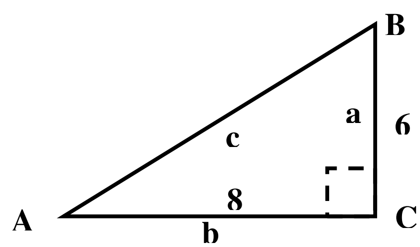
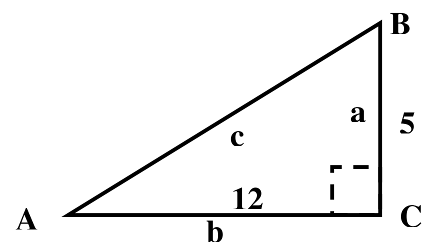
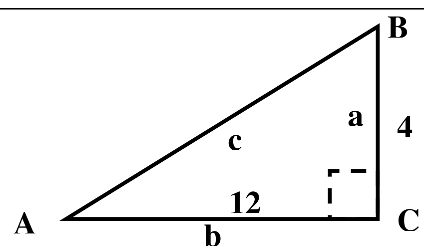
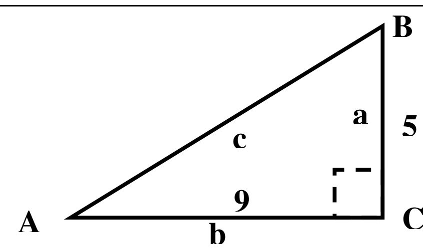
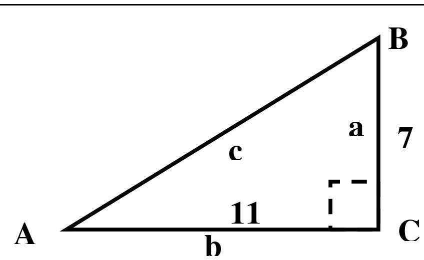
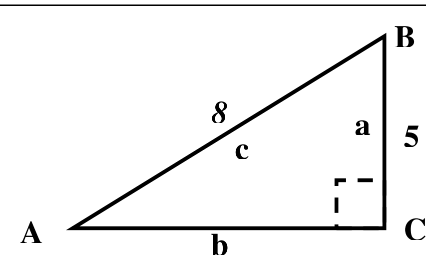
TRIG REVIEW AND EXTRA PRACTICE

Triangles in this review are not necessarily to proper scale or size. Trust what the labels say.

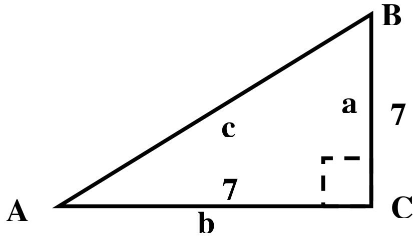
1. **Pythagorean calculations**. Pythagoras discovered that the square of the long side equals the sum of the squares of the other two sides **if and only** if it is a right angle triangle.

Or $c^2 = a^2 + b^2$ if c is considered the long side.

Find the missing side: the answers will not always be nice round whole numbers. Round to one decimal. (or better yet, just leave the answer as an exact answer with a radical)

a.  Length of side c is missing. But $c^2 = a^2 + b^2$ So $\mathbf{c^2 = 8^2 + 6^2}$ So $\mathbf{c^2 = 100}$ So $\mathbf{c} = \sqrt{100} = 10$	b. 
c.  	d. 
e.  	f. 

g.



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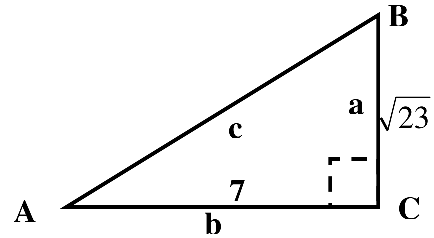

2. Find the sine, cosine and tangent of the indicated angle in these right–angled triangles. (the way the ancient Greeks did, just with a triangle). Try to keep the answers exact!!

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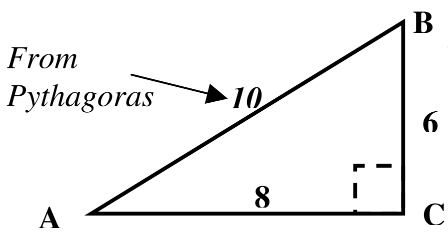
a.



