Algebra

This chapter was written by Gene Schaul (Math & Science Instructor, SWTC)
Section 11 was written by John Pluemer (Math & Science Instructor, SWTC)

INTRODUCING SIGNED NUMBERS

Signed numbers are used every day. You use them when you talk about temperature (20 degrees below zero is -20 while 72 degrees above zero is either +72 or just 72 and you use them when you talk about money (+ $150.00 means that you made $150.00 while - $150.00 means that you lost $150.00).

Positive numbers: 13, $2.95, 6.125", +12.65, +87, 3 3/4

Negative numbers: -15, - $13.85, -4.785", - 2 1/2

Zero: is neither positive nor negative and can be written as 0, +0, or -0

Positive numbers can be written either with a positive sign (+) or without any sign. Negative numbers must be written with a negative sign (-).

Positive 63 is written as +63 or just 63. Negative 63 is written as -63.

Practice Set 1 Write each of the following as a signed number.

1) negative 23.45 = _____ 2) positive 19.75 = _______

3) positive 16 3/4 = _______ 4) negative 95% = _______ (write as a decimal)

5) 42 degrees below zero = _______ 6) 32 degrees above zero = _______

7) a net gain of $146.34 = _______ 8) a net loss of $23.95 = _______

9) 450 pounds more = _______ 10) 275 pounds less = _______

Practice Set 2 Identify each of the following as either positive, negative, or neither.

1) -12 = _________ 2) +68 = _______

3) -1.53 = _________ 4) 0 = _______

5) 4 2/3 = _________ 6) 123 = _______

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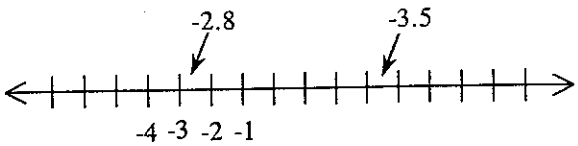
Section 1: Introducing Signed Numbers

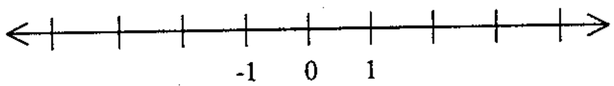
THE NUMBER LINE

A number line is used to picture all the positive and negative numbers. Positive numbers are written to right of zero while negative numbers are written to the left of zero. Whenever you look at a number line the smallest number is on the far left and the largest number is on the far right.

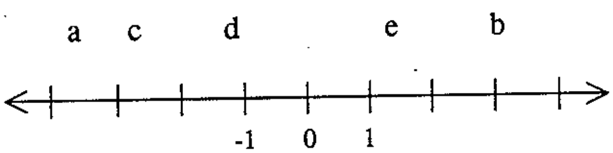


Whenever you compare two numbers on a number line, the larger number is always to the <u>right</u>.

<u>Practice Set 3</u> Locate the following numbers on the number line: -3, 2, 3.25, -4.75, -1 1/2, 3/4



<u>Practice Set 4</u>



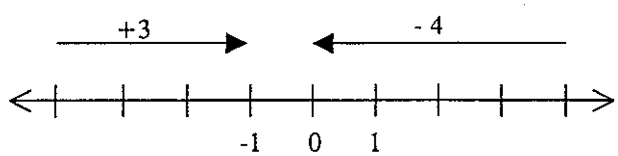
On the number line, a, b, c, d, and e represent numbers.

1) Comparing a and e, which is larger? ___ 2) Why? ____________________

3) Comparing c and b, which is larger? ___ 4) Why? ____________________

NUMBER ARROWS

Positive numbers can be pictured as arrows which point to the right and negative numbers can be pictured by arrows which point to the left. The length of the arrow indicates the size of the number. Number arrows can start anywhere on the number line.



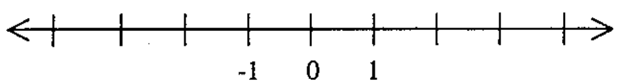
The number arrow on the left represents the number +3 because the arrow is 3 units long and it points to the right. It starts at -5 on the number line and ends at -2.

The number arrow on the right represents the number -4 because the arrow is 4 units long and it points to the left. It starts at +6 on the number line and ends at +2.

Practice Set 5 Draw a number arrow for each of the following:

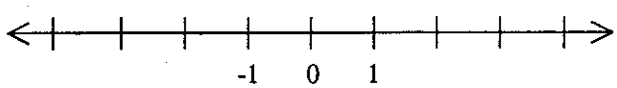
1) positive 2, starting at -4

2) positive 3, starting at 0



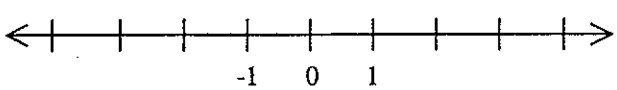
3) negative 5, starting at 3

4) negative 2, starting at - 3



5) negative 6, starting at 0

6) positive 7, starting at -3.5



SECTION 2

ADDING SIGNED NUMBERS

In mathematics, the same symbol can have more than one meaning.

In the expression 6 + 3, the + means 'add the six and the three', while in the expression + 8, the + means that the eight is a positive number.

In the expression 9 - 5, the - means 'subtract the five from the nine', while in the expression - 2, the - means that the two is a negative number.

Parentheses can be used to make sure that the expressions are easy to understand.

For example:
(+6) + (+9) -12 + (-6) 2 - (+12) -4 - (-7)

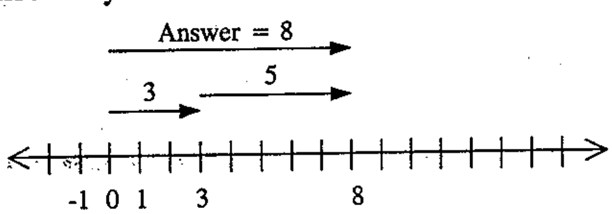
There are three rules for adding signed numbers. Number arrows can be used to help understand and remember these rules.

ADDITION RULE #1: When adding two or more numbers with the same sign, ignore the signs, add the numbers, and give the answer the sign of the original numbers.

EXAMPLE: Add +3 and +5

Solution: Add the numbers without the signs. (3 + 5 = 8)
Give the answer the sign of the original numbers: Answer = + 8

Number line answer:
Start at zero.
Draw the first number arrow.
Draw the second number arrow beginning at the tip of the first number arrow.
Draw a number arrow starting at zero and ending at the end of the second number arrow; this number arrow is your answer.



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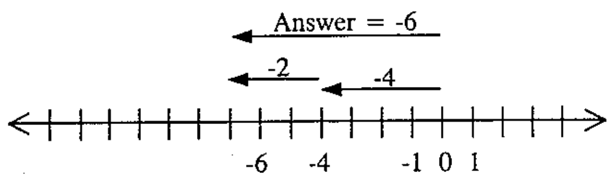
Section 2: Adding Signed Numbers

EXAMPLE: Add -4 and -2.

Solution: Add the numbers without the signs. (4 + 2 = 6)
Give the answer the sign of the original numbers: Answer = - 6

Number line answer:

Draw the first number arrow starting at zero.
Draw the second number arrow starting at the tip of the first number arrow.
Draw a number arrow starting at zero and ending at the tip of the second number arrow; this number arrow is your answer.



EXAMPLES:

7	+ 19	- 5	+ 20	- 101	16
+ 6	+ 23	- 17	+ 6	- 4	4
13	42	- 22	26	- 105	20

-6 + (-4) + (-2) = -12 12 + (+4) + (+2) = + 18 (-3) + (-4) = -7

<u>Practice Set 6</u> Add the following positive numbers:

1.) 9+8+7 = 2.) 12+7+3 = 3.) 12.35+8.65+10.90 =

4.) 5 1/2 + 2 3/4 = 5.) (+12)+(+9)+(+21) =

6.) (+6.234)+(+12.45)+(+8.185) =

Add the following negative numbers:

7.) -4 + (-2) = 8.) -1.25 + (-0.50) = 9.) -1200 + (-876) =

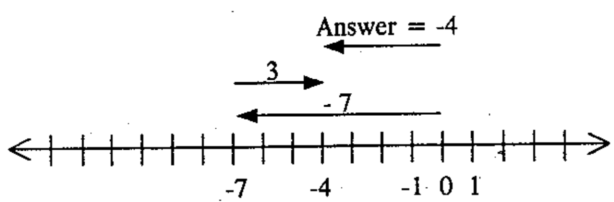
10.) -4 1/2 + (-1 1/4) =

ADDITION RULE #2: When adding two numbers which have different signs, ignore the signs, subtract the "smaller" number from the "larger", and give to the answer the sign of the "larger" number.

EXAMPLE: Add -7 and 3.

Because every positive number is larger than every negative number, 3 is a larger number than -7. However, when adding numbers with different signs, pretend that the numbers do not have signs (so the -7 becomes a "larger" number of 7 and the 3 becomes a "smaller" number of 3), subtract the "smaller" number from the "larger" (7 - 3 = 4), and give the answer the sign of the larger. The answer is -4.

The number arrow drawing for the previous problem is:



EXAMPLE: Add 5 and -19.

Because 19 is larger than 5 and -19 is negative, the answer will be negative. Do the subtracting: 19 - 5 = 14. So, the answer is -14.

EXAMPLE: Add 14 and -3.

Because 14 is larger than 3 and 14 is positive, the answer will be positive. Do the subtracting: 14 - 3 = 11. So, the answer is 11.

Practice Set 7 Add the following numbers:

1) 14 + (-21) =
2) -4 + 12 =
3) 9 + (-11) =
4) -18.765 + 4.982 =
5.) 12,300 + (-567) =
6) 12.75 + (-14.9)
7) 12 + (-8) =
8) -19 + (12) =
9) 8 + (-21) =

continued...

10) -9 + 21 = 11) -27 + 27 = 12) -19 + 34 =

13) -12 + 7 = 14) 3 2/3 + (-4 1/2) = 15) - 6 1/2 + 4 3/4 =

ADDITION RULE #3: When adding several signed numbers, add the positive numbers together, add the negative numbers together, then add the positive number result to the negative number result.

EXAMPLE: Add: -14, 6, 18, -7, -21, and 4

Solution: Add: 6 + 18 + 4 = 28 Add: -14 + (-7) + (-21) = -42

Add the results together: 28 + (-42) = -14 Answer: **-14**

<u>**Practice Set 8**</u> Find the sum for each of the following problems:

1) 21 + (-9) + (-13) = 2) -12 + (-102) + 86 =

3) $14.75 + 8.55 + (-24.60) = 4) (-1 7/8) + (-12 5/8) + 5 1/4

5) -12.75 + (-8.12) + 36.24 + (-0.76) + (-76.34) =

6) 18 + (-4) + (-22) + 12 + (-6) = 7) (+2) + (-17) + (-8) + (+12) =

8) (-12) + 17 + 42 + (-47) = 9) 13 + (-3.75) + 21 + (-81.25) =

10) $ 12.95 + $ 4.79 + - $ 21.62 + $ 18.97 + (- $ 42.12) + $ 21.45 =

SUBTRACTING SIGNED NUMBERS

SUBTRACTION RULE: To subtract signed numbers, change the number being subtracted, and change the problem into an addition problem.

EXAMPLES:

1) Subtract + 7 from + 12 ⟶ +12 - (+7) = +12 + (-7) = **+5** or **5**

2) Subtract + 18 from + 6 ⟶ +6 - (+18) = +6 + (-18) = **-12**

3) Subtract - 5 from + 11 ⟶ +11 - (-5) = +11+(+5) = **+16** or **16**

4) Subtract - 12 from - 6 ⟶ -6 - (-12) = -6 +(+12) = **+6** or **6**

5) (+3) - (+12) = (+3) + (-12) = **-9**

6) (+8) - (+5) = (+8) + (-5) = **+3**

7) (+21) - (-9) = (+21) + (+9) = **+30**

8) (-18) - (+6) = (-18) + (-6) = **-24**

9) (-5) - (-12) = (-5) + (+12) = **+7**

10) (+4) - (-17) = (+4) + (+17) = **21**

<u>Practice Set 9</u> Find the answers to the following subtraction problems.

1.) + 12 - (+7) =

2.) 19 - 31=

3.) -12 - (-90) =

4.) 1.25 - 19.15 =

5.) -12.75 - (-6.90) =

continued...

6) -4 1/5 - 8 3/5 =

7.) -21.35 - (-8.75) =

8.) 125 - (-2080) =

9.) -4 5/8 - (+5 1/2) =

10) (+15) - (+18) =

11) (-87) - (67) =

12) (-21) - (-46) =

13) (-34) - (8.75) =

14) (-4) - (3 3/4) =

15) (126) - (-98) =

16) (-19 3/4) - (-4 1/2) =

17) (12) - (-14.75) =

18) (-1.375) - (18 5/8) =

19) (12.75) - (12 3/4) =

Using signed numbers, it is possible to subtract larger numbers from smaller numbers.

