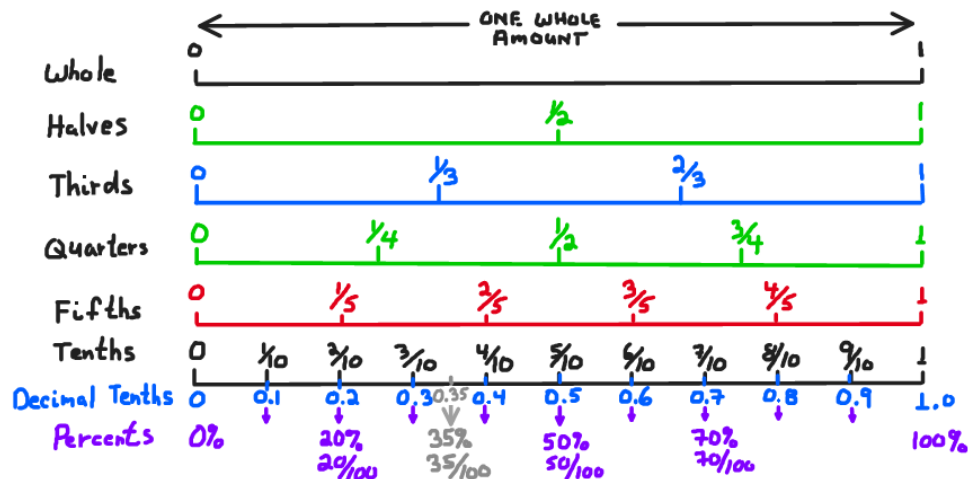


NODDY'S GUIDE RATIOS, FRACTIONS, PERCENT, DECIMALS

INTRODUCTION

1. Ratios (Rates), Fractions, Percents and Decimals are rather fundamental to all arithmetic and mathematics. In effect, they are **all the same thing!**



So let's do a quick 'Noddy's Guide'! (An old Air Force phrase for a quick review)

2. **Ratio.** A comparison of two numbers having no units. Eg: I lost half my marbles; I lost one for every two I had. Often expressed using a colon, for example: **1 : 2**. More often as a fraction: $\frac{1}{2}$

3. **Rate:** Same as a ratio but units of measure being compared differ. Eg: 60 km / 1 hour; \$60 / 1 day; 3.2 pickles per child, etc.

The words rate and ratio are often interchanged without too much confusion.

4. **Practice.** Complete blanks below:

a. Write the ratio comparisons as a fraction and a ratio using a colon:

Four boys for every three girls: _____ : _____

5 blue for every 6 red: _____ : _____

7 slices out of a total of 8 slices: _____ : _____

b. Write the rates as a fraction (make sure you show the units!):

Five metres for every second: _____

\$18 for each hour: _____

12 pickles for every 6 kids: _____

Did you know that 12 pickles for 6 kids is the same as 2 pickles for 1 kid? More about 'simplify' shortly.

5. **Fractions.** A measure of a portion of a whole amount, so it is a ratio comparing equal sized pieces of some amount to the whole amount. Five equal pieces of an eight piece pizza; $\frac{5}{8}$. We do not use the colon to represent fractions (not for the last few hundred years at least)

a. **Proper fractions** amounts that are less than one whole amount.




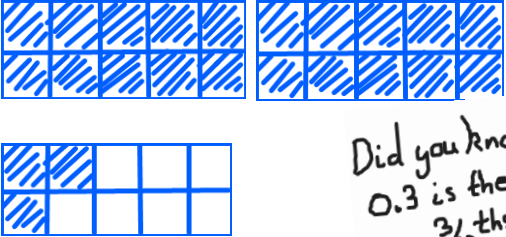
I have one slice of an 8 slice pizza: $\frac{1}{8}$ pizza. Examples: $\frac{3}{5}$; $\frac{10}{13}$; $\frac{413}{1,000}$.

b. **Improper fractions** are fractions that are more than a whole amount; eg: 11 slices of an 8 slice pizza: $\frac{11}{8}$ of a pizza. The top **numerator** numbers is larger in value than the bottom **denominator** number.

c. **Mixed numbers** (mixed fractions) combine whole amounts with fraction amounts. One whole pizza plus 3 slices of an 8 slice pizza is $(1 + \frac{3}{8})$ pizzas, "one an three-eighths pizzas", written as $1\frac{3}{8}$.

