GRADE 12 ESSENTIAL
UNIT I - PROBABILITY
EXPECTED VALUE

Name:_____
Date: _____

1. The expected value of some process involving probability and risk and reward is given by the formula:

EV = P(Win) * $NetGain – P (Loss) * $Loss

Where **EV** means the **E**xpected mean **V**alue that one can expect, on average if you play often enough, for each play (trial) of some game (experiment).

In the formula **P(Win)** is the probability of a Win, **P(Loss)** is Probability of *not win*, so $P(\overline{Win})$ or 1 – P(Win) identically.
(Notice this formula does not readily allow for ties, you could adapt one though)

In the formula '**$NetGain'** is the net amount gained with a Win (the prize less any 'stake' or bet that it may cost to play, bearing in mind some games you do not always lose your stake)[In roulette you do not lose your stake if you win]

$Loss is the amount wagered (bet), cost to play each play.

2. The formula can be adjusted for more than just one winning event, for example in a card game perhaps a win is a face card giving one $Gain **and** a 'seven' giving a different $Gain. This involves **compound** probabilities.

3. An example: Say you pay $1 to have a 20% chance of winning $4. Can you see that you can expect to lose money after a few plays? Can you expect to walk away a winner? How much do you expect to win or lose if you play 20 times?

Remember, probability does not tell you what will happen on any particular play (trial), but it will tell you what will happen on many plays. Probability does not predict the outcome of any single play, but it does for many!

An Expected Value is just a 'mean outcome' if you like to think of it that way and you recall your statistics.

3. The expected value can be better visualized by using a simple table to express the related values. So, you *pay* $1 to have a 20% chance of *winning* a prize of $4:

Event	Prob	Amount Won	Cost to play	Net Gain or Loss [Payoff]	Probability * Payoff	=
Win	1/5	4	1	4 – 1 = 3	1/5 *3 =	**3/5**
Loss	4/5	0	1	1	4/5 * 1=	**4/5**
				EV→	Difference: 3/5 – 4/5 =	**⁻1/5 or** ⁻$0.20

Recall that 'Net', as in 'Net Income', means the final result after considering all the contributing values.

So you *expect* to *lose* 20 cents, *on average*, each play, over many plays.
After 50 plays you will lose a total of : _$_____ .

4. You are aware that tables are a great way to organize your methodical calculations but formulae may be easier. [Easier is not better!]

Try the example above with the formula:

EV = P(Win) * $NetGain – P (Loss) * $Loss

Let us do it using decimals for those who detest fractions

EV = P(Win) * $NetGain – P (Loss) * $Loss
EV = 0.2 * (4 – 1) – 0.8 * 1 = ⁻ 0.2;

notice the result is negative (**–**). A negative value means you can expect to **lose** $0.20 every play on average.

Which of course means the person running the game can expect to win from you +$0.20 per game. **Your loss is their gain!**

Problems to work

Show work!

5. **Game 1**. A certain game involves rolling a single die. It costs $1 each play (to roll the die). If it comes up five or more you win $3.

a. Determine the expected value of this game.
b. Calculate how much can you expect to win (or lose) each play.
c. If you play 50 times, determine how much will you win (or lose).

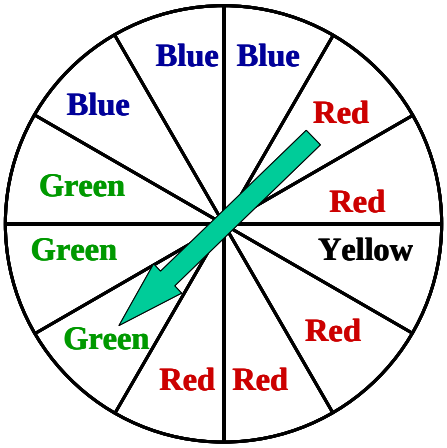
Event	Prob	Amount Won	Cost to play	Net Gain [Payoff]	Probability times Payoff	
Win						
Loss						
					Difference: (win – loss)	

Using the EV Formula:

6. **Game 2**. A certain game involves drawing a card from a standard deck of 52 cards. It costs $5 each play. If you draw a face card (*Jack to King*) you win $20.

a. Determine how much you can expect to win or lose each play. By calculating the player's expected value (EV)
b. If you play this game 30 times, calculate how much can you *expect* to lose total.

7. **Game 3**. A certain game involves a spinner at the right. It costs **$2.00** to play. We assume the spinner is **fair**! (ie: It gives random outcomes). All the sectors are the same size also; so each is equally likely. You **Win $5** if the spinner comes up **BLUE**. You play the game 60 times.



a. how many times *should* it come up **BLUE**? _____

b. How much will you have spent for those 60 plays? _____.

c. How much will you have *likely* won in prizes on those 60 plays? _____

d. On those 60 plays, determine the **difference** of what you spent and the prizes you won: _____
[ie: total spent – total prizes won]

e. Calculate the Expected Value (EV) of playing **BLUE:**
_____

f. Using the EV, calculate how much you can expect to win or lose total after the 60 plays.
_____

8. **Game 4**. Now re-calculate the game 3 above if you pay **$2** per play and you win a prize of won $6 if it comes up **RED.**

a. determine the EV.
b. calculate if you can expect to be a net winner or loser and by how much after 60 plays.

7. **Game 5**. Here is a fun (?) game! You bet (wager) a dollar, $1. A single play is you roll a normal [fair] six-sided die. If the die comes up a five you win $1.00 and you keep your wager. If it comes up a six you win $2 and you keep your wager. If neither a five or a six is rolled you lose your $1 wager.

a. determine the expected value (EV) for a play

b. determine how much you should expect to win or lose after playing 40 times.

*you see games like this at the Red River Ex (crown and anchors) or roulette, games where you keep your wager if you win.

8. Not all risk and profit calculations involve gambling. You risk lots of things!

You risk ruining your $300 suede coat to look cute despite there is a chance of rain. You risk your safety running through traffic catch your bus and to save eight minutes. What do you gain, what do you lose? What is the chance you will gain and what is the chance you will lose? You gamble all the time!

Try this example of risk management and benefit analysis if you own a construction company.

Project Management. You run an engineering firm that builds custom-metal storage sheds. You have seen a request for proposal (RFP) from Underhill College to build a large shed for storing equipment. Several companies are likely to bid on the project. It will cost you (your engineering form) $3,000 to design the custom-made shed (correspondence, drawings, travel, meetings, testing, licence applications, etc). Your net profit (ie: net gain, what you earn compared to what you spend) for the project will be $20,000. There is a one in ten chance you will be selected to do the job. Should you bid on the job? (Show the EV calculation)