a fence around her rectangular yard. Her yard is 30 m wide and 45 m long. How many metres of fence will Joy need?

Joy's yard is in the shape of a _____.

Joy should use the formula for _____.

Joy needs _____ m of fencing.

2. A hole was dug 11.2 m long, 10.2 m wide, and 3 m deep. How many metres of dirt were removed?

The hole has the shape of a _____.

The formula for _____ should be used.

There were _____ m³ of dirt removed.

3. A picture frame has a length of 18.5 cm and a width of 14.5 cm. What is the perimeter of the picture frame?

The picture frame has a perimeter of _____ cm.

PROBLEM-SOLVING STRATEGIES

Use Logical Reasoning

Aki, Botan, and Chris collect rocks, baseball cards, and postcards, although not necessarily in that order. Aki is the sister of the baseball card collector. Chris once went to the beach with the rock collector and the baseball card collector. What does each person collect?

Aki collects rocks.

Botan collects baseball cards.

Chris collects postcards.

Use a table to keep track of the facts. First, begin by writing 'no' in the table to mark what each person does not collect.

	Rocks	Baseball cards	Postcards
Aki	yes	no	no
Botan	no	yes	no
Chris	no	no	yes

Indicate your conclusions on the table with 'yes'.

Solve each problem.

SHOW YOUR WORK

1. Brian has three solid objects: a cube, a sphere, and a cylinder. The solids are red, green, and blue, although not necessarily in that order. The blue solid is not the cube. The green solid has no edges. What colour is each solid object?

The cube is _____.

The sphere is _____.

The cylinder is _____.

2. Four letters, A, B, C, and D, are each written with a number 1, 2, 3, or 4, although not necessarily in that order. (For example, B3.) The letter A is written with a number greater than 2. The letter B is written with a number less than 2. Neither A nor D is written with an odd number. The letters A–D are written with what numbers?

Letter A is written with the number _____.

Letter B is written with the number _____.

Letter C is written with the number _____.

Letter D is written with the number _____.