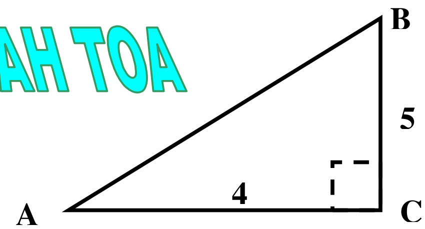
Find side c:

$c^2=8^2+6^2=100$. So **c = 10**

Sin(∠A) = $\frac{Opposite}{Hyptenuse}=\frac{6}{10}=\frac{3}{5}=0.6$

Cos (∠A) = $\frac{Adjacent}{Hyptenuse}=\frac{8}{10}=\frac{4}{5}=0.8$

b.



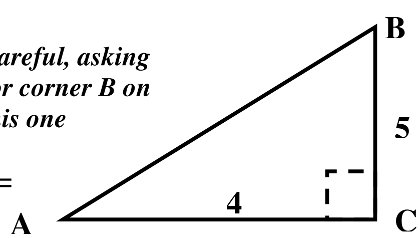
Sin(∠A) =

Cos (∠A) =

Tan (∠A) =

c.

Careful, asking for corner B on this one

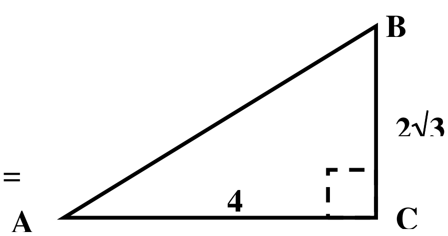


Sin(∠B) =

Cos (∠B) =

Tan (∠B) =

d.

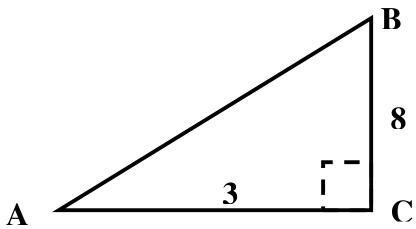


Sin(∠A) =

Cos (∠A) =

Tan (∠A) =

e.

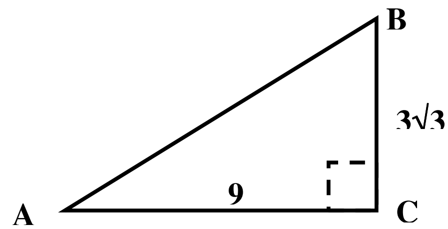


Sin(∠B) =

Cos (∠B) =

Tan (∠B) =

f.



Sin(∠A) =

Cos (∠A) =

Tan (∠A) =

3. Use your calculator to complete the table of values here (fill in the blanks!)Round angles to the nearest whole degree, round numbers to three decimals:

Angle θ	**Ratio**
30°	Sin(30°) = ___
	Sin (θ) = 0.5000
45°	Cos(45°) = ___
	Cos(θ)=0.707
30°	Cos(30°) = ___
	Cos(θ) = .866
75°	Sin(75°)=
22°	Tan(22°)=
15°	Cos(15°)=
32.5°	Sin(32.5°)=
	Cos(θ)=.555
	Tan(θ)=1
	Sin(θ)=.123

Ans: 0.5000

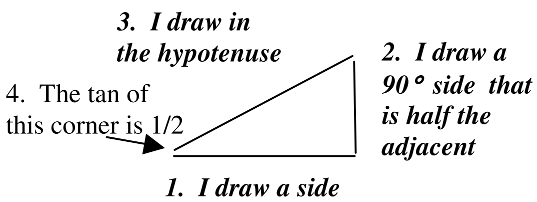
Ans: $\sin^{-1}(0.5) = 30°$

4. Try filling in this table, the white blanks only: (angles to nearest degree, trig ratios to 3 decimals)

Angle θ	**Sine**	**Cos**	**Tan**
24°			
	0.345		
35°			
20°			
		0.866	
			1.000
50°			
130°			

5. Draw a right angle triangle! Given the trigonometric ratio of a corner of a triangle, draw it approximately. Remember the trig ratio just tells you how point a corner is.

