Geometry Review

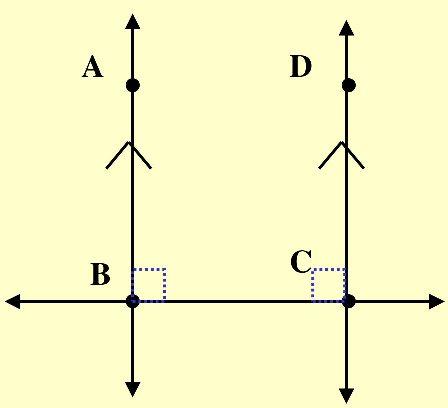
Geometry is the original mathematics!

Way before x^2 and y^3s and all that algebra stuff!

You will find that geometry has many definitions and exact vocabulary! Knowing the meaning of words is the beginning of wisdom (*Confucius*)

Guide your self through this review. Select the small slideshow animation icon to animate the review. You can advance quickly in the window by hitting the space bar. You can advance to the next slide by hitting the arrows in the top right corner.

Lines



A *line* runs through two points; it is endless. Lines are drawn with arrowheads on the ends.

We name this line AB, or we can use symbol $\overleftrightarrow{AB}$

A *perpendicular* line is one that makes a 90° angle with another line.

Line BC is *perpendicular* to Line AB. We show that with a small square.

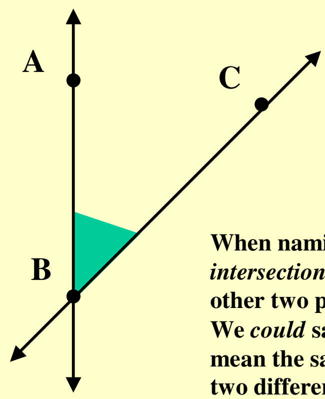
Line CD is drawn *perpendicular* to Line BC.

Lines AB and CD go in the exact same direction, they are *parallel*.

We show that two lines are parallel by giving them similar *chevrons*.

A line that crosses *any* two lines is called a *transversal*. So line BC is called a *transversal*.

Labelling Angles 1



The point where two lines *intersect,* or *cross,* is called the *vertex* of an angle.

The inside corner vertex at point B formed by lines AB and BC is called *angle ABC*. We use the symbol ∠ABC to say '*angle'* ABC.

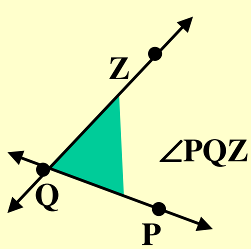
When naming an angle we always put the *point of intersection* (vertex) in the *centre* of the label and the other two points on either side but in alphabetical order. We *could* say ∠CBA , which folks would understand to mean the same as ∠ABC , but then it looks like there are two different names for the same angle. So put the outside points in alphabetical order.

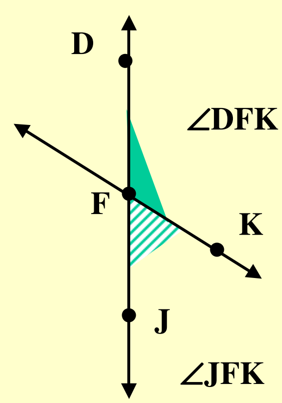
∠A**B**C

The centre letter is always the vertex point

Label Angle - Practice

Name the shaded angle!