For instance, if the temperature is now 14 degrees and it is predicted that the temperature will fall another 20 degrees, the temperature may fall to -6 degrees, which is 6 degrees below zero. (14 - 20 = -6)

If the temperature is now 4 degrees below zero (-4) and it is predicted that the temperature will fall another 8 degrees, the temperature may fall to -12 degrees, which is 12 degrees below zero. (-4 - 8 = -12)

MULTIPLYING SIGNED NUMBERS

MULTIPLICATION RULE #1: **If you multiply two numbers which have the same sign, then the answer is positive.**

A **positive** number **times** a **positive** number is **positive**.

A **negative** number **times** a **negative** number is **positive**.

MULTIPLICATION RULE #2: **If you multiply two numbers which have different signs, then the answer is negative.**

A **positive** number **times** a **negative** number is **negative**.

A **negative** number **times** a **positive** number is **negative**.

MULTIPLICATION SYMBOLS: All the following symbols represent multiplication:

$6 \times 7 = 42$ $\quad$ $(6)(7) = 42$ $\quad$ $6 \cdot 7 = 42$ $\quad$ $6 * 7 = 42$

EXAMPLES:

$(6)(8) = 48$ $\quad$ $(-6)(8) = -48$ $\quad$ $(6)(-8) = -48$ $\quad$ $(-6)(-8) = 48$

Problem: If you lose $ 4.00 each day for five days, how much have you lost?

Solution: $-4.00 \times 5 = -20.00$ or You have lost $ 20.00.

Practice Set 10 Solve the following multiplication problems.

1) (8)(12) =	2) (-2)(12) =	3) (-4)(-12) =
4) (4)(-11) =	5) 6(-12) =	6) (+3)(-2) =
7) (-9)(5) =	8) (-1)(-1) =	9) (-12)(0) =
10) (-2.25)(12.75) =	11) (3/4)(- 2/3) =	12) (8.75)(- 3 1/2) =
13) 4(-12.75) =	14) -4 x -12.25 =	15) -1.25(4.75) =

What happens if a multiplication problem has more than two numbers? Let's look at two examples.

Example #1: Multiply (-4) x (3) x (-5) x (2)

(-4) x (3) x (-5) x (2)	
	neg x pos = neg
- 12 x (-5) x (2)	
	neg x neg = pos
60 x (2)	
	pos x pos = pos
120	

Example #2: Multiply (-3) x (2) x (-4) x (-5)

(-3) x (2) x (-4) x (-5)	
	neg x pos = neg
(-6) x (-4) x (-5)	
	neg x neg = pos
(+ 24) x (-5)	
	pos x neg = neg
-120	

If you do not want to work with the signs during each step, you can ignore the signs until the final step, if you follow the next rule.

MULTIPLICATION RULE #3: If there are several signs in a multiplication problem, the answer will be positive if there is an even number of negative signs and the answer will be negative if there is an odd number of negative signs.

EXAMPLES:

(-1)(-5)(4)(3)(-2) = -120 since there are 3 negative signs

2(-2)(-1)(-1)(3)(-3) = 36 since there are 4 negative signs

<u>**Practice Set 11**</u> Find the answers to the following multiplication problems.

1) (+2)(-3)(+4)(-5) =

2) 3(-2.5)(-1)(-3.75) =

3) 3(2 1/2)(- 3 1/4)(-2) =

4) (-1)(2)(-3)(4)(-5)(6) =

5) (-2)(-4)(-6)(-8) =

6) (2.15)(3.23)(-6.78)(0)(-9.17) =

7) (12)(2)(-3.2)(1.11) =

8) 4(-4)(-2)(2) =

DIVIDING SIGNED NUMBERS

The rules for division with signed numbers are the same as the rules for multiplication.

DIVISION RULE #1: If you divide two numbers which have the same sign, then the answer is positive.

A **positive** number **divided by** a **positive** number is **positive**.

A **negative** number **divided by** a **negative** number is **positive**.

DIVISION RULE #2: If you divide two numbers which have different signs, then the answer is negative.

A **positive** number **divided by** a **negative** number is **negative**.

A **negative** number **divided by** a **positive** number is **negative**.

EXAMPLES:

$+28 \div +4 = +7$ $\quad$ $-28 \div -4 = +7$

$+28 \div -4 = -7$ $\quad$ $-28 \div +4 = -7$

__Practice Set 12__ Find the answers to the following division problems.

1) $+27 \div -9 =$
2) $-18 \div +3 =$
3) $-36 \div -9 =$
4) $+36 \div +12 =$
5) $-81 \div -9 =$
6) $72 \div -9 =$
7) $-28 \div +16 =$
8) $+16 \div -32 =$
9) $-12 \div -16 =$
10) $-28 \div 18 =$
11) $-84 \div -42 =$
12) $+21 \div -14 =$
13) $-3.75 \div 1.25 =$
14) $3.5 \div -2 =$
15) $-12.5 \div 0 =$

SIGNED NUMBER INVENTORY

This section is a review of all the properties for adding, subtracting, multiplying, and dividing signed numbers.

Practice Set 13

1) The highest point in North America is the tip of Mount McKinley, which is approximately 20,320 feet above sea level. The lowest point is in Death Valley, which is approximately 280 feet below sea level. What is the difference in height between these two points?

2) If at noon yesterday the temperature was 12 degrees above zero while at 5:00 am this morning the temperature was 7 degrees below zero, what was the difference in temperature between these two times?

3) Freezing occurs at 32^{o} F. How many degrees below freezing is -20^{o} F?

4) On June 1, you owed $232.67 on your credit card. During June, you make the following charges and payments. What is your balance on July 1?

6/4	charge	$ 23.54	6/15	finance charge	$ 8.75
6/7	charge	$ 35.12	6/21	charge	$ 42.55
6/10	payment	$ 112.00	6/27	charge	$ 34.97
6/12	charge	$ 27.85			

5) You make and sell widgets. The material for each widget costs you $2.55. It takes 1 and 1/2 hour to make each widget. Labor cost you $5.85 per hour. You sell each widget for $17.95. In September, you made 121 widgets and you sold 101 widgets. What was your profit (or loss) for September?

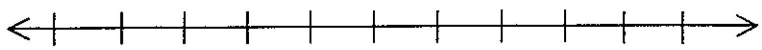
6) If North latitude is considered positive, how should 52^{o} South latitude be written?

7) What is the difference in latitude between 47^{o} North latitude and 23^{o} South latitude?

8) If the temperature at 6:00 am is -4^{o} F and the temperature at noon is 12^{o} F, how much has the temperature risen?

9) Each box of drinking straws is supposed to contain 200 straws. However, when you check 25 boxes, you find that only 15 have exactly 200 straws. Of the others, you find that 3 are short 4 straws, 2 have 3 straws too many, one is short 7 straws, one is short 3 straws, and the rest have 5 too many straws each. When finished checking, do you have too few or too many straws? How many too few or too many?

10) Locate the following points on the number line: -3, 0, 4.5, -1.75, 5, 3 3/4



11) Tom weighed 210 pounds when he went for his annual check-up. His doctor told him to lose weight. So, he started his diet in March and he did well, losing 12 pounds that month. The next few months did not go so well, as he lost only 3 pounds in April, then gained 4 pounds in May, gained 5 pounds in June, lost 2 pounds in July, and lost 4 pounds in August. At the end of this time, how much did he weigh? Did he have a net loss (a negative number) or a net gain (a positive number)? Express this loss/gain as a signed number.

12) -221
- 32

13) - 8
+ 13

14) 16
- 64

15) - 124
- - 68

16) 36 - (-54) =

17) -87 + 45 - (-23) - 12 + (-12) =

18) 4(-12)(8) =

19) (-2)(-3)(-4)(5) =

20) -3(-12) =

21) -12 ÷ -6 =

22) 45 ÷ - 12 =

23) -18 ÷ 12 =

24) (-18 2/3) - (-18 2/3) =

25) 2(-3)(15)(0)(-5)(6)(-7) =

26) What is the opposite of 23 1/2?

Getting Acquainted with Algebra

The equation **d = r·t** expresses the relationship between distance (d), rate (r), and time (t). For instance, if you are driving your car at the average rate of 50 miles per hour for the time of 4 hours, you can determine the distance traveled by putting those numbers into the equation.

$$d = r \cdot t \longrightarrow d = (50) \cdot (4) \longrightarrow d = 200 \text{ miles}$$

In algebra, letters are used to represent numbers. These letters are called **variables.** Whenever these variables (letters) are used, they have all the properties of regular numbers.

The symbols for addition, subtraction, division, powers, and roots are the same in algebra as they are in arithmetic. The only symbol that is different is the one for multiplication. In algebra, multiplication is indicated by writing the number and the variable together (thus, "5x" means "5 times x") and by writing two or more variables together (thus "D = rt" means "distance equals rate times time").

OPERATION	ALGEBRAIC EXPRESSION	WORD EXPRESSION
addition	x + 2	x plus 2 or the sum of x and 2
subtraction	x - 2	x minus 2 or the difference of x and 2
subtraction	2 - x	2 minus x or the difference of 2 and x
multiplication	2x	2 times x or the product of 2 and x
division	$\frac{2}{x}$	2 divided by x or the quotient of 2 and x
division	$\frac{x}{2}$	x divided by 2 or the quotient of x and 2
powers	x^2	x squared
powers	x^3	x cubed
roots	$\sqrt{x}$	the square root of x

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Section 7: Getting Acquainted with Algebra

Many algebraic expressions contain more than one operation. When they do, you must be careful to follow the rules of operations. When parentheses are needed, they are indicated by using the word "**quantity**".

ALGEBRAIC EXPRESSION	WORD EXPRESSION
$2x + 4$	2 times x plus 4 or the sum of 2 times x and 4
$2(x + 4)$	2 times the quantity x plus 4 or the product of 2 and the sum of x and 4
$4(x^2 - 6)$	4 times the quantity x squared minus 6 or the product of 4 and the difference of x squared and 6
$4(x - 6)^2$	4 times the quantity of x minus 6 squared
$(4x - 6)^2$	the quantity 4x minus 6 squared

<u>**Practice Set 14**</u> Write an algebraic expression for each of the following:

1) a number x plus 7

2) a number y minus 5

3) three times the number x

4) seven times x

5) x divided by 17

6) two times x minus 14

7) 21 plus three times x

8) the number d decreased by 12

9) three times the sum of 2 and x

10) x plus y minus -2

11) five times the quantity x minus y

12) x squared minus y squared

13) three-fourths the quantity of x plus 7

14) three-fourths x plus 7

15) two-thirds the quantity of x minus 4

16) two-thirds x minus 4

17) the product of z and the quantity z plus 8

18) the square of the quantity 2x + 7

19) the sum of the square of 2x and 7

20) three times the quantity 2x plus 3 squared

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Section 7: Getting Acquainted with Algebra

EVALUATING ALGEBRAIC EXPRESSIONS

To find the value of an algebraic expression, substitute a number for each variable, and do the arithmetic.

Remember the order of operations:

1) inside parentheses first, then
2) do the powers, then
3) do the multiplications and divisions, and
4) do the additions and subtractions last.

EXAMPLES:

PROBLEM: evaluate $17 - 2x + 12$ when $x = 8$

SOLUTION:

$17 - 2x + 12$	problem
$17 - 2(8) + 12$	substitute
$17 - 16 + 12$	multiply first
$1 + 12$	subtract because it is on the left
13	add

PROBLEM: evaluate $3x^2 - 4x + 6$ when $x = -2$

SOLUTION:

$3x^2 - 4x + 6$	problem
$3(-2)^2 - 4(-2) + 6$	substitute
$3(4) - 4(-2) + 6$	do the powers first with $(-2)^2 = 4$
$12 - 4(-2) + 6$	do the multiplication on the left
$12 - -8 + 6$	multiply
$20 + 6$	subtracting -8 is the same as adding +8
26	add

PROBLEM: evaluate $\frac{x}{y} - xy$ when $x = 6$ and $y = 2$

SOLUTION:

$\frac{x}{y} - xy$	original problem
$\frac{6}{2} - (6)(2)$	substitute
$3 - (6)(2)$	
$3 - 12$	
-9	

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Section 7: Getting Acquainted with Algebra

PROBLEM: evaluate $3(x - 4)^2$ when $x = -3$

SOLUTION:	
$3(x - 4)^2$	problem
$3(-3 - 4)^2$	substitute
$3(-7)^2$	work inside the parentheses first
$3(49)$	do the power
147	multiply

PROBLEM: evaluate $\frac{2(x+3)^2}{x-4} - \frac{x}{3}$ when $x = 6$

SOLUTION:

$\frac{2(x+3)^2}{x-4} - \frac{x}{3}$ problem

$\frac{2(6+3)^2}{6-4} - \frac{6}{3}$ substitute

$\frac{2(9)^2}{6-4} - \frac{6}{3}$ work inside parentheses first

$\frac{2(9)^2}{2} - \frac{6}{3}$ numerators and denominators can be considered to have parentheses around them.