a. draw a right-angle triangle whose corner has an opposite side that is one half the length of the adjacent side. (ie: a corner that has a tan of ½, the opposite side is half as long as the adjacent side)



b. draw a right-angle triangle that has a corner angle whose tangent is 1. (that means the opposite side is the same length as the adjacent side)

c. draw a right-angle triangle that has a corner angle whose adjacent side is one third as long as its hypotenuse. (ie: the corner has a cosine of 1/3)

d. draw a right-angle triangle that has a corner angle that has a sine of 0.7. (that is the side opposite from the corner is 7/10ths the length of the hypotenuse)

e. draw a right-angle triangle whose corner angle has a tangent of 2.

f. draw a right-angle triangle whose corner angle has a cosine of 3. (trick question)

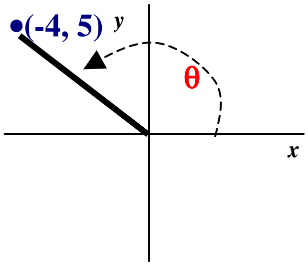
6. In examples 2(a) to 2(f) above you calculated the sine, cosine, and tangent ***ratios*** of a given corner. Now using the exact same diagrams in question 2, calculate the ***actual angle***.

a. $\angle A = \sin^{-1}(0.60) = 36.870°$ ***or***; $\angle A = \cos^{-1}(0.80) = 36.870°$ ***or***; $\angle A = \tan^{-1}(0.75) = 36.870°$ ***There are three ways to get the answer.*** ***Don't forget a sin^{-1} function (ie: an' inverse trigonometric function') is like a sine but just asking the question backwards.***	b.
c.	d.
e.	f.

7. For the terminal arm of a ferris wheel getting to the given point, find the sine, cosine, and tangent of the angles involved. Angles are measured in the ‘standard’ way from the positive x-axis. Remember that unlike the Ancients, we now use the modern trigonometric ratios of:

$$\sin(\theta) = \frac{y-coordinate}{radius};\quad \cos(\theta) = \frac{x-coordinate}{radius};\quad \tan(\theta) = \frac{y-coordinate}{x-coordinate}$$

a.



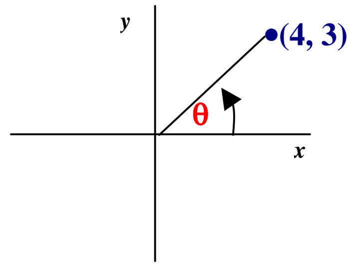
Find the radius first. $r^2=(-4)^2+5^2$
So $r^2 = 16 + 25 = 41$
So $\mathbf{r = \sqrt{41}}$
Ok, now find sin, cos, tan of angle θ

$$\sin(\theta) = \frac{y}{r} = \frac{5}{\sqrt{41}} = \frac{5}{\sqrt{41}} * \frac{\sqrt{41}}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$$

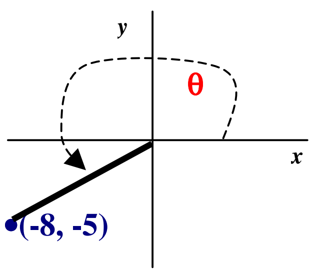
$$\cos(\theta) = \frac{x}{r} = \frac{-4}{\sqrt{41}} = \frac{-4\sqrt{41}}{41}$$

$$\tan(\theta) = \frac{y}{x} = \frac{5}{-4} = -\frac{5}{4}$$

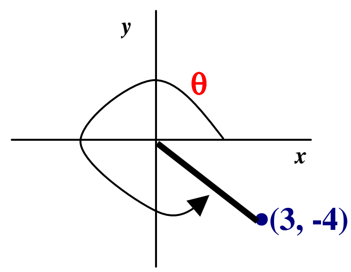
b.



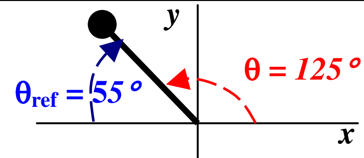
c.



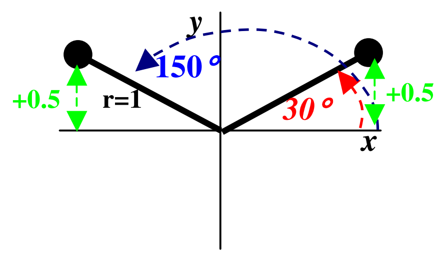
d.