Did you know you likely have an $a\frac{b}{c}$ button on your calculator that does fractions?

6. **FRACTION PRACTICE.** Use fractions to represent the shaded amounts

<p>How much pizza? _____</p>  <p>$\frac{3}{8}$</p>	<p>How much birthday cake? _____</p>  <p>$\frac{3}{10}$</p>
<p>How much pizza? _____</p>  <p>Improper fraction: _____ Mixed fraction: _____</p> <p>$\frac{27}{8}$; $3\frac{3}{8}$</p>	<p>How much birthday cake? _____</p>  <p>Did you know? 0.3 is the same as $\frac{3}{10}$ths</p> <p>Improper fraction: _____ Mixed fraction: _____</p> <p>$\frac{23}{10}$; $2\frac{3}{10}$</p>

SIMPLIFY (REDUCE) RATIOS AND FRACTIONS

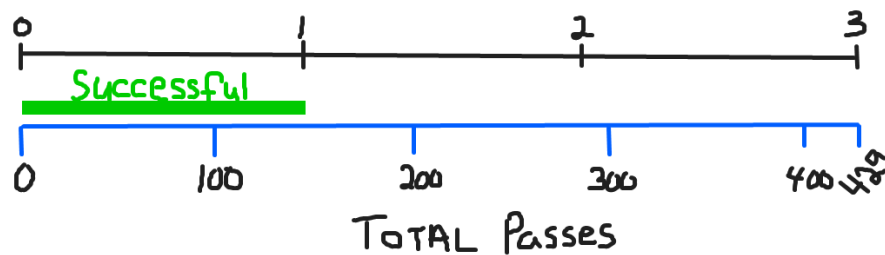
7. Josh played football. In his career he made 429 passes, 143 were completed! What fraction of his total passes were complete? _____

Would you believe that $\frac{143}{429}$ is identically the same ratio or fraction as $\frac{1}{3}$. So out of every three passes Josh made one was complete. A much simpler fraction to picture.

You get the idea that making the numbers in the fraction or ratio smaller to turn it into an equivalent fraction is much tidier! Making the numbers small is called simplifying (or also known as reducing)

DEMONSTRATION

8. Drawing a 429 slice pizza is a bit awkward, let's do a number line:



It just so happened that 429 is just 3 equal packages of 143!

So for three equal packages of 143 passes; one package of 143 was successful. So $143 : 429$ is the same as $1 : 3$; 143 successes to every 429 passes is the same as one success for every three passes.

$$\frac{143}{429} = \frac{1}{3}$$

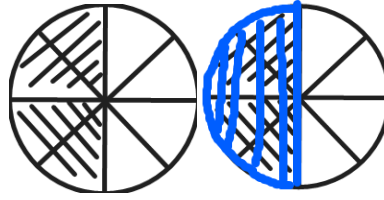
9. You will seldom have to reduce fractions and ratios that are that crazy (besides there is a button on most calculators that does it anyway!)

10. Of course, knowing how to break big numbers down into small packages means that **you** can readily divide (*'un-multiply'*) in your head knowing instantly your multiplication tables! If you do not know your **multiplication facts**, then fractions will certainly be brutal!

11. If you do not know the multiplication table just quit here **or** else use a calculator!

Or better yet, commit to spending six hours over a couple weekends memorizing the multiplication tables!

12. **Example:** 4 slices of an 8 slice pizza is exactly the same as 1 slice of a 2 slice pizza;



I saw above that I could make equal packages of 4 slices!

$$\text{So } \frac{4 \text{ slices}}{8 \text{ slices in the whole amount}} = \frac{1 \text{ group of 4 slices}}{2 \text{ groups of 4 slices}} = \frac{1}{2}$$

I have half a pizza.

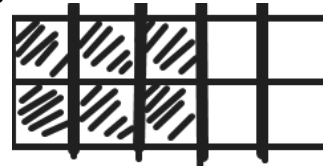
13. **Try these.** Convert the picture as indicated by making 'larger packages'. Boldly outline your conversion of the smaller packages into larger equal packages.

a. 2 slices of an 8 slice pizza into 1 slice of a 4 slice pizza:



$$\frac{2}{8} = \frac{1}{4}$$

b. 6 slices of a 10 slice birthday cake into 3 slices of a 5 slice birthday cake.



$$\frac{6}{10} = \frac{3}{5}$$

14. Reduce the following fractions:

$$\text{Example: } \frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1 \text{ pack of 3}}{4 \text{ packs of 3}} = \frac{1}{4}$$

$$\frac{5}{15} =$$

$$\frac{2}{12} =$$

$$\frac{16}{20} =$$

1/3, 1/6, 4/5

PERCENTS

15. Percent are fractions! The denominator is always 100. Exactly as they mean in French: 'per **cent**', ie: for each **hundred**. The secret symbol to write a percent is '%'.
There are 100 years in a century
There are 100 cents in a dollar

That was easy!