$\frac{2(81)}{2} - \frac{6}{3}$ do the power

$\frac{162}{2} - \frac{6}{3}$ multiply

$81 - 2$ do the two divisions

79 subtract to get your answer

Practice Set 15 Evaluate each of the following:

1) $x + 12$ when $x = -4$

2) $2x - 12$ when $x = -3$

3) $2xy$ when $x = 4$ and $y = 3$

4) $4 - xy + 3xy$ when $x = 2$ and $y = -3$

5) $3x^2 + 2x - 4$ when $x = 5$

6) πr^2 when $\pi = 3.14$ and $r = 6$

7) $12 - 2y$ when $y = -3$

8) $3xy - 2x$ when $x = -3$ and $y = 5$

Algebra

Section 7: Getting Acquainted with Algebra

9) $2x - 9$ when $x = 3.5$

10) $2x + 3y - 6$ when $x = 3$ and $y = 4$

11) $\frac{-x}{7} + 1$ when $x = 14$

12) $\frac{2x}{y} + \frac{y}{x}$ when $x = 5$ and $y = 10$

13) $3(x - 4) + 2(x + 5)$ when $x = 3$

14) $2(x + 3)^2 - 3x^2$ when $x = -4$

15) $-4(x - y)$ when $x = 6$ and $y = 3$

16) $2l + 2w$ when $l = 10$ and $w = 8$

17) $x(x + y)$ when $x = 4$ and $y = 2.5$

18) $3x(x + 2)$ when $x = 5$

19) $2x^3 + 5x + 1$ when $x = 4$

20) $(-x)^2 + -(x)^2$ when $x = 9$

21) $\frac{2(x + 7)}{3(x - 6)}$ when $x = 8$

22) $\frac{2(x + 3)^2}{5x^2}$ when $x = 2$

23) $\frac{3(x - y)^2}{5x^2}$ when $x = 3$ & $y = -1$

24) $\frac{-2(x - 1)^2}{3x(y + 1)^2}$ when $x = 4$ & $y = 1$

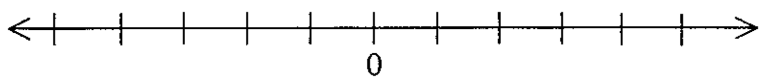
25) $\sqrt{x^2 + y^2}$ when $x = 4$ & $y = 3$

26) $3x\sqrt{\frac{2x}{9}}$ when $x = 2$

REVIEW

Practice Set 16

1) On the following number line, locate: -4 1/2, 5.75, .25, -3, 4



Simplify:

2) -3 + (-5) - (-6) - 7 =

3) 3(4 - 7) =

4) (-7)(6)(-5) =

5) -2(4 + 5) =

6) $\frac{-6}{-3}$ =

7) $\frac{12}{-4}$ =

8) -2 + (-3) + -4 =

9) -2 · -3 · -4 =

10) $6\frac{2}{3} - 12\frac{1}{3}$ =

11) 12 - (-9) =

12) $3\frac{1}{2} - 2\frac{1}{4} + 4\frac{3}{4} - 5\frac{1}{2}$ =

13) $12\frac{2}{5} - -12\frac{2}{5}$ =

If v = 0, w = -1, x = 1, y = 2, and z = -3, find the value of each of the following:

14) 8 + 4z =

15) 2x - 3y + 2z =

16) wxyz =

17) $2y^2 - 4z^2$ =

18) $2y^2 + 3y + 4$ =

19) $2(w - y)^2 - 3(z - y)^2$ =

20) (y + z)(y - z) =

21) $-2(y - w)^3$ =

22) $\frac{y}{x} - xy$ =

23) $(y + z^2) / (2y^2)$ =

24) 2(y - 2z)(2x - y) =

25) $\frac{v}{w} + \frac{x}{y} - \frac{y}{z}$ =

26) At noon, the temperature was 63^{o} F. It lost 6^{o} between noon and 2 pm. Then the temperature rose 7^{o} between 2 pm and 6 pm. At midnight, the temperature 59^{o}. Did the temperature rise or fall between 6 pm and midnight and by how many degrees did it change?

27) What is farther from zero: 14 degrees above zero or 12 degrees below zero?

28) If the temperature started at 21 degrees above zero and fell to 4 degrees below zero, by how many degrees did the temperature fall?

In the following formula, C represents the Celsius (or centigrade) temperature and F represents the Fahrenheit temperature.

$$C = 5/9 \cdot (F - 32)$$

Find the corresponding centigrade temperatures for the following Fahrenheit temperatures.

29) 32^{o} F =

30) 180^{o} F =

31) 212^{o} F =

32) 148^{o} F =

33) You can calculate the amount of (simple) interest that you will earn by using this formula: I = prt, where I represents interest, p represents principal, r represents rate, and t represents time in years. How much interest is earned if the original amount invested (the principal) is \$450.00, the rate is 0.075 (7 1/2 %), and the time is 0.5 (1/2 year)?

34) The formula for the volume of a sphere is: $V = \frac{4}{3}\pi r^3$. (A sphere is a three dimensional circular object like a basketball.) If the radius of a sphere is 6 inches, what is its volume?

Algebra

Section 9: Introduction to Equations

INTRODUCTION

You have already seen many equations: $2 + 3 = 5$ is an equation; so is $A = LW$ and all the other formulas that you have used. In fact, anytime that you have an equal sign, you have an equation.

Mathematical Operation:	Arithmetic-style equation:	Algebraic equation:
addition	$12 + 23 = 35$	$x + 6 = 14$
subtraction	$18 - 6 = 12$	$x - 10 = 32$
multiplication	$9 \times 6 = 54$	$3x = 12$
division	$\frac{12}{6} = 2$	$\frac{x}{3} = 10$
exponent (powers)	$3^2 = 9$	$x^2 = 25$
roots	$\sqrt{36} = 6$	$\sqrt{x} = 49$

Solving an addition equation

In the equation: $x + 2 = 5$ what number can be substituted for x so that both sides will be equal to 5? Finding this number is called "solving the equation". In this section, you will begin to learn the processes that you can use to solve equations.

You might be able to look at the equation and decide that 3 is the number. However, just by looking at the following problem, you might not be able to see that the answer to $3x + 14 - (x - 2) = 7x - 2(x + 4) + 15$ is also 3.

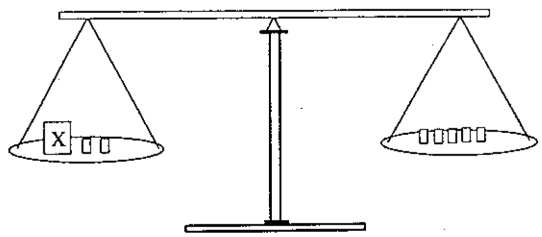
So we will begin with small problems and find the simple steps necessary to solve them. Then, we will be able to see that long problems can be solved through a series of these small steps.

Algebra

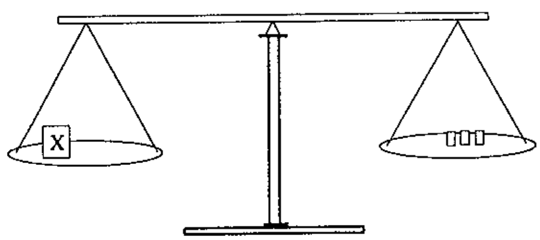
Section 9: Introduction to Equations

Visualize Equations

A good way to picture an equation is to use a balance scale. The equation $x + 2 = 5$ would look like this on the balance scale:



The way to find the value of the weight marked X is to remove the two known weights on the left-hand side. But for the scale to remain in balance, two weights *from the other side* must also be removed. After the weights have been removed the scale this is what the scale looks like:



When you get the unknown alone on one side, what remains on the other side is the answer.

This example is an **addition equation.** What we did to solve it was to subtract (remove).

On paper, it is written like this:

$$\begin{array}{cccccc} x & + & 2 & = & 5 \\ & & -2 & & -2 \\ \hline x & & & = & 3 \end{array}$$

You can make sure this is the correct answer by putting it back into the problem. If the left side is equal to the right, then you will confirm that 3 is the correct answer.

$$\begin{array}{ccccc} \overset{3 \searrow}{x} & + & 2 & = & 5 \\ 3 & + & 2 & = & 5 \\ & & 5 & = & 5 \end{array}$$

Because the left side is equal to the right side, the answer $x = 3$ must be correct.

Algebra

Section 9: Introduction to Equations

EXAMPLE: Solve and check the following addition equations.

1.)

$$\begin{array}{l} x + 16 = 3 \\ \underline{\quad - 16 = -16} \\ x + 0 = -13 \\ x = -13 \end{array}$$

(Answer)

Check:

$$\begin{array}{l} \boldsymbol{-13} + 16 = 3 \\ 3 = 3 \end{array}$$

(This confirms that x = -13 must be correct.)

2.)

$$\begin{array}{l} x + 8 = -4 \\ \underline{\quad -8 = -8} \\ x + 0 = -12 \\ x = -12 \end{array}$$

(Answer)

Check:

$$\begin{array}{l} \boldsymbol{-12} + 8 = -4 \\ -4 = -4 \end{array}$$

(This confirms that x = -12 must be correct.)

3.)

$$\begin{array}{l} 12 = x + 6 \\ \underline{-6 = \quad -6} \\ 6 = x + 0 \\ 6 = x \end{array}$$

(Answer)

Check:

$$\begin{array}{l} 12 = \boldsymbol{6} + 6 \\ 12 = 12 \end{array}$$

(This confirms that x = 6 must be correct.)

4.)

$$\begin{array}{l} 12 + x = 8 \\ \underline{-12 \quad = -12} \\ 0 + x = -4 \\ x = -4 \end{array}$$

Check:

$$\begin{array}{l} 12 + \boldsymbol{-4} = 8 \\ 8 = 8 \end{array}$$

(This confirms that x = -4 must be correct.)

Algebra

Section 9: Introduction to Equations

Practice Set 17 Solve and check each of the following addition equations.

1) $x + 12 = 32$

2) $y + 8 = 23$

3) $a + 14 = 20$

4) $r + 21 = 12$

5) $x + 3 = 1$

6) $x + 18 = 18$

7) $x + 12 = -12$

8) $12 + x = 40$

9) $2 + x = 8$

10) $6 + x = 3$

11) $4 + x = -4$

12) $x + 3.45 = 12.25$

13) $4.75 + x = 19.95$

14) $x + \frac{3}{4} = 3\frac{1}{2}$

15) $2\frac{1}{4} + x = 5\frac{1}{8}$

16) $x + 1.5 = 3\frac{3}{4}$

Algebra

Section 9: Introduction to Equations

Solving a Subtraction Equation

Subtraction is the operation used to solve addition problems because subtraction "undoes" addition. Subtraction cancels addition. Subtraction and addition are ***inverse operations***.

OPERATION	INVERSE OPERATION	SOLVING EQUATIONS
addition	subtraction	Use subtraction to solve an addition equation.
subtraction	addition	Use addition to solve a subtraction problem.

Basic Rule for Solving Equations
Whatever inverse operation you apply to one side of an equation, you must apply to the other side.

How to solve a subtraction equation:

Equations such as: $x - 8 = 14$ are called **subtraction equations**. To solve a subtraction equation, use addition, because addition *undoes* subtraction.

EXAMPLES

1.)

$$\begin{array}{rcl} x - 8 &=& 14 \\ +8 &=& +8 \\ \hline x + 0 &=& 22 \\ x &=& 22 \end{array}$$

(Answer)

Check:

$$\begin{array}{rcl} 22 - 8 &=& 14 \\ 14 &=& 14 \end{array}$$

(Since both sides are equal, the solution $x = 22$ must be true.)

2.)

$$\begin{array}{rcl} x - 21 &=& 6 \\ +21 &=& +21 \\ \hline x + 0 &=& 27 \\ x &=& 27 \end{array}$$

(Answer)

Check:

$$\begin{array}{rcl} 27 - 21 &=& 6 \\ 6 &=& 6 \end{array}$$

(Since both sides are equal, the solution $x = 27$ must be true.)

Practice Set 18

1.) $x - 3 = 7$

2.) $x - 124 = 634$

3.) $x - 12 = -4$

4.) $x - 6 = -6$

5.) $x - 3 = -19$

6.) $x - 12.45 = 45.85$

7.) $x - 3.65 = 2.54$

8.) $x - \frac{3}{4} = 4\frac{1}{4}$

9.) $x - \frac{1}{4} = 5.75$

10.) $x - 3\frac{1}{4} = 6\frac{1}{2}$

11.) $x - 3.55 = 4\frac{1}{4}$

12.) $x - 8.75 = 8\frac{3}{4}$

Algebra

Section 9: Introduction to Equations

Solving a multiplication problem

In an expression like **6*x***, the **6** is multiplied with ***x***. Numbers that you see written in front of a variable are called coefficients. In the expression **6*x***, we would say that **6** is the ***coefficient*** of ***x***. Likewise, the coefficient of **-8*x*** is **-8**. If the expression is just plain-old ***x***, the coefficient is understood to be **1**. Similarly, if the expression is **-*x***, the coefficient is **-1**.

Equations such as: 5x = 35 are multiplication equations. In this case, the variable x is multiplied by 5. The solve a multiplication equation, apply the inverse operation: ***division***.

OPERATION	INVERSE OPERATION	SOLVING EQUATIONS
multiplication	division	Use division to solve a multiplication equation.
division	multiplication	Use multiplication to solve a division problem.

EXAMPLES:

1.) $7x = 28$

$$\frac{7x}{7} = \frac{28}{7}$$ (Applying the inverse operation: division)

$1x = 4$

$x = 4$ (Answer)

Check:

$7(4) = 28$

$28 = 28$ (Both sides are equal, therefore $x = 4$ must be the correct answer.)