8. **Reference Angles**. Find the reference angle, $\theta_{Ref,}$ for the following angles. Use a sketch if you want.

<table>
<tr><td>a. θ = 125° → θ_{Ref} = 55°</td><td rowspan="7">
Find the nearest x-axis to the terminal arm and measure the acute angle that the triangle there would make</td></tr>
<tr><td>b. θ = 225°</td></tr>
<tr><td>c. θ = 330°</td></tr>
<tr><td>d. θ = 30°</td></tr>
<tr><td>e. θ = –45°</td></tr>
<tr><td>f. θ = 95°</td></tr>
<tr><td>g. θ = 172°</td></tr>
</table>

9. **Solving Trigonometric Equations Analytically**. Solve like normal algebra, but isolate the trig ratio so that you can do the inverse function to find the angle. Also: there are multiple answers for trigonometry equations. A calculator only gives you one answer. You will want to draw a sketch of the problem to find the other angles. *Sometimes there is no solution*!!

<table>
<tr><td>a. Given sinθ = 0.5; solve for θ.
There are an infinite number of angles that are a solution. The calculator only tells you one of them. So assume we are only interested in the ones going around the circle once only.
So the answer is 30 and 150, and 390 and 510, and 750 and 870, and….
So we will normally stick in the domain [0 , 360] for our answers.</td><td>Solution. $\sin^{-1}$(0.5) = 30° according to the calculator. But there is another angle where the y-coordinate on a unit circle is 0.5. It is at 150°.
</td></tr>
<tr><td>b. Given cosθ = 0.5, solve for θ In the domain [0 , 360]</td><td>c. $\frac{4\sin\theta}{3}=1$, solve for θ in the domain [0 , 360]</td></tr>
<tr><td>d. $\frac{1+4\sin\theta}{3}=1$, solve for θ in the domain [0 , 360]</td><td>e. $\frac{4\cos\theta-5}{3}=-2$ solve for θ in the domain [0 , 360]</td></tr>
</table>

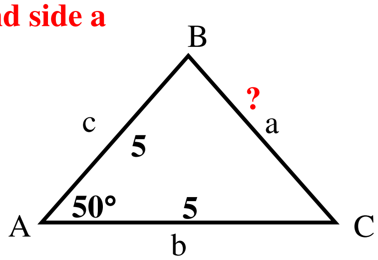
f. $\frac{2\sin\theta}{3} + 1 = -4$ solve for θ in the domain [0 , 360]

g. $5\sin\theta = \sin\theta - 1$

10. **Cosine Law**. Works on ***any*** triangle, not just a right-angle triangle. Used when you know:

- two sides and an included angle; or
- all three sides.
- See your notes or the course website for more complete information

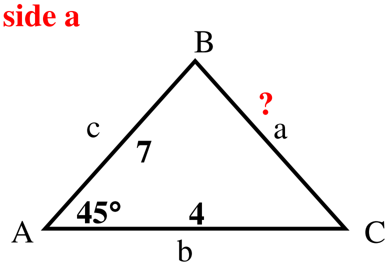
a. **Find side a**



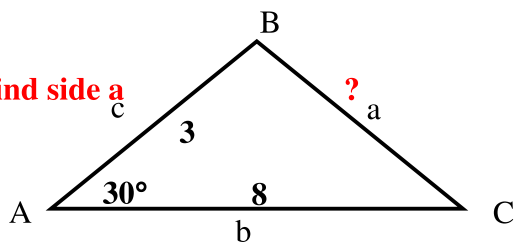
$a^2 = b^2 + c^2 - 2*b*c*\cos(A)$
$a^2 = 5^2 + 5^2 - 2(5)(5)\cos(50°)$
$a^2 = 25 + 25 - 50 * 0.643$
$a^2 = 50 - 32.15$
$a^2 = 17.85$