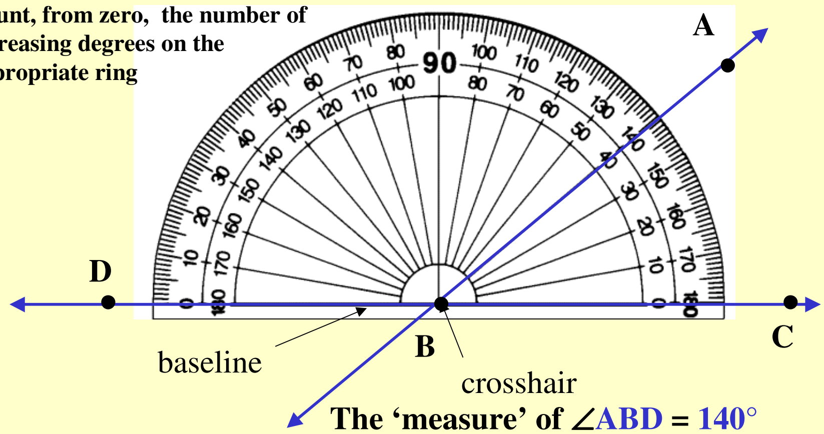




Measure angles with a protractor

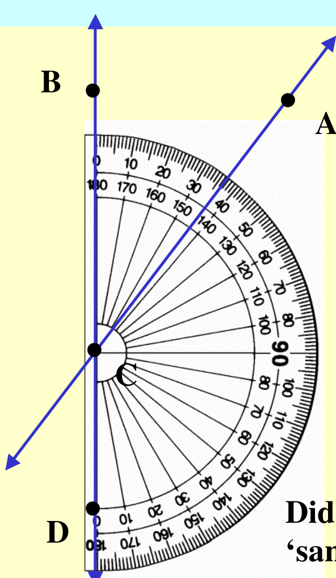
Put the *cross hair* of the protractor on the *vertex* of the angle with one of the lines along the *baseline* of the protractor. Count, from zero, the number of increasing degrees on the appropriate ring

The 'measure' of ∠ABC = 40°
We also say: m ∠ABC = 40°



The 'measure' of ∠ABD = 140°
We also say: m ∠ABD = 140°

Measure Angles Practice



Q: Find the measure of angle ACB (find m∠ACB) : ______

Ans: 37°

Q: Find the measure of angle ACD (find m∠ACD) : ______

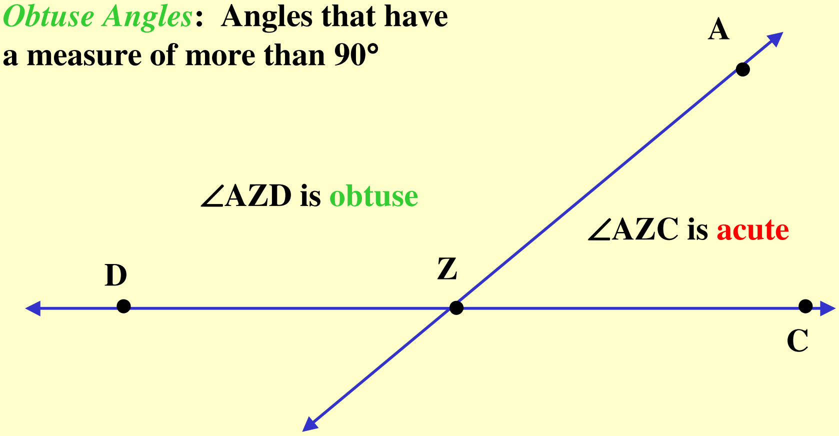
Ans: 143°

Did you notice that the angles on the 'same side' of an intersection of lines always seems to add up to 180?

Obtuse and Acute Angles

***Acute angles*: Angles that have a measure of less than 90°**

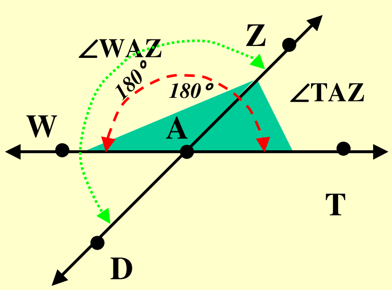
***Obtuse Angles*: Angles that have a measure of more than 90°**



∠AZD is obtuse

∠AZC is acute

Supplementary Angles Law



Supplementary Angles Law:
The two 'adjacent' angles formed by two intersecting lines on the same side of either line are *supplementary*; they add up to 180°

m∠TAZ + m∠WAZ = 180°

m∠WAZ + m∠DAW = 180°

Adjacent means: 'next to'

m∠DAT + m∠DAW = 180°

m∠DAT + m∠TAZ = 180°

The actual 'law' is not really expressed this way, but this is close. In books you might actually see this called a 'postulate', not a 'law'. A 'postulate' is like an idea that we have never found to be 'untrue'.

Supplementary Law Practice

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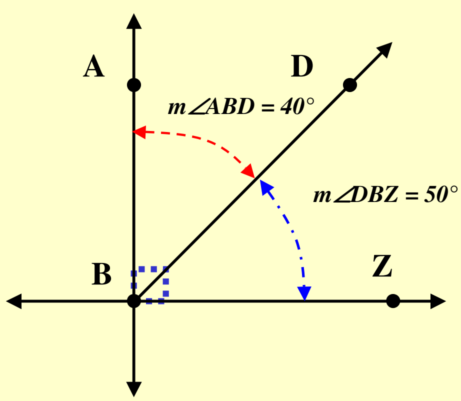
Complementary Angles



Complementary Angles

1. Two angles are said to be *complementary* if they add up to 90°.

2. ∠ABD is *complementary* to ∠DBZ because they add up to make 90°.