2.) $-8x = 24$

$$\frac{-8x}{-8} = \frac{24}{-8}$$ (Applying the inverse operation: division)

$1x = -3$

$x = -3$ (Answer)

Check:

$-8(-3) = 24$

$24 = 24$ (Both sides are equal, therefore $x = -3$ must be the correct answer.)

Algebra

Section 9: Introduction to Equations

EXAMPLES:

3.) $-2x = -60$

$$\frac{-2x}{-2} = \frac{-60}{-2}$$

$1x = 30$

$x = 30$ (Answer)

Check:

$-2(30) = -60$

$-60 = -60$ (Both sides are equal, therefore $x = 30$ must be the correct answer.)

4.) $-x = 27$

$$\frac{-1x}{-1} = \frac{27}{-1}$$ (Remember that $-x$ is equivalent to $-1x$)

$1x = -27$

$x = -27$ (Answer)

Check:

$-(-27) = 27$

$27 = 27$ (Both sides are equal, therefore $x = -27$ must be the correct answer.)

Practice Set 19

1.) $2x = 14$

2.) $3x = 39$

3.) $1x = 17$

4.) $5x = 1.25$

5.) $-4x = 28$

6.) $-7x = -56$

7.) $-1x = 98$

8.) $-x = 87$

9.) $-x = -123$

10.) $1.25x = 625$

11.) $5x = 23$

12.) $1.2x = 7.74$

Solving a Division Equation

Equations such as: $\frac{x}{5} = 12$ are **division equations**. To solve a division equation, use multiplication.

Solve: $\frac{x}{5} = 12$

Note: Multiplying $\frac{x}{5}$ by 5 will force the "cancellation" of the 5's on the left side. Also be sure to multiply the right side by 5 to keep the equation in balance.

$$\cancel{5}\left(\frac{x}{\cancel{5}}\right) = 5(12)$$

$1x = 60$
$x = 60$ (Answer)

Check:
$\frac{60}{5} = 12$
$12 = 12$ (Since both sides are equal, the solution $x = 60$ must be correct.)

<u>EXAMPLES</u>

1.) $\frac{x}{5} = 8$

$\cancel{5}\left(\frac{x}{\cancel{5}}\right) = 5(8)$ (Again we multiply by 5 since that is the inverse operation of division.)

$1x = 40$

$x = 40$ (Answer)

Check:
$\frac{40}{5} = 8$
$8 = 8$ (Both sides are equal, therefore $x = 40$ must be the correct solution.)

the next example is on the following page...

EXAMPLES:

2.) $\frac{x}{-2} = -4$

$-2\left(\frac{x}{-2}\right) = -2(-4)$ (Multiply by -2, since that is the inverse operation to division.)

$1x = 8$

$x = 8$ (Answer)

Check:

$\frac{8}{-2} = -4$

$-4 = -4$ (This confirms that the answer, $x = 8$ must be true.)

Practice Set 20

1.) $\frac{x}{8} = 12$

2.) $\frac{x}{7} = 56$

3.) $\frac{x}{-5} = 10$

4.) $\frac{x}{-6} = -2$

5.) $\frac{x}{4} = -9$

6.) $\frac{x}{8} = \frac{3}{2}$

Solving a Fractional Equation

If the coefficient of x (the number that x is being multiplied by) is a fraction, then the fraction is called a fractional equation. For example, $\frac{3x}{5} = 12$ is a fractional equation. The easiest way to solve a fractional equation is to multiply both sides by the **reciprocal** of the fraction. The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. The reciprocal is found by inverting (turning upside-down) the fraction.

It is worth noting that fractions with variables can be written in more than one way.

For example: $\frac{2}{3}x = \frac{2x}{3}$ also: $-\frac{3}{4}x = \frac{-3x}{4} = \frac{3x}{-4}$

<u>**EXAMPLES**</u>

1.) $\frac{3x}{7} = 9$

$\frac{\cancel{7}}{\cancel{3}}\left(\frac{\cancel{3}x}{\cancel{7}}\right) = \frac{7}{3}(9)$

$1x = 21$

$x = 21$ (Answer)

Check:

$\frac{3(21)}{7} = 9$

$9 = 9$ (The left and right sides are equal so that means $x = 21$ is correct.)

2.) $\frac{-6x}{5} = 12$

$\frac{\cancel{5}}{\cancel{-6}}\left(\frac{\cancel{-6}x}{\cancel{5}}\right) = \frac{5}{-6}(12)$

$1x = -10$

$x = -10$ (Answer)

Check:

$\frac{-6(-10)}{5} = 12$

$12 = 12$ (The left and right sides are equal so that means $x = -10$ is correct.)

Algebra

Section 9: Introduction to Equations

Practice Set 21

1.) $\frac{4x}{5} = 36$

2.) $\frac{3x}{10} = 9$

3.) $\frac{-9x}{2} = 27$

4.) $-\frac{3}{4}x = 18$

5.) $-\frac{4}{5}x = -4$

6.) $\frac{2}{3}x = 6$

Algebra

Section 9: Introduction to Equations

The following problems represent all of the different equation-solving skills presented in this section.

Practice Set 22

1.) $x + 9 = 12$

2) $12 + x = 18$

3.) $x + 23 = 11$

4.) $32.5 + x = 11.4$

5.) $x - 7 = 21$

6.) $x - 19 = 10.5$

7.) $x - 3\frac{1}{2} = 18$

8.) $x - 4\frac{1}{2} = 16\frac{5}{8}$

9.) $3x = 36$

10.) $-5x = 35$

11.) $-x = -98$

12.) $-4x = 18$

13.) $-2x = 148.76$

14.) $-x = 43$

15.) $2x = 14.45$

16.) $2x = 3\frac{3}{4}$

17.) $\frac{x}{5} = 14$

18.) $\frac{w}{32} = -4$

continued...

19.) $\frac{x}{-4} = -7$

20.) $\frac{9d}{4} = 729$

21.) $\frac{2x}{5} = -91$

22.) $\frac{x}{-7} = -23$

23.) $\frac{-2x}{5} = 14$

24.) $\frac{-3x}{29} = 6$

25.) y - (-4) = 12

26.) x + (-16) = -20

Algebra

Section 9: Introduction to Equations

Solving Problems with Algebra

The purpose of equations is to help you solve problems. In order to do this, you will need to be able to state the problem and then translate the problem into an equation.

For this section, the problem will be stated for you. Your job is to "translate" the problem into an equation and then solve that equation. In each case, let x be the number to be found and replace the verb ("is", "is equal to", "paid", etc.) with an equal sign.

EXAMPLES

1.) Problem: Seven times a number is 784. What is the number?

Replace "a number" with the variable x and replace "is" with =. Since "times" means multiplication, this will be a multiplication problem.

Seven times a number is 784. What is the number?

$$7x = 784$$

Since this is a multiplication equation, you will solve it using division. You should get an answer of ***112***.

2.) Problem: You are a salesperson who receives a 12% commission on every sale. If you earned \$879.96 during the month, what was the total value of the sales for the month?

Since commission is calculated by taking the percentage times the amount of the sale, the appropriate equation is:

(commission percentage) x (total amount of sales) = total commission earned

The commission percentage must be written as a decimal: 12% = 0.12
The total amount of sales is unknown, therefore, it is replaced with an x.
The total commission earned: \$879.96

The equation is: $0.12x = 879.96$

Solving this by dividing shows that the answer is: **\$7331.33**

Practice Set 23

1.) If you subtract 6 from my sister's age, you will get my age. I am 37 years old. How old is my sister?

2.) When my father was 78 years old, he was twice as old as I was. How old was I then?

3.) I plan to pay off one-half of my VISA bill this month. This means that I will pay VISA $134.95 this month. How much is my VISA bill?

4.) I pay 1/3 my salary each month for rent. My rent costs $325.00 each month. What is my monthly salary?

5.) If I were to take $400.00 out of my check each month to pay rent, I would be left with only $372.35 each month. How much is my net salary each month?

6.) I traveled 330 miles in 6 hours. What was my average rate of speed?

7.) Last month I earned $778.00 in commissions. I earn commissions at a rate of 8%. What was the total sales for the month?

8.) If I were to lose 42 pounds, I would be back at my old weight of 169 pounds. How much do I weigh now?

SOLVING MULTI-STEP PROBLEMS

Each equation that you solved in section 9 had only one operation. Most equations have more than one operation, however. This section will teach you how to handle these more complex equations.

Since solving an equation is a peeling away of numbers until only the variable remains, it is an "undoing". You have already learned this in the simpler equations, because you always "did the opposite". When you were given an addition equation, you solved it by subtracting; when you were given a multiplication equation, you solved it by dividing, etc.

In Chapter 5, Section 1, you learned the Order of Operations -- especially that you were to do the multiplications and divisions before you did the additions and subtractions. Since solving equations is doing the opposite, when solving an equation, you do the opposite here also.

You undo the additions and subtractions before you undo the multiplications and divisions.

EXAMPLE: Solve: 3x - 13 = 29

Since this problem contains both multiplication (3x) and subtraction (-13), you must do two "undoing" steps: you must add 13 to undo the - 13 and you must divide by 3 to undo the $3x$. Since you must undo additions before multiplications, the solution looks like this:

$$\begin{array}{rcr} 3x - 13 & = & 29 \\ +\ 13 & & +\ 13 \\ \hline 3x & = & 42 \end{array}$$ (Add 13 to both sides.)

$$\frac{3x}{3} = \frac{42}{3}$$ (Divide both sides by 3.)

$x = 14$ (Answer)

Check:

$3x - 13 = 29$
$3(14) - 13 = 29$
$42 - 13 = 29$
$29 = 29$

Algebra

Section 10: Solving Multi-step Problems

EXAMPLE: Solve: $\frac{x}{6} + 12 = 48$

Since this problem contain both division (x/6) and addition (+ 12), you must again do two undoing steps: multiplication, to undo the division, and subtraction, to undo the addition. Since the addition must be undone before the division is undone, the step are these:

$\frac{x}{6} + 12 = 48$

$\underline{\quad -12 \quad -12}$ (Subtract 12 from each side.)

$\frac{x}{6} = 36$

$6\left[\frac{x}{6}\right] = 6\ (36)$ (Multiply both sides by 6)

$x = 216$ (Answer)

Check:

$\frac{216}{6} + 12 = 48$

$36 + 12 = 48$

$48 = 48$

<u>Practice Set 24</u> Solve and check each of the following.

1) $3x + 18 = 36$

2) $2x - 10 = 48$

3) $4x - 12 = -10$

4) $8 + 3x = 5$

5) $7 + 5x = 21$

6) $4 - x = 6$

7) $12 - 3x = 60$

8) $40 = 3x + 19$

9) $\frac{x}{2} - 12 = 48$

10) $12 - \frac{x}{3} = 72$

Algebra

Section 10: Solving Multi-step Problems

You need not write all the steps when you solve an equation. If you can do the additions, subtractions, multiplications, and divisions in your head or with a calculator, all you need to write is the results of doing those steps.

For instance, for the problem: $4x + 16 = 48$

Showing all steps:

$$\begin{array}{rcl} 4x + 16 &=& 48 \\ -16 && -16 \\ \hline 4x &=& 32 \end{array}$$

$$\frac{4x}{4} = \frac{32}{4}$$

$$x = 8$$

Showing only the results:

$$4x + 16 = 48$$
$$4x = 32$$
$$x = 8$$

(Doing the subtraction in your head and then doing the division in your head while writing only the results.)

Check:

$$4(8) + 16 = 48$$
$$32 + 16 = 48$$
$$48 = 48$$

If you feel more comfortable writing all the steps, do so! Also, if you are making several errors while solving equations the short way, go back to showing all the steps!

Practice Set 25 Solve and check the following equations:

1) $2x + 27 = 59$

2) $5x - 7 = 42$

3) $12 + 3y = 36$

4) $-6 + 7y = 36$

5) $-12 - w = 23$

6) $18 - 3t = 24$

7) $2.5 + 4.1x = 12.75$

8) $-1.1x + 2.31 = 71.94$

9) $\frac{x}{6} - 19 = 59$

10) $\frac{-2x}{3} - 16 = 22$

11) $12 - \frac{3x}{4} = -33$

12) $6 - \frac{x}{7} = -2$

13) $1 - x = -1$

14) $1.6 - 2x = -0.45$

SOLVING EQUATIONS WITH MORE THAN ONE VARIABLE TERM

A common command is "Don't mix apples and oranges!" If you have 2 apples and 3 oranges in one bag, 5 apples in another bag, and 3 apples and an orange in a third bag, and if you combine the contents of the bags together, you'll have 10 apples and 4 oranges, not 14 apple-oranges. This sounds simple and obvious; the same idea applies to algebra.

In algebra, the command is ***"Don't mix unlike terms!"***

Examples of number terms: 12, -6, 0, 1, -12.25, 2 3/4

Examples of *x*-terms: 3x, -2x, 1.25x, x, -x, $\frac{-3}{2}x$

Examples of *y*-terms: 2.75y, -y, 12y, y, $\frac{2}{3}y$

Like terms can be combined (added together or subtracted from each other); unlike terms can't be combined.