therefore a = 4.22

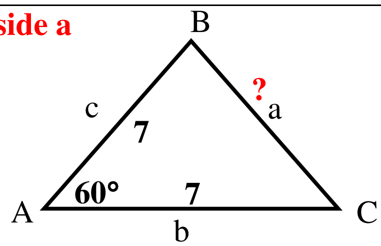
b. **Find side a**



c. **Find side a**



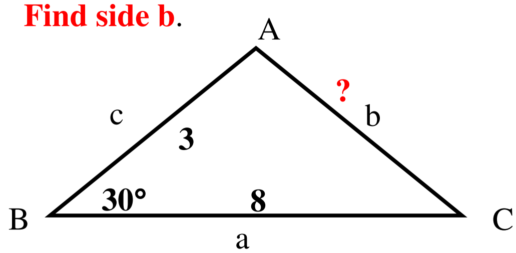
d. **Find side a**



e. **Find side b.**

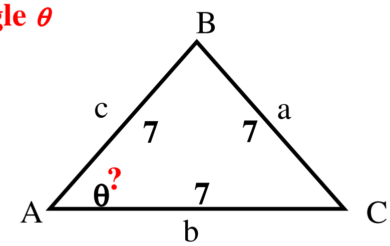
f. **Find angle θ**

e. **Find side b.**



This one is exactly the same as above, but the corners have different names. So the formula you use will look a bit different.

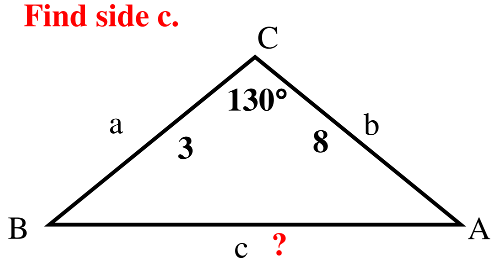
f. **Find angle θ**



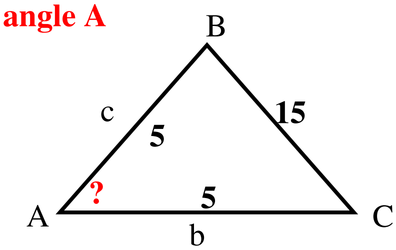
In this one, you need to find angle θ.

11. **More Cosine Law.**

a. **Find side c.**

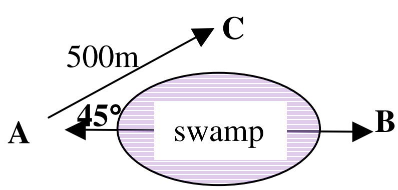


b. **Find angle A**



This one is rather tricky! Why?

c. Kevin needs to get through the bush from A to B (800 meters). But he doesn't want to walk through the swamp. So he goes 45° left to point C first. How far is it from C to B.

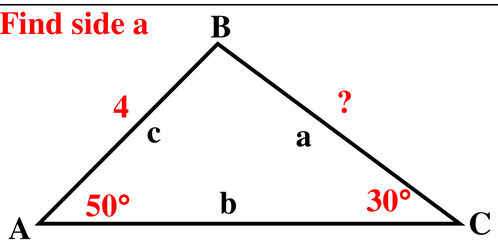


d. You invent your own problem here. Give the solution too of course!

12. **Sine Law – Finding a side**. Find an unknown side if given the angle opposite it and any other side with its opposite angle. Check your notes. But it says that for any given triangle the following ratios are all equal.

$$\frac{a}{SinA} = \frac{b}{SinB} \quad and \quad \frac{a}{SinA} = \frac{c}{SinC} \quad and \quad \frac{b}{SinB} = \frac{c}{SinC}$$

a. **Find side a**



Select a formula that works (where you know everything except one thing):

$\frac{a}{SinA} = \frac{c}{SinC}$ so $\frac{a}{Sin50^\circ} = \frac{4}{Sin30^\circ}$

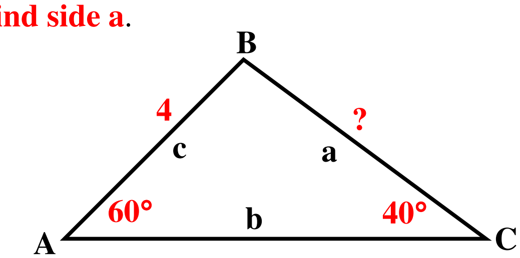
so $\frac{a}{0.766} = \frac{4}{0.500}$ so by cross multiplying:

$a = \frac{0.766 * 4}{0.500} = 6.128$

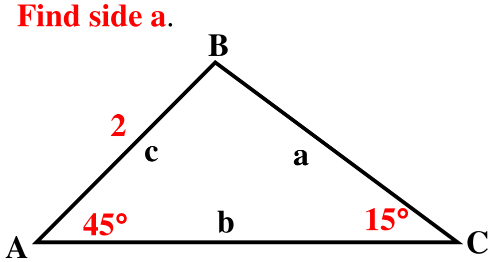
So side 'a' is 6.128 units long.