14% means $\frac{14}{100}$. 91% means $\frac{91}{100}$. 37% means? _____

That was easy.

$$x\% \equiv \frac{x}{100}$$

Some, x , percentage means identically some, x , amount per hundred.

CONVERT PERCENT TO FRACTION

16. Of course, you recall, we always simplify (reduce) fractions. Even when we convert a percent to a fraction.

13% means $\frac{13}{100}$. It cannot be made simpler (reduced) since what size packages (aka: factors) make a 13 that also make 100? There are no such factors!

Recall. Numbers that you cannot break down are called 'prime'. Example 3, 5, 7, 11, 13, 23, 37, 41, ... are all prime.... have no factors that make them except themselves. 13 is prime and cannot be broken down further.

28% means $\frac{28}{100}$. But I see packages of 4 in each top and bottom number.

28 is 7 groups of 4 and 100 is 25 groups of 4. So the proper reduced fraction is answer is $\frac{7}{25}$. (pronounced 'seven twenty-fifths').

$$\frac{28}{100} = \frac{7 * 4}{25 * 4} = \frac{7}{25} * \frac{4}{4} = \frac{7}{25} * 1 = \frac{7}{25}$$

Note: This quick Noddy's Guide is not meant to replace your proper full 15 hours of lessons, but if you were to sit and doodle and draw pizzas and birthday cakes you could readily use the ideas here to explain fractions to yourself!

17. Convert the following percents to (reduced) fractions.

a) 50% =

b) 60% =

c) 80% =

d) 75% =

e) 15% =

f) 95% =

$\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{3}{20}$, $\frac{19}{20}$

18. If you really want to see how to see what size package (what size 'factor') you use to reduce (*ie*: simplify) a fraction go to the Appendix at the end of these notes.

CONVERT FRACTIONS TO PERCENT

19. Did you know $\frac{2}{5}$ is the same as 40%, since 40% means $\frac{40}{100}$?

This time we made the numbers in the fraction bigger, since we had to make the bottom number 100. We had to 'un-simplify'.

20. Percents are simply fractions with the bottom number (denominator) always 100.

A percent is a fraction!
The bottom number is always 100. When we write '%'
it is the same as writing '/100'.

21. The easiest and most reliable way to convert a **fraction to a percent** is to divide to a decimal form and then multiply by 100 and put the % symbol at the end of the result.

Example: $\frac{17}{20}$ means $17 \div 20 = 0.85$, and $0.85 * 100 = 85$; **so 85%**

So $\frac{17}{20}$ is the same as 0.85 as a decimal which is the same as 85% which is the same as $\frac{85}{100}$. It is all the same amount of pizza!

Recall that multiplying by 100 just moves a decimal two places right.

Further, the proper way to pronounce the decimal amount 0.85 is 'eighty-five hundredths' which is a good clue what is it as a percent.

22. Convert the following fractions to a percent:

$$\frac{3}{5} =$$

$$\frac{3}{4} =$$

$$\frac{3}{8} =$$

$$\frac{13}{20} =$$

60%, 75%, 37.5%, 65%

DECIMALS

23. Decimals are really just fractions, we are just too lazy to write them properly!

When you see 3.7 it really means $3\frac{7}{10}$! In fact you are supposed to pronounce it that way: “three and seven tenths”

When you see 8.12 it really means $8\frac{12}{100}$! You are actually supposed to properly pronounce it as: “eight and twelve hundredths”

When you see 36.135 it really means $36\frac{135}{1,000}$. So 36 whole pizzas plus 135 slices of a 1,000 slice pizza! *Pretty tiny slices!*

24. Notice with decimals the *denominator* is always a power of 10. ie: 10, 10^2 , 10^3 ,.... The prefix ‘**dec**’ in **decimal** means 10! **10** years in a **dec**ade! You earth people have 10 fingers so you prefer counting by 10’s and 10s of 10s, ...etc.