Here are some examples:

3x + 2x + 4x simplifies to 9x	(since 3 + 2 + 4 = 9)
5y - 3y - y simplifies to y	(since 5 - 3 - 1 = 1)
3x + x - 3x simplifies to x	(since 3 + 1 - 3 = 1)
4x - x - 3x simplifies to 0	(since 4 - 1 - 3 = 0)

In simplifying terms: 1x is just x and 0x is just 0.

If a problem has more than one kind of term, simplify each kind, but do not try to combine their results together (there are no apple-oranges!).

For example:
2x + 4 + 3x - 5 *simplifies to* 5x - 1 (Combine the x-terms; combine the numbers; but don't combine the answers.)

3x - 2y + 16 - y + x *simplifies to* 4x - 3y + 16 (Combine the x-terms; combine the y- terms; combine the numbers; but don't combine the answers.)

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EXAMPLES: Solve each of the following equations:

1) $2x + 5x = 56$
   $7x = 56$ — combine the like terms
   $x = 8$ — divide both sides by 7

2) $12 - 3x + 5 - x = 7$
   $17 + -4x = 7$ — combine the like terms
   $-4x = -10$ — subtract 17 from both sides
   $x = 2.5$ — divide both sides by -4

<u>**Practice Set 26**</u> Solve and check each of the following equations.

1) $4x + 3x = 28$

2) $7y - y = 30$

3) $2x + 3 + 4x = 11$

4) $6z - z = 16 - 12$

5) $3 - x + 12 = 23.5$

6) $4x - 9 - x = 18$

7) $12 = 4x - 3 + 5x$

8) $21 - 19 = 6x - x$

9) $\frac{3}{4}x + 12 - \frac{x}{2} = 17$

10) $4 - \frac{3}{8}x + 6 + \frac{5x}{8} = 22$

11) $x + 6 - 3.5x = 12$

12) $2.25 - x + 6.5 + 3x = 29.75$

13) $6 - 2x + 12 = 17 - 5 - 12$

14) $2x - 4 + 3x + 9 = 5$

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SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

So far, every equation had variables only on one side of the equal sign. However, the equations may have variables on both sides of the equal sign. The following rules are appropriate for those equations. Following the rules is a set of examples which shows how the rules are applied.

RULES FOR EQUATIONS WITH VARIABLES ON BOTH SIDES	
RULE #1:	**If there is a number on only one side of the equation, get all the variables to the other side.**
RULE #2:	**If there are numbers and variables on both sides of the equation and the variables are positive on both sides, then subtract the smaller variable from both sides.**
RULE #3:	**If there are numbers and variables on both sides of the equation and one of the variables is positive and the other is negative, remove the negative variable by adding the correct amount to both sides.**
RULE #4:	**If there are numbers and variables on both sides of the equation and the variables are negative on both sides, add the correct amount to remove the larger negative number.**

EXAMPLES:

1) $6x = 2x - 5$
$\underline{-2x \quad -2x}$
$4x = -5$
$x = -1.25$

Since the only side that has a number is the right side, get rid of the 2x by subtracting it from both sides. (Rule #1) Then, divide both sides by 4.

2) $3x + 14 = -4x$
$\underline{-3x \qquad\quad -3x}$
$14 = -7x$
$-2 = x$

Since the only side that has a number is the left side, get rid of the 3x by subtracting it from both sides. (Rule #1) Then, divide both sides by -7.

3) $2x + 18 = 6x - 9$
$\underline{-2x \qquad\quad -2x \quad}$
$18 = 4x - 9$
$27 = 4x$
$6.75 = x$

Since both sides have positive variables, get rid of the smaller one (the 2x) by subtracting it from both sides. (Rule #2) Then add 9 to both sides and divide by 4.

4)
$$\begin{array}{rcl} 6x - 12 &=& 18 + 3x \\ \underline{-3x} & & \underline{-3x} \\ 3x - 12 &=& 18 \\ 3x &=& 30 \\ x &=& 10 \end{array}$$

Since both sides have positive variables, get rid of the smaller one (the 3x) by subtracting it from both sides. (Rule #2) Then add 12 to both sides and divide both sides by 3.

5)
$$\begin{array}{rcl} 2x - 5 &=& 10 - 3x \\ \underline{+3x} & & \underline{+3x} \\ 5x - 5 &=& 10 \\ 5x &=& 15 \\ x &=& 3 \end{array}$$

Since this equation has one positive variable and one negative variable, add 3x to both sides to get rid of the negative variable. (Rule #3) Then add 5 to both sides and divide by sides by 5.

6)
$$\begin{array}{rcl} -x + 7 &=& 3x - 9 \\ \underline{+x} & & \underline{+x} \\ 7 &=& 4x - 9 \\ 16 &=& 4x \\ 4 &=& x \end{array}$$

Since this equation has one positive variable and one negative variable, add x to both sides to get rid of the negative variable. (Rule #3) Then add 9 to both sides and divide both sides by 4.

7)
$$\begin{array}{rcl} 8 - 4x &=& -7x - 10 \\ \underline{+7x} & & \underline{+7x} \\ 8 + 3x &=& -10 \\ 3x &=& -18 \\ x &=& -6 \end{array}$$

Since this equation has negative variables on both sides of the equation add 7x to both sides to get rid of all negative variables. (Rule #4) Then subtract 8 from both sides and divide both sides by 3.

8)
$$\begin{array}{rcl} 12 - 5x &=& 8 - x \\ \underline{+5x} & & \underline{+5x} \\ 12 &=& 8 + 4x \\ 4 &=& 4x \\ 1 &=& x \end{array}$$

Since this equation has negative variables on both sides of the equation, add 5x to both sides to get rid of all negative variables. (Rule #4) Then subtract 8 from both sides and divide both sides by 4.

If there is more than one number or more than one variable on either side of the equation, you will need to simplify the sides of the equation before you continue.

9)
$$\begin{array}{rcl} 7 + 2x + 9 &=& 5x + 6 - 7x + 8 \\ 16 + 2x &=& -2x + 14 \\ \underline{+2x} & & \underline{+2x} \\ 16 + 4x &=& 14 \\ 4x &=& -2 \\ x &=& -0.5 \end{array}$$

Since each side has more than one number or more than one variable, the sides must be simplified first. Then 2x is added to both sides, 16 is subtracted from both sides, and finally both sides are divided by 4.

Practice Set 27 Solve and check each of the following equations.

1) $17x = 2x - 30$

2) $5x = 15 + 2x$

3) $12 + 3x = 9x$

4) $38 - 2x = 17x$

5) $14z + 26 = 9z - 12$

6) $12 + 2x = 8x - 6$

7) $x - 12 = 4 - x$

8) $3 - 4x = 5x - 24$

9) $8 - 2x = 22 - 9x$

10) $9d - 12 = 3d + 21$

For the following, you may have to simplify each side first.

11) $12 + 3x - 8 = 4 - 2x$

12) $3x = 9 - 4x - 2$

13) $7 + 2x - 3 - 5x = 4 - x + 6$

14) $5 - 4x + 3 + 4x - 8 = 12 - 4x$

15) $6 - 2x + 9 = 3x + 7 - 9x$

16) $4x + 9 + 3x = 12 - x - 5$

17) $1.2 + 2x - 0.8 = 3x$

18) $x + 12 = 18.6 - 3x + 3.6$

19) $x + x + 2 = 3 - x - x$

20) $3x - 6 = 5x - 6$

HANDLING PARENTHESES

A minus sign means "opposite"; so -4 means "the opposite of 4".
The opposite of negative 6 is positive 6: $-(-6) = +6$ or $--6 = 6$

Looking at the examples $18 + -6 = 12$ and $18 - 6 = 12$ shows us that "adding a negative" is the same as "subtracting a positive".

Therefore, a minus sign in front of a set of parentheses means the opposite of what is inside those parentheses:

$-(x + 9)$	means $-x + -9$	which is	$-x - 9$
$-(x - 4)$	means $-x - -4$	which is	$-x + 4$
$-(3 + 4x)$	means $-3 + -4x$	which is	$-3 - 4x$
$-(-3x - 9)$	means $- -3x - -9$	which is	$3x + 9$

A plus sign means that everything is kept the same: $+ (2x - 5)$ means $2x - 5$

If a number is placed directly in front of a set of parentheses, you must multiply.

(When the sign in front of the parentheses is positive, the sign inside the parentheses does not change.)

$$2(x + 6) = 2(x) + 2(6) = 2x + 12$$
$$3(2x - 7) = 3(2x) - 3(7) = 6x - 21$$
$$4(-3x + 8) = 4(-3x) + 4(8) = -12x + 32$$

(When the sign in front of the parentheses is negative, the sign of the first number inside the parentheses and the sign inside the parentheses change.)

$$-5(x + 8) = -5(x) + -5(8) = -5x + -40 = -5x - 40$$
$$-3(5 - 2x) = -3(5) - -3(2x) = -15 - -6x = -15 + 6x$$
$$-2(-3x - 9) = -2(-3x) - -2(9) = 6x + 18$$

EXAMPLES: Solve the following equations.

1) $3(x + 12) = 60$ — Remove parentheses first; multiply out: $3(x + 12) = 3(x) + 3(12) = 3x + 36$
$3x + 36 = 60$
$3x = 24$ — Subtract 36 from both sides.
$x = 8$ — Divide both sides by 3.

2) $2x - (x - 14) = 3$ — Remove parentheses by changing:
$2x - x + 14 = 3$ — $-(x - 14) = -x + 14$
$x + 14 = 3$ — Simplify the left side.
$x = -11$ — Subtract 14 from both sides.

3) $2(2y + 4) - 3(5 - 3y) = 19$ Remove parentheses by multiplying out:
$4y + 8 - 15 + 9y = 19$ $2(2y + 4) = 2(2y) + 2(4) = 4y + 8$
$13y - 7 = 19$ $-3(5 - 3y) = -3(5) - -3(y) = -15 + 3y$
$13y = 26$ Simplify the left side. Add 7 to both sides.
$y = 2$ Divide both sides by 13.

4) $3(x - 7) - 6 = 16 - 4(x - 5)$ Remove parentheses by multiplying out:
$3x - 21 - 6 = 16 - 4x + 20$ $3(x - 7) = 3(x) - 3(7) = 3x - 21$
$3x - 27 = 36 - 4x$ $-4(x - 5) = -4(x) - -4(5) = -4x + 20$
$7x - 27 = 36$ Simplify each side. Add 4x to each side.
$7x = 63$ Add 27 to each side.
$x = 9$ Divide both sides by 7.

RULE #1:	**Remove all parentheses.**
RULE #2:	**Simplify each side of the equation so that there is at most one variable term and one number on each side of the equal sign.**
RULE #3:	**If there are variables on each side of the equation, remove the variables from one of the sides.**
RULE #4:	**Finish solving the equation by doing the appropriate additions, subtractions, multiplications, and divisions.**

<u>**Practice Set 28**</u> Solve and check each of the following equations.

1) $3(x + 2) = 12$

2) $4(x - 5) = 80$

3) $2(x + 9) = 18$

4) $5(x - 3) + 12 = 22$

5) $-4(x + 6) = 60$

6) $3 - (x - 12) = 35$

7) $2(x - 4) - 2(x + 4) = 2x$

8) $3(2x + 6) = 2(5x + 2)$

9) $8 - (3 - x) = 6 + (3x + 1)$

10) $3x - (7 - x) = 21$

11) $x - (18 - x) = x - (x - 18)$

12) $2(x - 2) - 3(x - 3) = 4(x - 4) - 1$

13) $1 - 2 + 3 = x - 2x + 3x$

14) $4(x - 5) = 3 + 2(9 + 2x) - x$

15) $6(2x - 4) - 3 = 3 - 6(2x - 4)$

16) $2 + 3x = 4x - (2x - 1)$

17) $5x = 3 - 2(x - 4) + 3$

18) $x - 6 = 4 - 2(x + 8)$

19) $2 - 3(x - 4) = x$

20) $9(2x - 4) = 3 - 3(5x - 7) + 3x$

APPLYING EQUATIONS TO WORD PROBLEMS

In a word problem, if it says:

"three times the sum of a number and five"	use $3(x + 5)$
"four times the difference of a number and six"	use $4(x - 6)$
"the product of two and the quantity x minus nine"	use $2(x - 9)$
"one number is twice as big as another"	use x and $2x$.

EXAMPLE:

Problem: Five times the sum of some number and eighteen is one hundred. What is the number?

Solution:

$$\begin{aligned} 5(x + 18) &= 100 \\ 5x + 90 &= 100 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Each problem is individual and you must read each one carefully to see what kind of equation applies.

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EXAMPLE:

Problem: You and a friend intend to share a two bedroom apartment which rents for \$295.00 per month. The bedroom are differently sized and since you have the larger bedroom, the two of you agreed that you should pay pay \$25.00 more per month. How much will be your monthly rent?

Solution: Let x represent how much your friend will pay each month. Then, since \$25.00 more than your friend does, you x + 25 represents how much you will pay every month. Adding these two together will be your total rent.

$$x + (x + 25) = 295$$
$$x + x + 25 = 295$$
$$2x + 25 = 295$$
$$2x = 270$$
$$x = 135$$ (This is the amount that your friend will pay.)
$$x + 25 = 160$$ (This is the amount that you will pay.)