Which makes sense because the longest side has to be across from the bigger angle.

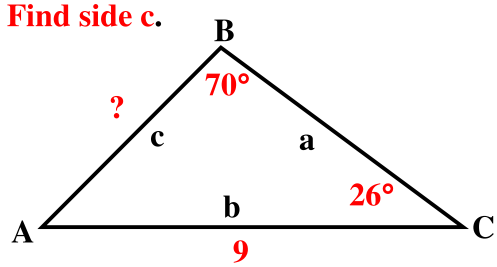
b. **Find side a**.



c. **Find side a**.



d. **Find side c.**



13. **Sine Law – Finding an angle**. Use the sine law to find an angle of any triangle if you are given a side opposite from the angle and any other side with it's opposite angle.

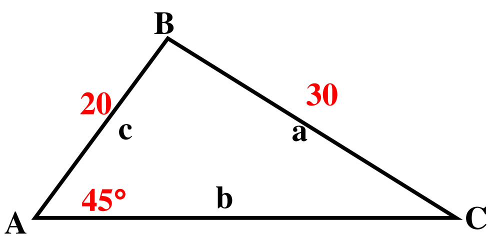
$$\frac{a}{SinA}=\frac{b}{SinB} \quad and \quad \frac{a}{SinA}=\frac{c}{SinC} \quad and \quad \frac{b}{SinB}=\frac{c}{SinC}$$

or sometimes the math is easier if you just write the ratio the other way (reciprocal):

$$\frac{SinA}{a}=\frac{SinB}{b} \quad \text{and} \quad \frac{SinA}{a}=\frac{SinC}{c} \quad \text{and} \quad \frac{SinB}{b}=\frac{SinC}{c}$$

Sine law finding angles practice.

a. **Find angle C**.



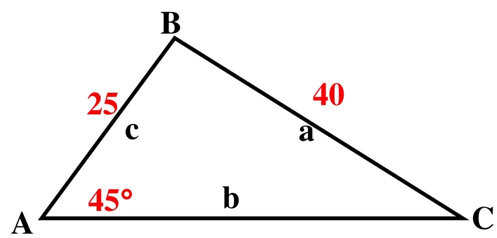
Select a formula so that there is only one unknown: $\frac{SinA}{a}=\frac{SinC}{c}$. Plug in!

$$\frac{Sin45^\circ}{30}=\frac{SinC}{20}$$

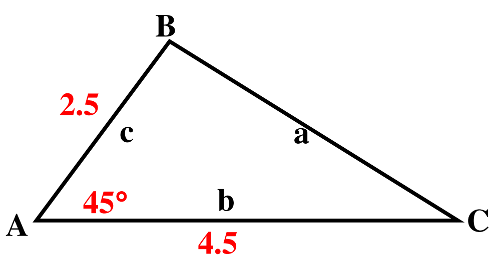
$$SinC=\frac{20*\sin 45^\circ}{30}=0.4714...$$

therefore angle C = $\sin^{-1}(0.4714)$ = 28.13°

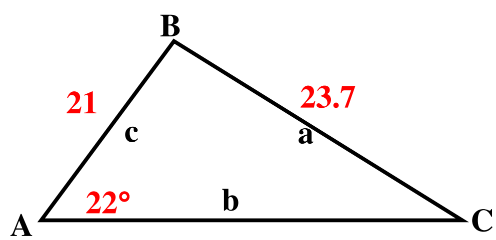
b. **Find angle C**



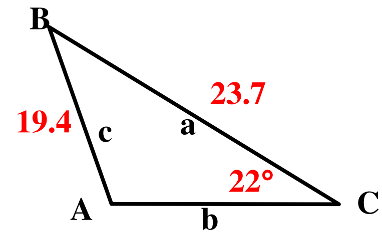
c. **Find Angle B**



d. **Find Angle C**



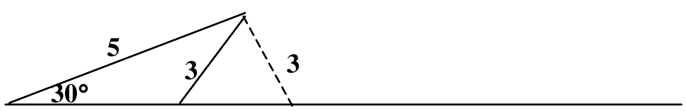
e. **Find Angle A**



There are always two answers when solving for an angle if its side opposite is a long side and the third side is unknown.