PLACE VALUES FOR WHOLE AMOUNTS AND FRACTION AMOUNTS

25. A useful (familiar) chart.

DECIMAL PLACE VALUE CHART								
One Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	One-Thousandths	
		2	1	.	7			$21\frac{7}{10}$ Twenty-one and seven tenths
		4	4	.	3	9		$44\frac{39}{100}$ Forty-four and thirty-nine hundredths
		9	8	.	1	2	6	$98\frac{126}{1,000}$ Ninety-eight and one hundred twenty six thousandths
				.				

CONVERT FRACTION TO A DECIMAL (EASY WAY)

You will often need a calculator for these!

26. $3\frac{5}{8} \equiv 3 + (5 \div 8)$, on your calculator (or otherwise) do $3 + 5 \div 8$; the result is 3.625. Easy. You generally will favour using a calculator to convert.

27. You Try:

a) $5\frac{3}{4} =$

b) $12\frac{1}{8} =$

c) $6\frac{3}{5} =$

d) $13\frac{15}{16} =$

e) $4\frac{7}{9} =$

(this one goes forever!)

Ans: 5.75, 12.125, 6.6, 13.9375, 4.7777777777777777

Note the problem with decimals, sometimes they go on and on forever, fractions do not!

CONVERT DECIMAL TO A FRACTION

30. '123.45' means 123 whole amounts plus 45 hundredths of an amount

So $123\frac{45}{100}$ would be correct. But reduce it to be more correct!

I see packages (factors) of 5 in the top and bottom;

So $123\frac{9}{20}$ when properly reduced.

0.45 really means 4 tenths and 5 hundredths, which if you know how dimes and pennies work is 45 pennies. So 0.45 is 45/100

(since $\frac{4}{10} + \frac{5}{100} = \frac{45}{100}$). But will not burden you here with how to do operations on fractions like that.

See the Decimal Value Place Chart in this guide.

PRACTICE DECIMAL TO FRACTION

0.25 =	4.75 =	2.08 =
3,466.72 =	0.008 =	1.43 =

Ans: $\frac{1}{4}$; $4\frac{3}{4}$; $2\frac{1}{25}$; $3,466\frac{18}{25}$; $\frac{8}{1,000}$; $1\frac{43}{100}$

CONCLUSION

31. There! You are done the Noddy's Guide.

Hopefully this was enough to *remind* you how simple fractions are or to at least convince you that they are useful and somewhat simple and worthy of further study.

32. This guide was not meant to replace the normal 15 to 20 hours it would take to learn the entirety of fractions.

FINAL PRACTICE

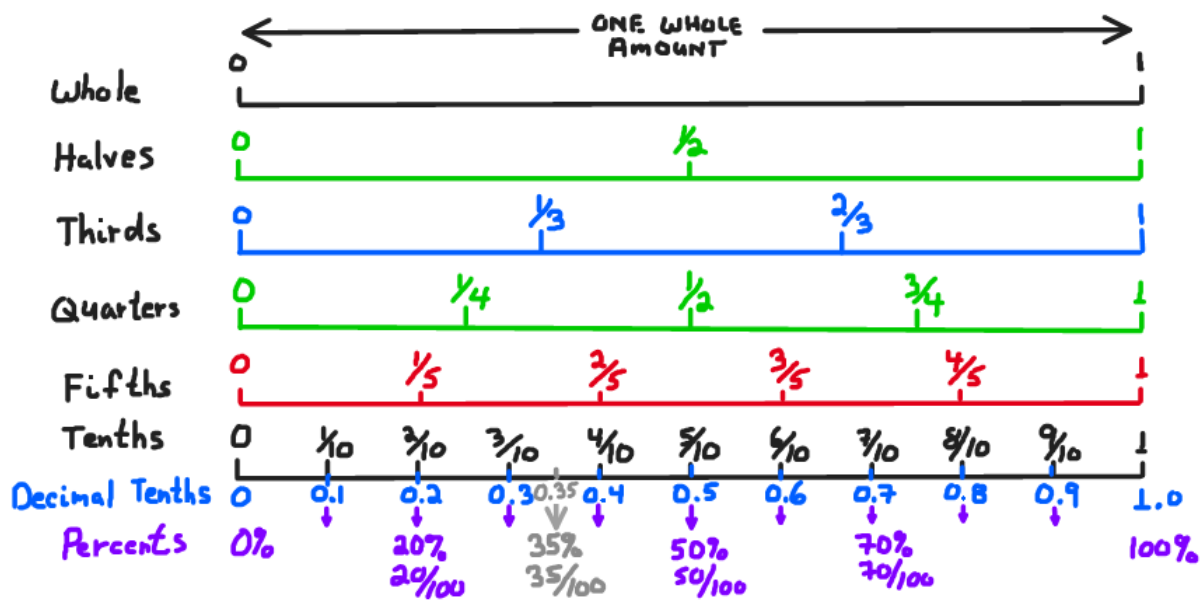
Now complete the following table in the blanks areas.