If you analyze these steps, you might see that you can do these without writing anything on paper. From \$295.00, you subtracted \$25.00, giving you \$270.00. Then you divided that by two, giving your friend's amount: \$135.00. Your amount was found by adding \$25.00 to that.

EXAMPLE:

Problem: Larry, Curly, and Moe went out to eat. Their bill came to \$22.20. They didn't want to go through all the work of figuring out each person's meal, dessert, and drink, so they estimated it this way: Curly's entree was about 50 cents more than Larry's and Moe's was about \$1.00 more than Larry's. If the desserts and the drink were about the same for each person, how much should each person pay?

Solution: Since Larry's meals was the cheapest, let x represent Larry's amount.
Then Curly will pay x + 0.50 (50 cents more than Larry)
and Moe will pay x + 1.00 (\$ 1.00 more than Larry).

Since Larry's cost + Curly's cost + Moe's cost = total cost

$$x + (x + 0.50) + (x + 1.00) = 22.20$$
$$x + x + 0.50 + x + 1.00 = 22.20$$
$$3x + 1.50 = 22.20$$
$$3x = 20.70$$
$$x = 6.90$$ (Larry's amount)
$$x + 0.50 = 7.40$$ (Curly's amount)
$$x + \$1.00 = 7.90$$ (Moe's amount)

EXAMPLE continued

If you analyze these steps, you might see that you can solve this problem without writing anything on paper. Start with the $22.20, and subtract the $ 0.50 and the $1.00, leaving you with $20.70. Then divide this by three and you will have $6.90, Larry's amount. Finish by adding $ 0.50 to get Curly's amount and by adding $ 1.00 to get Moe's amount.

Practice Set 29 Solve the following problems by writing and solving appropriate equations.

1) Ten times the sum of some number and twelve is 260. What is the number?

2) Four times the difference of some number and seven is 76. What is the number?

3) Three times the difference of eleven and some number is 15. What is the number?

4) There are two numbers. The first is twice as big as the second. The sum of the two numbers is 192. What are the two numbers?

5) John and Mary eat lunch together. John's entree is about $2.00 more than Mary's. If their total bill is about $14.00, how much should each pay?

6) Ann, Betty, and Carla share a two bedroom apartment. Ann and Betty share one bedroom while Carla has the other bedroom. Because of this arrangement, they agree that Carla should pay $30.00 per month more than either Ann or Betty. If the monthly rent on this apartment is $345.00, how much should each person pay?

7) I have three children, Andy, Bruce, and Chad. Andy is the oldest and gets \$1.50 per week more than Bruce for his allowance. Bruce gets \$1.00 more per week than Chad gets. If it costs me \$8.00 per week to give them their allowances, how much does each child get?

8) Of the three numbers, x, y, and z, x is the largest, being twice as big as y and three times as big as z. If the three numbers are added together, their sum is 66. What is the value of each number?

APPLIED PROBLEMS

WIND CHILL FACTOR

In winter, you feel colder when the wind blows then when it doesn't. This means that you are more likely to be frostbitten and your water pipes are more likely to freeze when its windy. The following table shows this factor.

wind speed **Fahrenheit Thermometer Temperature**

	-25	-20	-15	-10	-5	0	5	10	15	20	25
5	-31	-26	-21	-15	-10	-5	0	6	11	16	22
10	-52	-46	-40	-34	-27	-22	-15	-9	-3	3	10
15	-65	-58	-51	-45	-38	-31	-25	-18	-11	-5	2
20	-74	-67	-60	-53	-46	-39	-31	-24	-17	-10	-3
25	-81	-74	-66	-59	-51	-44	-36	-29	-22	-15	-7
30	-86	-79	-71	-64	-56	-49	-41	-33	-25	-18	-10
35	-89	-82	-74	-67	-58	-52	-43	-35	-27	-20	-12
40	-92	-84	-76	-69	-60	-53	-45	-37	-29	-21	-13

If you wish to find the wind chill temperature when the thermometer temperature is 10^o Fahrenheit and the wind speed is 30 miles per hour, find the 10^o temperature in the top row and place your right-hand index finger upon it. Now find the 30 mph speed in the left-most column and place your left-hand index finger upon it. As you bring your left hand to the right and your right hand down, they meet at -33^o. This means that when the temperature is 10^o and the wind is blowing at 30 mph, the effect is the same as when the temperature is -33^o (33 degrees below zero) and the wind is not blowing.

EXAMPLE: When the thermometer reading is -10^o and the wind speed is 20 mph, what is the wind chill reading and how many degrees below the actual temperature is this?

Answer: The temperature with the wind chill factor is -53^o. This results in lowering the temperature 43^o. $(-53 - -10 = -53 + 10 = -43)$

<u>Practice Set 30</u> Using the wind chill chart, answer these questions.

1) What is the wind chill temperature and how many degrees below the temperature reading is this reading when the temperature is 20^o and the wind speed is 10 mph?

2) What is the wind chill temperature and how many degrees below the temperature reading is this reading when the temperature is -15° and the wind speed is 15 mph?

3) What are the only two ways in which the wind chill temperature is -60°?

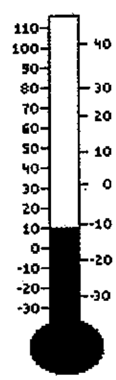
4) What are all the ways in which a wind chill temperature can be -44°?

5) What has a greater difference between the actual temperature reading and the wind chill temperature: when the temperature is 10° and the wind speed is 20 mph or when the temperature is -10° and the wind speed is 10 mph?

6) In which way does it seem colder: when the temperature is 15° with a 35 mph wind or when the temperature is -5° and the wind speed is 5 mph?

FAHRENHEIT AND CELSIUS TEMPERATURES

The most commonly used temperature scale in the United States is the Fahrenheit scale. Two of the more important numbers on this scale are 212°, the point at which water boils, and 32°, the point at which water freezes. The Celsius scale, also called the centigrade scale, is used in most of the other countries of the world, as well as in many scientific experiments in the United States. This scale has water boiling at 100° and water freezing at 0°.



If you want to change a Fahrenheit temperature into a Celsius (centigrade) temperature, use this formula: $C = 5/9\,(F - 32)$

If you want to change a Celsius (centigrade) temperature into a Fahrenheit temperature, use this formula: $F = 9/5\,C + 32$

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EXAMPLES:

72^{o} F is a pleasant temperature. What is the Celsius equivalent?

$$C = 5/9 (F - 32)$$
$$C = 5/9 (72 - 32)$$
$$C = 5/9 (40)$$
$$C = 22^{\circ} \text{ celcius (approximately)}$$

40^{o} C is hot! What is its Fahrenheit equivalent?

$$F = 9/5\,C + 32$$
$$F = 9/5(40) + 32$$
$$F = 72 + 32$$
$$F = 104^{\circ}$$

Practice Set 31

1) Show that 0^{o} C is equivalent to 32^{o} F.

2) Show that 212^{o} F is equivalent to 100^{o} C.

3) If you set your thermostat to 65^{o} F, what is the equivalent Celsius temperature?

4) The normal human body temperature is 98.6^{o} F. What is this in Celsius?

5) If your body temperature is 38^{o} C, what is that temperature in Fahrenheit?

6) You are baking a cake at 350^{o} F. What is that in Celsius?

7) You are slowing roasting a turkey, at 275^{o} F. What is that in Celsius?

8) Cookies are baked at 190^{o} C. What is that in Fahrenheit (round to the nearest 5^{o})?

9) Casseroles are baked at 150^{o} C. What is that in Fahrenheit (nearest 5^{o})?

10) Is 25^{o} C a pleasant outdoor temperature?

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DISTANCE - RATE - TIME PROBLEMS

The distance formula $D = rt$ tells the distance (D) that you have traveled if you know the rate (r) -- which is the speed -- and the time (t) that you have traveled.
The formula can be used three ways:

as a distance formula: $D = rt$

as a rate formula: $r = \frac{D}{t}$

as a time formula: $t = \frac{D}{r}$

EXAMPLES:

1) How far can you travel in 3 1/2 hours if you average 55 miles per hour?

Analysis:

3 1/2 hours is the time -- as a decimal: 3.5
55 miles per hour is the rate
You need to know the distance, so use the distance formula.

Solution:

$D = rt$
$D = (3.5)(55)$
$D = 192.5$ miles

2) I want to go 330 miles but I don't want to take more than 5 1/2 hours. At what speed must I drive?

Analysis: 330 miles is distance
5 1/2 hours is time -- as a decimal: 5.5
I want to know speed, which is rate, so I will use the second formula.

Solution:

$r = \frac{D}{t}$

$r = \frac{330 \text{ mi}}{5.5 \text{ hr}}$

$r = 60$ miles per hour

3) I want to still go the 330 miles. How long will it take me if I could average 65 mph?

Analysis: 330 miles is distance
65 mph is 65 miles per hour, which is rate
Since I want to know time, I will use the third equation.

Solution:

$$t = \frac{D}{r}$$

$$t = \frac{330}{65}$$

t = 5.077 hours or 5 hours and 5 minutes

Practice Set 32

1) How far can you travel if you travel at 60 miles per hour for 4 hours?

2) How far can you travel if your rate is 55 miles per hour and you travel for 3 hours and 15 minutes?

3) How long will it take to travel 300 miles at 60 miles per hour?

4) How fast would you have to travel to cover 150 miles in 3 hours?

5) What would have to be your average rate to cover 2200 miles in 4 days if you would be driving 10 hours per day?

6) To the nearest hour, how long would it take to travel 2200 miles if you could average 65 miles per hour?

SIMPLE INTEREST PROBLEMS

When you borrow money, you have to pay back more money than you borrowed. This extra amount is called the interest that you owe. Similarly, when you place money in a saving account at a bank, credit union, or savings and loan, you earn money; this is also called interest because the bank, credit union, or savings and loan is borrowing the money from you.

There are two kinds of interest -- simple interest and compound interest . Simple interest is used when there is only one interest earning period, when the interest is only calculated once, when there is an unchanging principal. Compound interest occurs when there are several interest earning periods, when the interest is calculated at the end of each interest earning period.

The simple interest formula is: I = PRT

The amount that is borrowed, or the amount put into a savings account, is called the principal (P). The interest rate (R) is a percentage and is expressed as a decimal; for example, it might be 8 1/2 % and would be 0.085. The time (T) is the number of years. The interest (I) is the number of dollars earned or owed.

EXAMPLES:

1) What is the interest earned on $500 deposited for 1/2 year in a simple interest account which pays 5 1/4 % ?

Analysis: principal: P = 500 time: T = .5
rate: R = .0525 interest: I = unknown

Solution: I = PRT
I = (500)(.0525)(.5)
I = $ 13.13 (rounded)

2) What is the simple interest rate if you borrow $500 and pay back $525 in 3 months?

Analysis: principal: P = 500 time: T = 0.25
(3 mo = 3/12 yr = 1/4 = 0.25)
rate: R = unknown interest: I = 25 (525 - 500 = 25)

Solution: I = PRT
25 = (500)(R)(0.25)
25 = 125R (multiply the 500 times the 0.25)
0.2 = R (divide the 25 by the 125)

Thus, the rate is 20%. (0.2 as a percent is 20%)

Practice Set 33

1) What is the interest earned on \$400 deposited for 1 year in an account which pays 6 3/4% simple interest?

2) What is the total amount of money (original principal plus interest) in a \$1200 account which pays 6 1/2% for 9 months?

3) You borrow \$1800 for a year-and-a-half at 17% simple interest. How much must you pay back (principal plus interest)?

4) How much do you owe on your credit card (principal plus interest) after one month if you bill was \$450 and the interest rate is 18%? (Total, principal plus interest.)

5) You borrow \$800 and agree to pay back \$850 after 9 months. What is the simple interest rate?

6) You borrow \$1000 and agree to pay back \$1050 after 1 month. What is the simple interest rate?

COMPOUND INTEREST RATE

Most savings accounts pay to you and loans require that you pay back at a compound interest rate. A compound interest account or loan requires an interest period; usually this is monthly (like on car loans and house loans), but this can be any amount of time: yearly, daily, or even instantaneously.

Compound interest is calculated by using the simple interest formula for each interest period. The principal is changed each period -- the principal for the second period is the principal plus the interest of the first period -- the principal for the third period is the principal plus the interest of the second period, etc.

As an example, say you deposit \$400.00 into a savings account that pays 6% compounded monthly.

For the first month: $P = \$400.00$, $R = 0.06$, $T = 1/12$ (monthly = 1/12 year)

$I = PRT$

$I = (\$400)(0.06)(1/12)$

$I = \$2$ (The amount of interest earned the first month is \$2.00. Now the amount in the account is \$402 (principal plus interest), and this number becomes the principal for the second month.)

For the second month: P = \$402.00, R = 0.06, T = 1/12

I = PRT

I = (\$402)(0.06)(1/12)

I = \$2.01 (The amount of interest earned the second month is \$2.01. Now the amount in the account is \$404.01 (principal plus interest), and this number becomes the principal for the third month.)