3. m∠ABD + m∠DBZ = 90°

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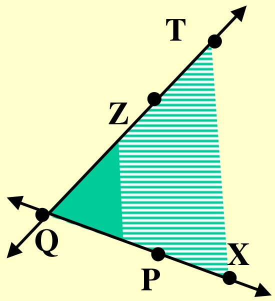
Congruent Angles

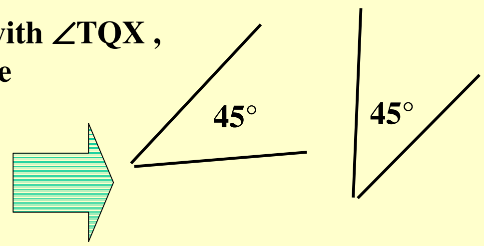


1. **Two angles are said to be *congruent* if they have the same angular measure.**

2. **The words *congruent* and equals are sort of the same, but *congruent* applies more for shapes. The symbol for congruence is: ≅**

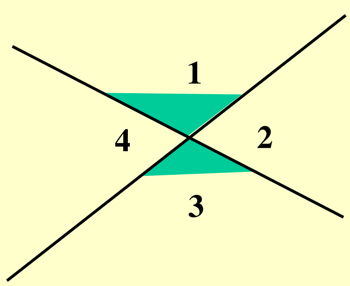
3. **∠PQZ is *congruent* with ∠TQX , or ∠PQZ ≅ ∠TQX, since m ∠PQZ = m ∠TQX**

4. **These two angles are *congruent***



Vertical Angles

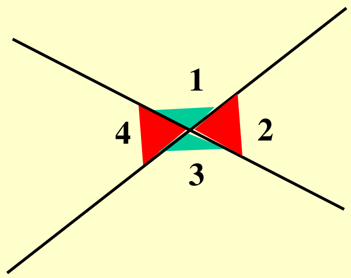
VERTICAL ANGLES are two *nonadjacent* (ie: not next to each other and not sharing a common side) angles formed by two intersecting lines. They are often called Opposite Angles instead.



1. **∠1 and ∠3 are *Vertical Angles*.**

2. **∠2 and ∠4 are *Vertical Angles*.**

Vertical Angles are Congruent



1. Angles 1 and 3 are congruent. ie: ∠1 ≅ ∠3

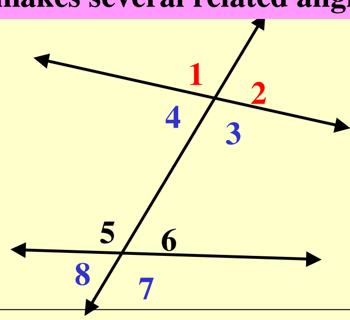
2. Angles 2 and 4 are congruent.
ie: ∠2 ≅ ∠4
Or: m ∠2 = m ∠4

Why? Well ∠2 is the supplement of ∠1 and ∠3 is the supplement of ∠2 . Since m∠2 + m∠1 = 180° and m∠2 + m∠3 = 180°; then m∠1 must equal m∠3.

EG: If Kevin's age plus Marc's age = 18, and If Kevin's age plus Charmaine's age = 18; then Marc and Charmaine must be the same age!

Transversals and Angles

A transversal cuts two lines and makes several related angles



Exterior Angles:
∠1, ∠2, ∠7, ∠8

Interior Angles:
∠3, ∠4, ∠5, ∠6

'Same Side' Interior Angles:
∠3 and ∠ 6, ∠4 and ∠5

Alternate Exterior Angles:
∠1 and ∠ 7, ∠2 and ∠8

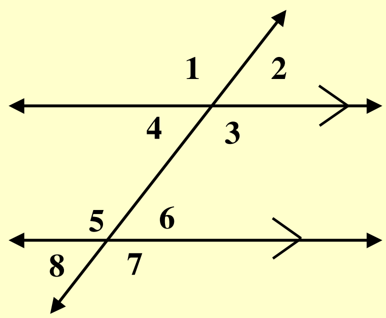
Alternate Interior Angles:
∠3 and ∠ 5, ∠4 and ∠6

'Alternate' indicates "other side of the cutting line".

Corresponding Angles (angles that match up after the 'cut'):
∠1 and ∠ 5, ∠4 and ∠8, ∠2 and ∠ 6, ∠3 and ∠7

Transversals and Parallel Lines

When any two lines are crossed by another line (which is called the transversal), the angles in matching corners are called *corresponding angles*. Eg: $\angle 2$ and $\angle 6$.