If the decimal goes on forever, just overbar the repeating portion

Fraction Or Mixed Number	Reduced Fraction or Mixed Number	Decimal	%
$\frac{2}{4}$			
$\frac{6}{8}$			
$5\frac{6}{18}$			
$\frac{9}{4}$			
$\frac{18}{7}$			
	$\frac{2}{3}$		
	$\frac{7}{8}$		
		0.90	
		0.95	
		0.7	
		0.75	
		0.02	
		0.035	
		1.375	
			17%
			34%
			7.5%
			16.8%
			1.25%
			140%
		0.2222222.....	
		4.65	

APPENDIX

Comparing Fractions, Decimals, Percents as parts of a whole amount



APPENDIX

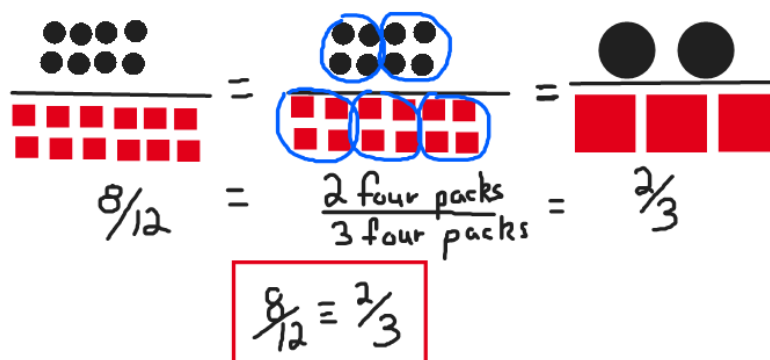
GREATEST COMMON FACTOR (G.C.F.)

ie: Detailed explanation of finding a 'package size' to reduce a fraction.

Reducing fractions means to make the numbers in the top (numerator) and bottom (denominator) proportionately smaller. If you chop the top number in half, you chop the bottom number in half, etc.

Common Factor. Finding a common size package (we shall call it a factor henceforth) is useful if you want to reduce (simplify) fractions.

Common Factor Diagram



In order to see that 8 and 12 could each be broken down into 'factors' of 4 you would of course need your times tables again! $8 = 2 \times 4$ and $12 = 3 \times 4$.

So $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$; were we divided by the common amount 4 to see how many packages of 4 there were.

Simple method to find the GCF.

List factors; find the common ones, find the greatest common one! Lol

Find the Greatest Common Factor of 12 and 18

Factors	Common	Greatest Common Factor (GCF)
12: 1, 2, 3, 4, 6, 12		
	1,2,3,6	6
18: 1, 2, 3, 6, 9, 12		

You find the Greatest Common Factor of 36 and 81

Factors	Common	Greatest Common Factor
36:		
81:		

So reduce the fraction: $\frac{36}{81}$: _____

REDUCING USING PRIME NUMBERS

This is the more rigorously correct way.

Just whittle down your numerator and denominator into all prime factors.

Eg: $\frac{12}{14} = \frac{3 \cdot 4}{7 \cdot 2}$ The '3', '2' and '7' are prime! But '4' is not

$\frac{12}{14} = \frac{3 \cdot 2 \cdot 2}{7 \cdot 2} = \frac{3 \cdot 2}{7} \cdot \frac{2}{2} = \frac{6}{7}$ A two in the top and bottom 'cancel'.

Examples:

$\frac{52}{24} = \frac{2 \cdot 2 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 3} = \frac{13}{6}$	$\frac{72}{100} = \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{18}{25}$
You try:	
$\frac{20}{35} = \frac{\quad}{\quad} = \frac{4}{7}$	$\frac{32}{48} = \frac{\quad}{\quad} = \frac{2}{3}$

Ask me for some divisibility rules. It makes reduction of even bigger fractions way easier!