Etc. This is summarized in the following table:

month	beginning balance	interest earned	ending balance
1	\$ 400.00	\$ 2.00	\$ 402.00
2	\$ 402.00	\$ 2.01	\$ 404.01
3	\$ 404.01	\$ 2.02	\$ 406.03
4	\$ 406.03	\$ 2.03	\$ 408.06
5	\$ 408.06	\$ 2.04	\$ 410.10
6	\$ 410.10	\$ 2.05	\$ 412.15
7	\$ 412.15	\$ 2.06	\$ 414.21
8	\$ 414.21	\$ 2.07	\$ 416.28
9	\$ 416.28	\$ 2.08	\$ 418.36
10	\$ 418.36	\$ 2.09	\$ 420.45
11	\$ 420.45	\$ 2.10	\$ 422.55
12	\$ 422.55	\$ 2.11	\$ 424.66

Thus compound monthly, \$400 at 6% is worth \$424.66.

If this were at simple interest: I = PRT ---> I = (400)(0.06)(1) ---> I = 24, the final amount would be only \$424.00.

This does not seem to be much difference: compounding is only worth 66 cents more. It makes more of a difference if this is extended over several years.

number of years	simple interest	compound interest
1	\$ 424.00	\$ 424.67
5	\$ 520.00	\$ 539.54
10	\$ 640.00	\$ 727.76
15	\$ 760.00	\$ 981.64
20	\$ 880.00	\$ 1324.08
25	\$ 1000.00	\$ 1785.99
30	\$ 1120.00	\$ 2409.03

Remember that this was only for \$400. So, for five years, the difference was only \$19.54. But make it a larger number, such as the cost of a car, perhaps \$12000. Since \$12000 is 30 times as large as \$400 (divide 12000 by 400), take the \$19.54 times 30 and it becomes \$586.20.

There is not much difference for one year, or even two. But, it does make a difference in five years (car loans) and a very large difference when it is applied to twenty and thirty year accounts (house mortgages) and even a larger difference when applied to retirement accounts of fifty years.

One does not have to make a month-by-month chart to figure these investments. There is a **compound interest formula:**

$$A = P\left[1 + \frac{R}{N}\right]^{NT}$$

A = final amount: principal plus interest
P = principal: beginning amount
R = yearly rate, as a decimal
N = number of compounding periods per year
T = number of years

EXAMPLE: Find the final value of \$400.00 at 6% per year, compounded monthly, for 5 years.

A = unknown P = \$400.00 R = 0.06 (6%) N = 12 (monthly) T = 5 (years)

$$A = P\left[1 + \frac{R}{T}\right]^{NT}$$

$$A = 400\left[1 + \frac{0.06}{12}\right]^{(12)(5)}$$

$$A = 400(1 + 0.005)^{60}$$

$$A = 400(1.005)^{60}$$

$$A = 400(1.348850153)$$

$$A = 539.54$$

If you have a calculator with a power key (y^x), you can solve the problem this way.
First, find the exponent by multiplying 12 x 5 to get 60
Then, 0.06 ÷ 12 + 1 = y^x 60 x 400 =

You might have noticed that the top table has a value of \$ 424.66 while the lower table has a value of \$ 424.67 for the same place, at the end of one year. This occurs because the top table was calculated monthly, with each month's total being rounded. When the formula is used, there is no rounding.

Practice Set 34

1) What is the final value of $500.00 placed in a savings account which pays 6 1/4% interest compounded monthly for 3 years?

2) What is the final value of $1200.00 placed in a savings account which pays 7 3/4% interest compounded monthly for 12 years?

3) What is the final value of $850.00 placed in a savings account which pays 7% quarterly for 5 years?

4) If you place $100.00 in a saving account when you are 18 years of age, how much will that be worth when you retire at age 65? Assume 6 3/4% interest compounded quarterly.

5) How much more will $1000.00 be worth after one year of 6% interest if the savings account is compounded daily instead of monthly for one (non-leap) year?

WORK PROBLEMS

If you know how long it takes one person, working alone, to paint a room and if you know how long it takes a second person, working alone, to paint the same room, how long will it take them to paint the room if they work together? This kind of problem is called a **work problem**, and assumes that the persons' working together will not cause the workers to work unusually slowly or quickly.

EXAMPLE: Mary can mow the lawn in 2 hours. It takes John 3 hours to mow the same lawn. How long will it take them if they can work together?

Analysis: Since Mary can mow the lawn in 2 hours, she mows 1/2 the lawn each hour. Thus, her rate is 1/2 lawn per hour.
Since John can mow the lawn in 3 hours, he mows 1/3 the lawn each hour. Thus, his rate is 1/3 lawn per hour.

The amount finished equals the rate times the time. $A = RT$

Mary's amount finished plus John's amount finished equals the total amount done, which is 1 lawn mowed.

If Mary and John both start at the same time and end at the same time, they both work for the same length of time. This is what needs to be found in this problem. Thus, the time for each is x.

For Mary: A = RT ---> A = (1/2)(x)
For John: A = RT ---> A = (1/3)(x)

Solution: For the two of them working together:

Mary's amount	+	John's amount	=	Total amount
(1/2)(x)	+	(1/3)(x)	=	1
		(5/6)(x)	=	1
		x	=	1.2 hours

Practice Set 35

1) Working alone, Carol can paint 1 room in 4 hours. David, working alone, can paint the same-sized room in 6 hours. If they worked together, how long would it take them to paint that sized room?

2) Using a smaller tractor and plow, it takes Erich 12 hours to plow a field. Using a larger tractor and plow, it takes Frieda 8 hours to plow the same field. How long will it take the two of them, working together?

3) Gene can wash and wax the car in 5 hours while it takes Helen 6 hours to wash and wax the same car. How long will it take the two of them, working together?

4) Irene takes 50 minutes to wash the windows while it takes Juan 70 minutes to wash the same windows. How long will it take the two of them, working together?

5) Karen can rake the leaves in 6 hours, Lena can rake them in 8 hours, but it takes Mark only 4 hours. How long will it take, working together?

6) Neil can paint 400 square feet in 2 hours while it takes Opal 3 hours to paint 400 square feet. How long will it take them to paint 1200 square feet, working together?

7) Pete can shingle 100 square feet in 2 hours; Quincy takes 3 hours to do the same amount; and Roberta can do it in 4 hours. Working together, how long will it take them to shingle 2400 square feet?

MIXTURE PROBLEMS -- WEIGHT

If cashews cost \$8.00 per pound and peanuts cost \$5.00 per pound, how much of each should be used in a mixture whose final cost is \$6.00 per pound?

Since the final cost (\$6.00) is closer to the cost of the peanuts (\$5.00) then it is to the cost of the cashews (\$8.00), you expect more peanuts will be used than cashews, but exactly how much of each?

Analysis: Since the final cost is \$6.00 per pound, let's assume that only one pound will be mixed.

Let the amount of cashews be x.
Then, since the total amount is 1 pound, the amount of peanuts is $1 - x$.

The cost of the cashews is $(8.00)(x) = 8x$ (price x amount)
The cost of the peanuts is $(5.00)(1 - x) = 5 - 5x$

The total cost can be found two ways -- it is
-- the cost of the cashews plus the cost of the peanuts: $8x + 5 - 5x$
-- the total price times the total amount: (6.00)(1 pound): 6

Putting these two together,
$$8x + 5 - 5x = 6$$
Solving:
$$3x + 5 = 6$$
$$3x = 1$$
$$x = 1/3 \text{ pound (of cashews)}$$

Solution: 1/3 pound of cashews, 2/3 pound of peanuts for each pound of mixture.

Practice Set 36

1) Walnuts cost \$ 4.00 per pound and peanuts cost \$ 3.00 per pound. How much of each should be used to produce a mixture of \$ 3.50 per pound?

2) If a nut mixture costs \$ 4.50 per pound and a dried fruit mixture costs \$ 3.00 per pound, how much of each should be used to produce a mixture of \$ 3.40 per pound?

3) Chocolate stars cost \$ 3.00 per pound and chocolate covered nuts cost \$ 4.00 per pound. How much of each should be used to produce a mixture of \$ 3.75 per pound?

MIXTURE PROBLEMS -- PERCENTAGE

Some mixture problems require mixing items which are of different strengths (different percentages). The procedure used to solve these problems is extremely similar to the procedure used in the last section.

When mixing something pure -- such as pure water or pure alcohol, you will need to use either 100% (the percentage of water in pure water, the percentage of alcohol in pure alcohol) or 0% (the percentage of water in pure alcohol, the percentage of alcohol in pure water).

EXAMPLE: How many gallons of a liquid that is 70% alcohol must be mixed with 10 gallons of water to produce a 20% alcohol solution?

Analysis: The amount of alcohol in all three items (the 70% alcohol solution, the water, and the 20% alcohol solution) is found by multiplying the percentage of alcohol times the amount of that solution.

The amount of 70% alcohol solution is unknown; call it:	x
The amount of water:	10
The amount of the final solution (70% solution + water):	x + 10

amount of alcohol in the:

70% solution:	(0.70)(x)	= 0.70x
water:	(0)(10)	= 0
final solution:	(0.20)(x + 10)	= 0.20x + 2.0

amount of alcohol in the 70% solution	+	amount of alcohol in the water	=	amount of alcohol in the final solution
0.70x	+	0	=	0.20x + 1.0
		0.70x	=	0.20x + 1.0
		0.50x	=	1.0
		x	=	2 gallons

Solution: (above)

Practice Set 37

1) How many gallons of a 40% alcohol solution must be mixed with 8 gallons of a 30% alcohol solution to produce a solution that is 34% alcohol?

2) How many gallons of water must be added to 10 gallons of a 30% alcohol solution to dilute the solution down to 5%?

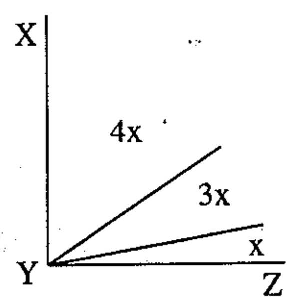
3) How many ounces of a 20% alcohol solution must be added to 8 ounces of pure water to produce a solution which is 5% alcohol?

4) How many gallons of an 80% alcohol solution must be added to 20 gallons of a 20% alcohol solution to produce a solution which is 40% alcohol?

ANGLE PROBLEMS

Many geometry problems are algebra problems. This section covers angle problems which relate to right angles (which have 90^o) and triangles (which have 180^o).

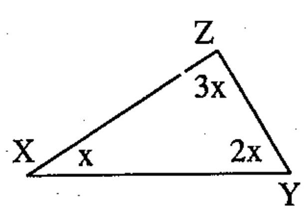
EXAMPLES:



Angle XYZ is a right angle. A right angle is 90°
How big are the smaller angles?

Therefore:
$$x + 3x + 4x = 90°$$
$$8x = 90°$$
$$x = 11.25°$$

So: x = 11.25°, 3x = 33.75°, 4x = 45°



There are 180° in a triangle. How large is each angle?

$$x + 2x + 3x = 180°$$
$$6x = 180°$$
$$x = 30°$$

So: X = 30°, Y = 60°, Z = 90°

Practice Set 38

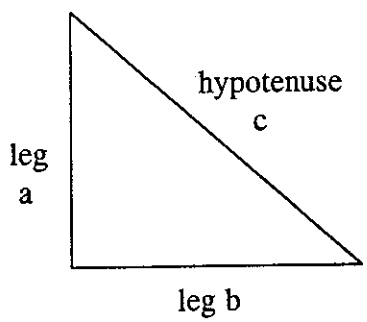
1) Right angle ABC is divided into three smaller angles. The smallest angle has size x. The middle-sized angle is 3 times bigger than the smallest angle and the largest angle is 5 times bigger than the smallest angle. How big is each angle?

2) Right angle DEF is divided into four smaller angles. The second smallest angle is three times bigger than the smallest while the second largest angle is twice as big as the second smallest.. The largest angle is ten times bigger than the smallest angle. How big is each angle.

3) In triangle GHI, angle G is twice as large as angle H, while angle I is five times as large as angle H. What is the size of each angle?

4) In triangle JKL, angle J is three times bigger than angle K and angle L is three times bigger than angle J. How large is each angle?

THE PYTHAGOREAN THEOREM



$$c^2 = a^2 + b^2$$

In a right triangle, the longest side, the side opposite the right angle, is calle the hypotenuse (side c in the diagram). The other two sides are called legs (sides a and b).
This formula is named for the Greek mathematician, Pythaogoras, who learned it from the Ancient Africans. The formula works for Right Triangles only.

EXAMPLES:

1) If side a = 5 in. and side b = 12 in., find the length of side c.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144$$
$$c^2 = 169$$
$$c = 13 \text{ in.}$$ (Use the square root key on your calculator.)

2) If side a = 16 in. and side c = 20 in., find the length of side b.

$$c^2 = a^2 + b^2$$
$$20^2 = 16^2 + b^2$$
$$400 = 256 + b^2$$
$$144 = b^2$$
$$12 = b$$

(Use the square root key on your calculator.)

Practice Set 39

1) The two legs of a right triangle are 3 feet long and 4 feet long. How long is the hypotenuse?

2) The hypotenuse of a right triangle is 26 feet long while one leg is 10 feet long. How long is the other leg?