Corresponding angles Postulate: If two *parallel* lines are cut by a transversal, then each pair of *corresponding angles* is congruent.

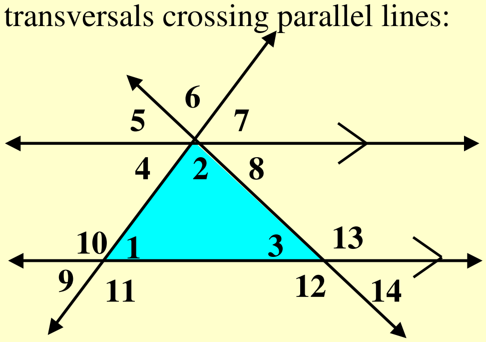
$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$,

$\angle 3 \cong \angle 7$, etc for corresponding angles of parallel lines cut by a transversal.

Transversal and Parallel Lines Practice

180° in a Triangle

If we think of a triangle being formed by two transversals crossing parallel lines:



We know:
$\angle 1 \cong \angle 9 \cong \angle 4$

And we know:
$\angle 3 \cong \angle 14 \cong \angle 8$

But:
$m\angle 2 + m\angle 4 + m\angle 8 = 180°$

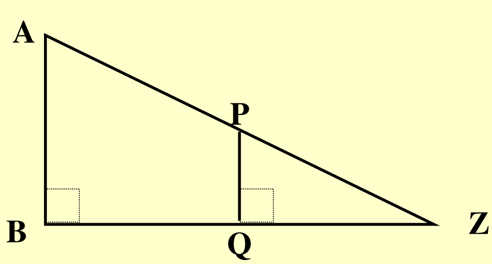
So:
$m\angle 2 + m\angle 1 + m\angle 3 = 180°$

So the inside corner angles of a triangle always add up to 180°

Similar Triangles

The study of similar triangles is important before the study of trigonometry. If you understand triangles you understand every shape!

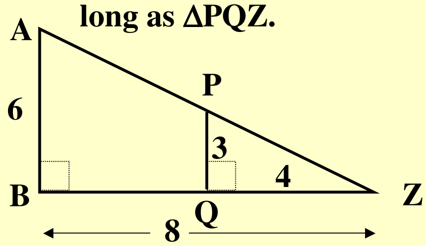
***Similar triangles* are triangles that have the same shape, but not necessarily the same size.**



We say that triangle ABZ (ΔABZ) is *similar* to ΔPQZ because they are the same shape. ΔABZ has sides that are each just twice as long as ΔPQZ and they both have the congruent corner angles.

Similar Triangles 1

Examine the two triangles. ΔABZ is 'similar' to ΔPQZ . Except ΔABZ just has all its sides twice as long as ΔPQZ.

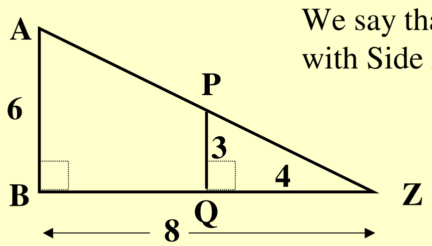


They have the same proportionate hypotenuses too. ΔPQZ has a hypotenuse of 5, ΔABZ has a hypotenuse of 10 (from Pythagoras of course)

You also know see that ∠PZQ is the same as ∠AZB

And if you think of line AZ as just being a transversal cutting the two parallel lines PQ and AB, then angles ∠BAP and ∠QPZ must be congruent too!

Similar Triangles – Corresponding Parts



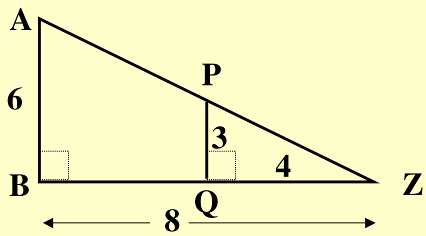
We say that side AB corresponds with Side AZ

We say that side QZ corresponds with Side BZ

We say that side PZ corresponds with Side AZ

Notice that all corresponding lengths are in the same ratio; 2:1. Each of the big triangle sides are all just twice the small triangles sides.