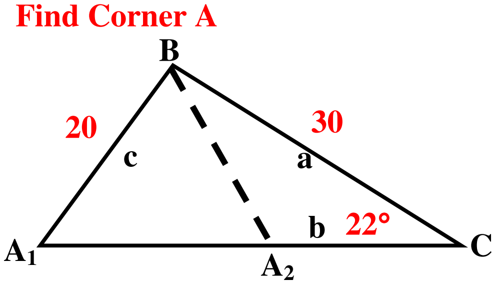
What are the solutions?

14. **Ambiguous triangles**. Ambiguous triangles have two possible situations. If you are asked for the shape of a triangle that is formed by a rod 5 meters long sticking out of the ground at a 30 angle with a 3 metre rod attached to the end of it there are two possible shapes.



Ambiguous cases happen only when you are given two sides and one opposite angle, but the angle given has the shorter side opposite from it.

a. **Find Corner A**



This is an ambiguous case because there is a short side opposite the given angle, arm AB can be placed in two different ways

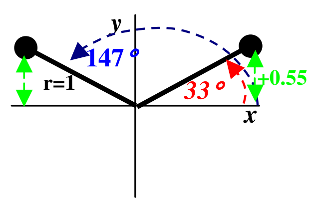
Find the first A, A_1.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \frac{\sin A}{30} = \frac{\sin 22^\circ}{20}$$

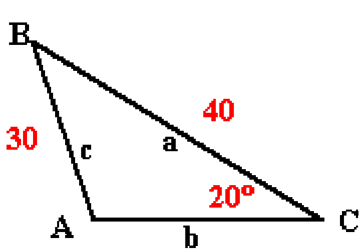
Cross multiply: $\sin A = \frac{\sin 22^\circ}{20} * 30$

$\sin A \cong 0.545$ so what angle has a sine of 0.545?

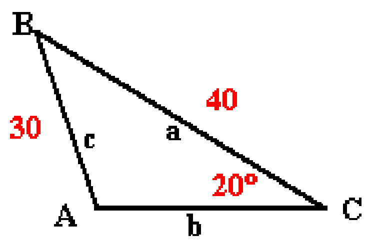
$A = \sin^{-1}(0.545) = 33^\circ$ but don't forget, your calculator only tells you half the answers, there is another angle that has a sine of 0.545 and that is 147°.



b. **Find Angle A**



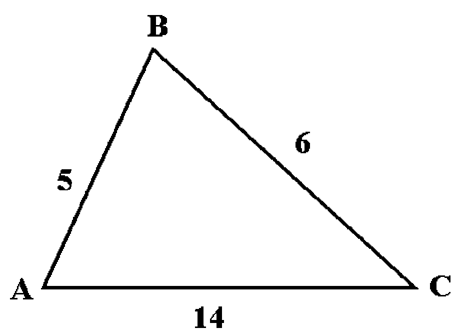
b. **Find Angle A**



Find **both** solutions for angle A above.

15. Impossible Triangles.

Find angle C.



You probably noticed that you get an equation that is impossible to solve. That is because the triangle is impossible. How can one side be longer than both other sides combined?

16. **Triangular Inequality**. The above example of an impossible triangle demonstrates the 'Triangular Inequality'. The Triangle Inequality Theorem states that:

'any side of a triangle is always shorter than the sum of the other two sides'

So for example if there is a triangle with side **a = 4**, **b = 5**, then side **c** must be less than **9**. Draw it to the right. Further, side **c** must be longer than **1**. A pure mathematical statement of this is beyond the scope of these notes.	