3) One leg of a right triangle is 2 in. long while the hypotenuse is 3 in. long. How long is the other leg? (Accurate to 4 digits.)

4) A rectangle is 12 in. wide and 32 in. long. How long is one of its diagonals? (Accurate to 4 digits.)

5) A baseball diamond is a square with each side 90 feet long. How far is it from home plate to second base?

6) A softball diamond is a square with each side 60 feet long. How far is it from first base to third base?

Algebra

Section 11: Rearranging Formulas

Rearranging Formulas

Review of Formulas
As you will recall from your reading in Ch 5, formulas are mathematical "tools" used to answer specific questions. You can tell what kind of answer you will get from a formula by noting which variable is isolated (by itself).

<u>**EXAMPLE:**</u> The formula **A = LW** is used to determine the area of a rectangle.

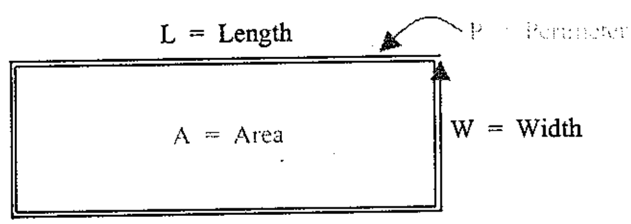
You can tell the formula is designed to give you an area answer because you see the variable **A** (which stands for "area") isolated on the left side of the formula.

In mathematical terms, we say that this formula is "*solved for A (area)*".

<u>**EXAMPLE:**</u> The formula **P = 2L + 2W** is used to determine the perimeter of a rectangle.

Since the variable **P** (perimeter) is isolated on the left side of the equation, you can tell what result will come from the formula.

In mathematical terms, we say that this formula is "*solved for P (perimeter)*".



Once you know the purpose of a formula, look on the other side of the equal sign to see what information is required and how that information is to be computed.

<u>**EXAMPLE:**</u> To calculate the area using this formula, **A = LW** you must know:

The length (**L**) of the rectangle.
The width (**W**) of the rectangle.

Since L and W are written next to each other, you know that L and W must be multiplied in order to get the area.

<u>**EXAMPLE:**</u> To calculate the perimeter using this formula, **P = 2L + 2W** you must know:

The length (**L**) of the rectangle.
The width (**W**) of the rectangle.

The arrangement of the formula tells you to determine P, you must multiply 2 and L together, multiply 2 and W together, then add those results.

Algebra

Section 11: Rearranging Formulas

Rearranging Formulas

It is possible and often desirable to change the **purpose** of a formula. To change the purpose of a formula is called "rearranging a formula". The algebra skills you have learned in this chapter will allow you to perform this important math skill.

For example, given the formula, $\boldsymbol{P = BR}$ (*Part = Base x Rate*), there are two ways it can be rearranged: $B = \frac{P}{R}$ $\quad R = \frac{P}{B}$

The above formula on the left has been solved for *B* (*Base*), the next one is solved for *R* (*Rate*). In each case, the original purpose of the formula, $\boldsymbol{P = BR}$ has been changed so that it can give you a different output.

EXAMPLE:

Let's say that you want to carpet a bedroom in your house. You will want to know the area of the room, so use the formula $\boldsymbol{A = LW}$. If the length of the room is 15 feet and the width is 10 feet, then the area is 150 square feet.

$\boldsymbol{A = LW}$
Area = Length times **Width**
Area = 15 ft x 10 ft
Area = 150 square feet

Now suppose you go to a home improvement store that sells carpet remnants. To save space, each piece of carpet is rolled up, tied, and stacked on end. The only dimension listed on the label is: ***Area is 192 sq ft.*** With a tape measure you are able to determine that the roll is 12 ft from bottom to top. What you want to know however is the **length.**

In the formula $\boldsymbol{A = LW}$, there are three variables. As long as you know any two of the three variables, you can calculate the unknown one. Since you know area (*A*) and width (*W*), you have enough information to calculate length (*L*).

To compute *L*, you must rearrange the formula so that it is "solved for *L*". This means the *L* must be isolated on one side of the equal sign.

Your equation solving skills will serve you well now:

Step 1. Write your original formula and circle the variable you want to solve for.

$$A = \textcircled{L}W$$

Step 2. Move any numbers or variables that are on the same side as the desired variable.

Width (W) needs to be moved. To move a variable, perform the inverse (opposite) mathematical operation. Currently L and W are tied together by multiplication. The inverse of multiplication is division. To continue, you must divide the right side of the equation by W.

$$A = \frac{\textcircled{L}\cancel{W}}{\cancel{W}}$$

Dividing by *W* on this side cancels out the *W*.

Continued...

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

$$\frac{A}{W} = \frac{LW}{W}$$

Since you divided the right side by W, you must divide the left side by the W.

Step 4. Rewrite the formula eliminating the cancelled variables.

If you find other variables or terms that have to be moved in order to isolate L, go back to Step 2 and continue the process.

In this example, we can say that the formula has been successfully rearranged since L is isolated.

$$\frac{A}{W} = L$$

The rearranged formula states that Length (L) equals Area (A) divided by Width (W).

Now let's finish the carpet problem. The roll of carpet had an area of 192 sq ft and it was 12 ft from end of the roll to the other end. How long is the carpet? That will be easy to determine because we now have a tool (the rearranged formula) to help us.

$$\frac{A}{W} = L$$

$$\frac{192\ sq\ ft}{12\ ft} = L$$

$$\boxed{16 \text{ ft} = L}$$

The Benefits of Formula Rearrangement

The ability to rearrange formulas is a valuable skill. You can take a given formula and adapt it to meet your needs. If you don't have this skill, you would have to look up a "new" formula every time you want to solve for a different variable. Often you will find that even a good reference book does not have the rearrangement you are looking for. Besides it is usually faster to make the rearrangement yourself than it is to track it down.

EXAMPLE:

Rearrange this formula so that it is "*solved for W*". $P = 2L + 2W$

Step 1. Write your original formula and circle the variable you want to solve for.

$$P = 2L + 2W$$

Continued...

Step 2. Move any numbers or variables that are on the same side as the desired variable.

The $2L$ term and the 2 in front of the W need to be moved. Start with the $2L$ term. The $2L$ term will be moved as a "package". When rearranging formulas, you move the lowest priority items first. Note that $2L$ is attached to the W term through addition. Addition (and subtraction) is the lowest priority math operation. To move the $2L$ term, you must perform the inverse mathematical operation. That means you must subtract $2L$ from the right side of the formula. This will cancel out the $2L$ from the right side.

$$P = \cancel{2L} + 2\textcircled{W}$$
$$\cancel{-2L}$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

Since you subtracted $2L$ from the right side, you must subtract $2L$ from the left side as well.

$$\begin{array}{rl} P = & \cancel{2L} + 2\textcircled{W} \\ -2L & \cancel{-2L} \\ \hline P - 2L = & 2W \end{array}$$

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

You moved the 2L, now you must move the 2 in front of the W. Repeat steps 2-4.

$$P - 2L = 2\textcircled{W}$$

Step 2. Move any numbers or variables that are on the same side as the desired variable.

Now the 2 in front of the W must be moved. The 2 is attached to the W through multiplication. The inverse of multiplication is division. Divide the right side of the formula by 2. This will cancel out the 2 from the right side.

$$P - 2L = \frac{\cancel{2}\textcircled{W}}{\cancel{2}}$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

*Since you divided by 2 on the right side, you must divide the left side by 2 as well. Important Detail: To do this step correctly, the **entire** left side of the equation must be divided by 2.*

$$\frac{P - 2L}{2} = \frac{\cancel{2}\textcircled{W}}{\cancel{2}}$$

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

$$\frac{P - 2L}{2} = W$$

> You have successfully rearranged the P = 2L + 2W formula so that it is "*solved for W*".

Algebra

Section 11: Rearranging Formulas

EXAMPLE:

Rearrange this formula so that it is "*solved for R*". $A = \pi R^2$

Step 1. Write your original formula and circle the variable you want to solve for.

$$A = \pi \textcircled{R}^2$$

Step 2. Move any numbers or variables that are on the same side as the desired variable.

The π and the exponent 2 need to be moved. Start with π. When rearranging formulas, you move the lowest priority items first. Note that π is multiplied with R^2. Multiplication has lower priority than a power. In order to move π, you must perform the inverse operation of multiplication; division. Dividing the right side of the formula by π will cancel it from the right side.

$$A = \frac{\cancel{\pi} \textcircled{R}^2}{\cancel{\pi}}$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

Since you divided by π on the right side, you must divide by π on the left.

$$\frac{A}{\pi} = \frac{\cancel{\pi} \textcircled{R}^2}{\cancel{\pi}}$$

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

You moved π, now you must move the exponent 2 from R. Repeat steps 2-4.

$$\frac{A}{\pi} = \textcircled{R}^2$$

Step 2. Move any numbers or variables that are on the same side as the desired variable.

*Now the exponent 2 on the R needs to be moved. The inverse operation of a power is a root. To be more specific for this example, the inverse of raising a number to the power of 2 is a **square** root. Take the square root of the right side of the formula. As a result, the exponent of 2 is cancelled out.*

$$\frac{A}{\pi} = \sqrt{\textcircled{R}^{\cancel{2}}}$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

Since you took the square root of the right side, you must take the square root of the left side as well.

$$\sqrt{\frac{A}{\pi}} = \sqrt{\textcircled{R}^{\cancel{2}}}$$

Continued...

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

$$\sqrt{\frac{A}{\pi}} = R$$

You have successfully rearranged the original formula so that it is "*solved for R*".

EXAMPLE:

Rearrange this formula so that it is "*solved for F*". $C = \frac{5}{9}(F - 32)$

Step 1. Write your original formula and circle the variable you want to solve for.

$$C = \frac{5}{9}(\textcircled{F} - 32)$$

Step 2. Move any numbers or variables that are on the same side as the desired variable.

The fraction 5/9 and the 32 both have to be moved. You need to decide which gets moved first. When rearranging formulas, you move the lowest priority items first. Even though 32 is attached to F through subtraction, it occurs inside a set of parentheses which represent the highest priority. Therefore, you will move the 5/9 first.

The mathematical operation that currently links the 5/9 and F together is multiplication. The inverse of multiplication is division. Note that when you divide fractions, you end ***up multiplying by the reciprocal****. So what you want to do here is multiply 5/9 by its reciprocal which is 9/5. This will cancel out 5/9 from the right side of the equation.*

$$C = \frac{\cancel{9}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{9}}(\textcircled{F} - 32)$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

Since you multiplied by 9/5 on the right side, you must multiply by 9/5 on the left side.

$$\frac{9}{5}C = \frac{\cancel{9}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{9}}(\textcircled{F} - 32)$$

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

Since there are no longer any operations being performed on the parentheses, you can drop them. Now repeat steps 2-4.

$$\frac{9}{5}C = \textcircled{F} - 32$$

Algebra

Section 11: Rearranging Formulas

Step 2. Move any numbers or variables that are on the same side as the desired variable.

Now the 32 must be moved. Since subtraction links F and 32, you will need to perform the inverse mathematical operation. The inverse of subtraction is addition. If you add 32 to the right side of the equation, the 32 will cancel out.

$$\frac{9}{5}C = \textcircled{F} - \cancel{32}$$
$$+\cancel{32}$$

Step 3. Whatever you do to one side of the equation, you have to do to the other side.

Since you added 32 to the right side, you must add 32 to the left side.

$$\frac{9}{5}C = \textcircled{F} - \cancel{32}$$
$$+32 \qquad +\cancel{32}$$

Step 4. Rewrite the formula eliminating the cancelled variables. Continue until the desired variable is alone on one side of the equal sign.

$$\frac{9}{5}C + 32 = F$$

> The original formula, C=5/9(F-32) has been "*solved for F*".

Practice Set 40 Rearrange the following formulas to solve for the indicated variable.

1.) Given $F = MA$, solve for M.

2.) Given $I = PRT$, solve for R.

3.) Given $C + M = S$, solve for M.

4.) Given $E = I\,R$, solve for I.

5.) Given $S = \frac{D}{T}$, solve for T.

6.) Given $A = S^3$, solve for S.

7.) Given $P = \frac{F}{A}$, solve for A.

8.) Given $R = \frac{I}{PT}$, solve for P.

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Section 11: Rearranging Formulas

9.) Given $A = \frac{1}{2} BH$, solve for B.

10.) Given $M = P(1 + RT)$, solve for R.

11.) Given $A = \frac{\pi D^2}{4}$, solve for D.

12.) Given $W = FD$, solve for F.

13.) Given $Y = MX + B$, solve for X.

14.) Given $P = \frac{V^2}{R}$, solve for R.

15.) Given $Y = \frac{PL}{AE}$, solve for E.

16.) Given $A = \frac{V_f - V_i}{T}$, solve for T.

17.) Given $F = \frac{9}{5} C + 32$, solve for C.

18.) Given $P = I^2 R$, solve for I.

19.) Given $S = \frac{V_f + V_i}{2}$, solve for V_f.

20.) Given $P = M - B$, solve for B.

21.) Given $W = \sqrt{\frac{1}{LC}}$, solve for C.

22.) Given $M = P(1 + I)^T$, solve for I.