Similar triangle formula

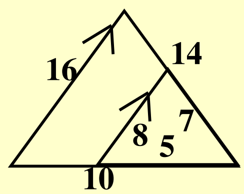


$$\frac{BZ}{QZ} = \frac{AB}{PQ} = \frac{AZ}{PZ}$$

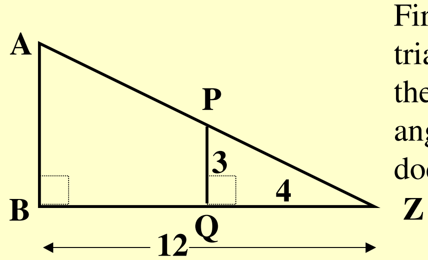
In this case:

$$\frac{8}{4} = \frac{6}{3} = \frac{10}{5} = 2$$

The relationship doesn't require just right angle triangles either, it works for all similar triangles.



Similar Triangle Problem 1



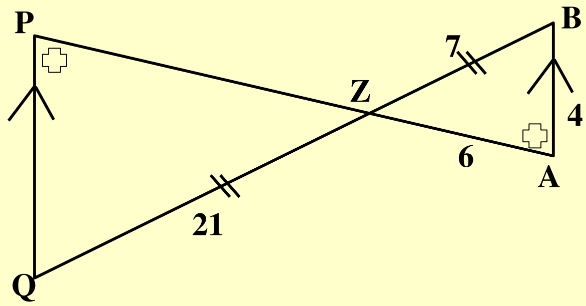
First you must ensure that the triangles are similar, that is: they have the same three corner angles. We know this one does.

Find side AB: $\frac{BZ}{QZ} = \frac{12}{4} = 3$

Therefore: $\frac{AB}{PQ} = 3 = \frac{?}{3} = \frac{9}{3}$

So length AB = 9; it has to be 3 times as long as its corresponding smaller sister

Similar Triangle Problem 2



We know they are similar triangles because: ∠AZB≅ ∠PZQ. And since lines AB and PQ are shown as parallel we know from the rules of transversals and parallel lines that ∠BAZ≅ ∠QPZ and that ∠ABZ≅ ∠PQZ . They are corresponding angles. So lines BZ and QZ are corresponding sides as are AZ and PZ.

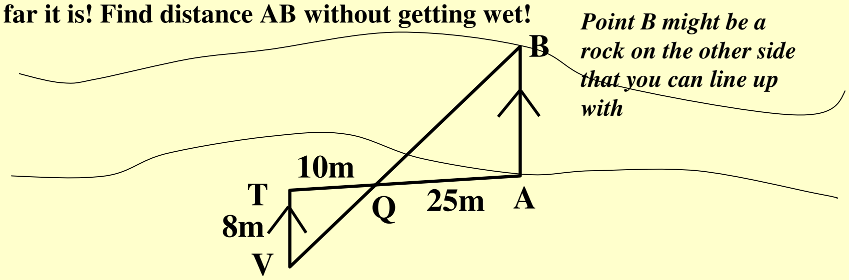
$$\frac{QZ}{BZ} = \frac{21}{7} = 3 \; so \; \frac{PZ}{AZ} = \frac{PZ}{6} = 3$$

So side PZ is 18 long

Similar Triangle Problem 2

You want to swim the river from Point A to Point B, but not sure how far it is! Find distance AB without getting wet!

Point B might be a rock on the other side that you can line up with

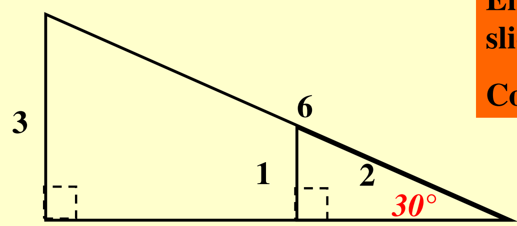


Solution: Put 3 stakes in the ground in a straight line at points T, Q and A. Put another stake in the ground at position V so that it lines up with Q and B and so that TV is running the same direction as AB (maybe straight North if you have a compass or maybe just make corners A and T 90° corners). You have made two similar triangles!

$$\frac{AQ}{TQ} = \frac{AB}{TV}, \; so \; \frac{25}{10} = \frac{AB}{8} \qquad So \;\; AB = \frac{8*25}{10} = 20$$

Trigonometry

When we study ***trigonometry*** we will still be comparing sides of triangles. **For example**: it turns out that every right angle triangle that has a *hypotenuse that is twice as long as one of its other sides* has one corner that is exactly 30°. So of course that means the other angle is 60°. But we will save those simple ideas for another unit.



End of the slideshow

Congratulations