ANSWERS

1a. 10	1b. c=13	1c. 12.6	1d. 10.3
1e. 13.0	1f. b= 6.2 Careful with this one, the hypotenuse is given in this one		
1g. c=9.9	1h. $c=\sqrt{72}$ or better yet: $c=6\sqrt{2}$		
2a. sinA = 3/5 or 0.6 sinA = 4/5 or 0.8 tanA = 6/8 or 0.75	2b. sinA = 0.781 or better yet the exact answer $\frac{5\sqrt{41}}{41}$ cosA = 0.625 or better yet: $\frac{4\sqrt{41}}{41}$ tanA = 1.2 or 5/4		
2c. sinB = 0.625 or better yet $\frac{4\sqrt{41}}{41}$ cosB = 0.781 or better yet $\frac{5\sqrt{41}}{41}$ tanB = 4/5 or 0.8		2d. $\sin A=\frac{\sqrt{21}}{7}$ $\cos A=\frac{2\sqrt{7}}{7}$ tanA=√3/2	
2e. $\sin B=\frac{3\sqrt{73}}{73}$ $\cos B=\frac{8\sqrt{73}}{73}$ tanB = 3/8		2f. $\sin A=\frac{1}{2}$ $\cos A=\frac{\sqrt{3}}{2}$ $\tan A=\frac{\sqrt{3}}{3}$	

3.

Angle θ	Ratio
30°	Sin(30°) = ***0.5***
30°	Sin (θ) = 0.5000
45°	Cos(45°) = ***0.707***
45°	Cos(θ)=0.707
30°	Cos(30°) = ***0.866***
30°	Cos(θ) = .866
75°	Sin(75°)= ***0.966***
22°	Tan(22°)= ***0.404***
15°	Cos(15°)= ***0.966***
32.5°	Sin(32.5°)= ***0.537***
56°	Cos(θ)=.555
45°	Tan(θ)=1
7°	Sin(θ)=.123

4.

Angle θ	Sine	Cos	Tan
24°	***0.407***	***0.914***	***0.445***
20°	0.345		
35°	***0.559***	***0.829***	***0.674***
20°	***0.342***	***0.940***	***0.364***
30°		0.866	
45°			1.000
50°	***0.766***	***0.648***	***1.192***
130°	***0.766***	***0.648***	***1.192***

5. answers will vary but they are easy to check with a ruler!

And of course question f was a trick question. How can a triangle have a cosine of 3, that would mean that its adjacent side was three times longer than the hypotenuse, but the hypotenuse is ***by definition*** the longest side, so it is impossible! You can have a side longer than the longest side.

6a. 36.87°	6b. 50.19° no matter how you do it	6c. ∠B = $\sin^{-1}(4/\sqrt{41})$ = 38.66° ∠B = $\cos^{-1}(5/\sqrt{41})$ = 38.66° or ∠B = $\tan^{-1}(4/5)$ = 38.66°
6d.	6e.	6f.

7. already done	7b. $r^2 = 4^2 + 3^2 \therefore r = 5$ $\sin(\theta) = \frac{y}{r} = \frac{3}{5}$ $\cos(\theta) = \frac{x}{r} = \frac{4}{5}$ $\tan(\theta) = \frac{y}{x} = \frac{3}{4}$
7c. $\sin\theta = \frac{-5\sqrt{89}}{89}$ $\cos\theta = \frac{-8\sqrt{89}}{89}$ $\tan\theta = \frac{5}{8}$	7d. $\sin\theta = \frac{-4}{5}$ $\cos\theta = \frac{3}{5}$ $\tan\theta = \frac{-4}{3}$
8. 55, 45, 30, 30, 45, 85, 8	
9b. 60° or 120°	9c. 48.59° or 131.41°
9d. 30° or 150°	9e. 104.48° or 255.52°
9f. Nil Solution	9g. 194.47° or 345.52°
10a. 4.22	10b. 5.04
10c. 5.61	10d. 7 exactly
10e. 5.61	10f. 60°
11a.	11b. Impossible triangle
11c. 569 metres	11d.

12a. 6.128	12b. 5.39	12c. 5.46	12d. 4.20
13a. 28.13°	13b. 26.23°	13.c 78.26° Need to find side a first	13d. 19.39°
13e. 27.23° or 152.77°			
14b. 27.13° or 152.87°			